THE SCIENCE OF LOGIC

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AN INQUIRY INTO THE PRINCIPLES OF ACCURATE THOUGHT AND SCIENTIFIC METHOD

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METHOD; OR THE APPLICATION OF LOGICAL PROCESSES TO THE ATTAINMENT OF TRUTH.

CHAPTER I.

GENERAL OUTLINE OF METHOD.

200. TRANSITION TO PART IV.—We have now completed our examination of the *formal* aspect of the reasoning process, and of the rules that guarantee its formal correctness or validity (Part III.). But the object of all reasoning, of all science and philosophy in fact, is to arrive at a *certain* knowledge of *truth*; and, to secure this, it is not enough that our reasoning processes be correct or valid formally: the judgments involved in them must, furthermore, be all both true and certain.

Truth is, as we saw (9, 79), contained in the mental act of judgment, to which the operations both of inference and of conception are thus subsidiary. An analysis of the material or "truth" aspect of inference will therefore, of necessity, direct our attention once more to the judgments of which our inferences are composed, and to the concepts or ideas which enter into our judgments (Parts I. and II.). After having separately examined each of the three mental operations, of conception, judgment, and inference, our next concern is to inquire how we reach *true* judgments, especially those true *universal* judgments which constitute *scientific* knowledge: how, in other words, we are to exercise those three mental operations on the data of knowledge to the best advantage for the acquiring of truth: how we are to regulate and co-ordinate those mental acts, conception, judgment, and reasoning, in exploring the various departments of the knowable universe. This portion of logical doctrine is variously described as applied logic, methodology, or the science of logical method.

In all logical inference, our reason for assenting to the conclusion is its evident connexion with premisses to which we have already assented. But how do we come to assent to these latter? Either because they are self-evident—like the universal axioms involved in all inference (193),—or derived by demonstrative evidence from such self-evident truths, or generalized by induction from observed facts. The general truths of the sciences may, then, be roughly divided into these three classes: (*a*) self-evident axioms or principles, such, for example, as "The whole is greater than its part": these are reached by a comparatively simple process of intellectual abstraction and intuition, involving *Definition* and *Division* of concepts, and their mutual comparison in judgment; (*b*) general truths that are not self-evident, but which have been generalized by *Induction* from observed facts; (*c*) conclusions inferred by *Demonstration* from truths of classes (*a*) or (*b*).

Before the inductive method was developed, attention was largely devoted, in the traditional Aristotelean logic, to definition, division, and demonstration—the *tres modi sciendi* as they were called.[1] *Definition*, by analysing our concepts of things into the simplest possible notions, gives rise to certain primordial, self-evident relations between these notions. These relations are formulated in judgments and propositions which furnish the foundations of the scientific edifice—*the principles of the sciences*. While definition thus analyses our concepts, and gives us information about the nature of their objects, it thereby also shows us wherein those objects agree in thought and wherein they differ from one another. The process of *differentiation*, or *classification*, or *division*, is thus the indispensable concomitant of definition.

According as the mind becomes equipped with its elementary ideas and judgments by means of sense observation, and intellectual abstraction and intuition, it has recourse to the third mode of procedure, *demonstration:* it draws certain and evident conclusions from self-evident principles, and from these conclusions still further conclusions, and so on. The employment of those various functions or factors of science, for the advance of knowledge, is what the Scholastics called METHOD.

The process of (real) *definition*, understood in the Scholastic sense as *an explanation of the nature of a thing*, and the concomitant process of (real) *division* or *classification*, were always regarded in Aristotelean philosophy as *material* processes, involving observation and analysis of facts, abstraction, generalization, comparison, and even inference and verification of hypotheses—in a word, all the processes nowadays described as "subsidiary to induction". These made up the analytic stage of the Scholastic method, as *demonstration* constituted its synthetic stage.

201. LOGIC AND METHOD.—Before investigating the method

[1] *Cf.* ZIGLIARA, *Logica*, (13), (44).

or methods of applying the mental processes we have been mentioning, to the pursuit of truth, it will be useful here to take a glance by anticipation at the main departments of human knowledge which the logician may have in mind, and from which he may draw his illustrations, in investigating such methods.

We have pointed out already, in common with all logicians, that it is not the function of logic to explore the provinces of the special sciences in order to expound the various modes of procedure peculiar to each. This is the function of the special sciences themselves: each has, or ought to have, its own special methodology. Logic ought to confine itself to an exposition of those guiding laws and principles of reasoning and research which are so universal that the mind must conform to them *always* and in *every* department of rational investigation.[1] In thus limiting its field, logic will not be aiding the study of the *special sciences* so directly as it will aid the study of *philosophy proper;* for philosophy presupposes a general knowledge of all the special sciences and endeavours to synthesize their results; and in this arduous work it is guided by no other "rules of philosophizing" than the general canons and laws laid down in logic. Indeed, if there be any science to which logic should serve as a special introduction, it is philosophy, the "general science," and not any of the special sciences.

But it is difficult to carry out in practice what is so simple in theory. Just because philosophy does take up, interpret, collate, and harmonize—as far as possible—the assumptions and conclusions of all the special sciences—mathematical, physical, natural, anthropological, social, economical, ethical, etc.—it is not easy in practice to say where the work of each special science ceases and that of philosophy begins. And so it is, too, with regard to the scope of logic. This may easily deviate into the investigation of methodological details proper to special sciences; or—which is a more serious mistake—it may, by losing sight of some departments of human experience and falling unduly under the influence of others, set forth, as general canons of philosophical investigation, methods that may be valid only within the narrower presuppositions of some special science or group of sciences. These

[1] "Logica tradit communem modum procedendi in omnibus aliis scientiis. Modus autem proprius singularum scientiarum, in scientiis singulis circa principium tradi solet."—St. Thomas, *In II. Metaph. lect.* 5.

are mistakes which writers on inductive logic since the time of Mill have not successfully avoided. Nor is it difficult to one looking back, to see why such mistakes were, humanly speaking, almost unavoidable.

At different epochs men engaged in the investigation of those higher and deeper problems which lie along the confines of philosophy and the special sciences, have been very differently impressed as to the relative values of these latter in advancing human knowledge. At one time the attention of scholars is drawn more exclusively to one group of sciences, and again to another group: and the logic of each period will be found to reflect faithfully the then prevailing attitude, by its fuller consideration of the methods and data of the dominating group.

Thus we see that, broadly speaking, the Middle Ages witnessed an exhaustive development of the logic of *Deductive Reasoning*. This was because men were then more satisfied with their principles of knowledge, and perhaps more religiously-minded; because they set greater store on a knowledge of man's nature and destiny than on a knowledge of the external universe; because for progress in the former they relied on (deductive) reasoning from great, broad, general principles and truths that were universally accepted at the time—some on the authority of God as being revealed by Him, others as self-evident, others again as sufficiently established partly by their intrinsic evidence and partly by the common assent and authority of the learned of past ages.

Then came the period of the Renaissance, a period of doubt about hitherto received principles, of revolt against authority and rejection of traditional views and methods. On the one hand, the hitherto accepted teachings of philosophy and religion were critically re-examined; and this new analysis had finally the effect of adding to the *traditional* logic an extensive discussion on the possibility and grounds of human certitude, and on the ultimate criteria or tests of truth (17). On the other hand, a closer attention to the study of external nature led to a wonderful progress in the domain of the physical sciences. The cultivation of this fertile field of research has been rewarded by rich and useful discoveries; the physical universe is being eagerly explored and made to yield up its secrets; and the general laws and conditions according to which its phenomena unroll themselves are the keys by which its most hidden agencies are brought to light and utilized by human enterprise. Hence the high degree of

importance that has been attached to *general truths of the physical order*—in contrast with these other general truths that have to do with man's religion, natural or supernatural, with his moral conduct in life, with the inner nature of his own mind and soul, with the ultimate purpose of his existence, and with his final destiny.[1] Hence, too, the very large and prominent place devoted in modern treatises on logic to an analysis of the method and processes by which *general truths about the physical universe* can be securely and certainly established: as if these were the only general truths of importance, or, anyhow, of most importance, to man; as if *physical* induction were the only or the chief method of reaching a certain knowledge of the weightiest truths to which the human mind can hope to attain.

The modern logician of induction invites us into chemical, physical and physiological laboratories; he familiarizes us with test-tubes and balances, with boilers and engines and dynamos, with microscopes and telescopes; he teaches us how to observe and experiment, how to detect analogies between physical phenomena, how to construct hypotheses foreshadowing the laws according to which these phenomena take place; he lays down canons which will help us to simplify our data by elimination of the unessential, and so to test or establish—or, it may be, to reject or to modify—our hypotheses, until we thus finally discover and generalize some abstract law about the conditions requisite for the occurrence and the recurrence of some physical event.

But the general truths we reach about the external universe, as distinct from man himself, by the application of such methods, constitute *only one* department of human knowledge—an important one, no doubt, yet by no means the most important. There is, for instance, the wide and fertile, if more difficult, department of human research which has for its object the phenomena of *human* activity in the individual, in the family, and in the State: the domains of anthropology and psychology, of the social, economic, and political sciences. The methods of discovering and establishing general truths in these sciences should have no smaller degree of interest for the logician than the method of reaching, say, the law of universal gravitation. Yet the modern logician tells us comparatively little about the former: about *statistics* and *averages* and the *canons of probability:* the various means of reaching another class of general truths or laws which may have immense practical interest for us, even though we can have only moral, and not physical or metaphysical, certitude concerning them.

And what about the innumerable truths, or supposed truths, some of which inform us of *particular* facts in human history, such as the conquest of Gaul by Cæsar, or the crucifixion of Christ; others of which embody *generalizations* such as that "Moral excellence in men and nations results from their possession of deep and true religious beliefs"; and all of which are accepted and believed, by nine-tenths of those who do accept and believe them, *on the authority of their fellowmen*, on the strength of *historical evidence?* If the

[1] *Cf.* JOSEPH, *op. cit.*, pp. 344, *sqq.*

logician thinks it a part of his duty to teach us how to measure *masses* and *motions* of *matter* by the "method of means," the "method of least squares,"[1] etc., may we not reasonably expect from him an equally detailed code of directions in the task, let us say, of estimating the value of the historical evidence for and against the alleged fact—so momentous in human history—that Christ rose from the dead after His crucifixion?

The logician is no more debarred from dealing with the methodology of "metaphysical," or "ethical," or "historical" truth, than he is from investigating the methods of discovering and establishing "physical" truths. Truths and theories, facts and phenomena, whether real or alleged, whether "religious" or "scientific," forming, as they all do, the common data of philosophy, fall equally within the sphere of logic. They are all subjects of human investigation: and it ought to be, therefore, the function of general logic, not to teach us how to explore the hidden recesses of any particular department, but rather to give us a general training in the method of discovering and proving truth: a training which will help us equally well all round, which will aid us in determining whether God exists and has spoken to us through Christ, no less than in determining whether radium cures cancer, or whether alleged "telepathic" phenomena are mere coincidences.

The logician must, of course, ultimately use his own discretion in determining whether he ought in a general way to indicate the main methods in use in this or that special department of science; and it is just here, in judging which departments are worthy of a more detailed attention, that he will be influenced, consciously or unconsciously, by the general trend of intellectual activity in his own time and country. In this way he is exposed to the danger of unduly emphasizing the scope and import of certain special methods of scientific research, or even of setting them up as the *only* methods of attaining to scientific truth.

Now, modern inductive logic shows pretty clear evidence of suffering from an undue bias of the sort just outlined: it has concerned itself somewhat too exclusively with the mathematically exact quantitative methods of the physical sciences, and it has thus fostered an unwholesome tendency to conceive and treat *all* human experience as amenable to the laws and methods of mechanics. It has been more or less obsessed by the rigid determinism of the "mechanical theory of the universe," which was so much in vogue about half a century ago.

There is something one-sided in this tendency to cultivate the positive, *physical* sciences, on the lines of mechanically exact, quantitative laws, and to develop, in logic, a corresponding methodology of them—to the exclusion of the *human* sciences, the knowledge of man's nature, origin, and destiny, of his conduct and religion, of his social activity and its history. The intellectually cogent evidence of the "exact" sciences—mathematics, whether pure or applied to physics—lends itself, of course, most readily to clear, logical treatment. But the "exact" sciences are not the only sciences, nor is the assent which is given on intellectually *cogent* evidence the only assent that deserves to be called scientific. Assents that are *freely* given may be scientific and certain, provided that the evidence is as strong as can be reasonably expected in the matter under consideration. And even where these assents do fall

[1] *Cf.* WELTON, *Logic*, ii., § 158; JOYCE, *Logic*, p. 368.

short of certitude, the general method of weighing the evidence on which they are based forms the proper object of logic.

It must be borne in mind that many of the processes to be hereafter described as subsidiary to induction find their application very extensively outside the merely *physical* sciences, although they are for the most part illustrated by examples drawn from the domain of these latter.[1]

202. SYNTHESIS AND ANALYSIS.—Method (μέθοδος) means mode or manner of procedure, and may be defined as *the proper arrangement of our mental processes in the discovery and proof of truth.* If a truth needs proving, we cannot be said to have fully discovered it until we have proved or established it as a truth; antecedently to this it is only a postulate or hypothesis. The method which thus *leads* to science is sometimes called *inventive* or *constructive*, to distinguish it from the method of *teaching* or expounding truths already established, this latter being known as *didactic* method (204).

In scientific method it is customary to distinguish the influence of two great mental functions, *analysis* and *synthesis*; and according to the predominance of either of these over the other in any department of scientific investigation, the latter is designated an *analytic* or a *synthetic* science.

When a science sets out from a few simple ideas and a few necessary, universal principles, and proceeds to combine these elementary notions and relations, in order to deduce from them other new, less simple, more complex relations, its progress is *synthetic* (σύν-τίθημι). It goes from the simple to the complex, from the more general to the less general. It employs the method of *composition*, the *synthetic* method. Such a science is called a *rational, deductive, abstract* science.

Pure mathematics, for example, sets out from a few necessary and universal principles ("*in materia necessaria*"), with which the mind equips itself by the simple abstraction of a few elementary concepts from the data of sense, and by direct intellectual intuition of certain self-evident relations between those concepts. These relations it combines and multiplies successively, thus gradually forming *definitions* of the various thought-objects with which it deals, *divisions* of these objects into groups or classes, and *demonstrations* which show the relations, ever more and more complex, between these objects. It is thus ever and always *discovering* new abstract objects of thought, *com-*

[1] *Cf.* JOSEPH, *Logic*, pp. 472 *sqq.*

pounding, or *building up* its conceptions, so to speak, into more and more complex wholes, *synthesizing* its gradually acquired truths into a logical, harmonious, and progressive system.

Throughout this whole work of elaboration, the student of the pure deductive sciences has no need to call in the aid of sense experience, of observation or experiment: he might conceivably become the greatest pure mathematician in the world without ever leaving his library. He would, of course, need charts or blackboards to aid his imagination in establishing the complex spatial or numerical relations he might desire to examine between the notions with which he deals. But it is *from the primary notions*, not from the figures or symbols before him, that he deduces even his remotest and most complex conclusions.

If, however, it is true that such quiet seclusion and abstract speculation can produce a great mathematician, is it not equally true that they can never produce a great physical scientist? A knowledge of the physical world implies actual, positive contact with Nature and its activities. The discovery of its laws is conditioned by the observation of its concrete phenomena, and even by experimenting with these latter. It is the result of a long analytic process that has been called *Induction*: hence the designation, *physical* or *positive* or *inductive* sciences.

When a science thus starts with concrete facts, with the data of observation and experiment, and aims at discovering general truths and formulating general laws, about those facts, its progress is from the complex to the simple, from the particular to the general. This is the *analytic* method (ἀναλύω); it finds its place mainly in the *experimental* sciences.

We have already distinguished the reasoning by which we thus ascend to higher and wider laws, as *regressive*, in opposition to the *progressive* reasoning which is characteristic of the deductive sciences (187). Professor Welton[1] thus illustrates the distinction: "Instead of starting from an axiom of the widest generality, in physical science it more frequently happens that the highest and most general principles are the last to be discovered. 'Certain general propositions are first discovered (*e.g.* the laws of Kepler) under which the individual facts are syllogistically subsumed. The highest principles are discovered later (*e.g.* the Newtonian law of Gravitation) from which those general propositions are neces-

[1] *Logic*, i., p. 392.

sary deductions' (Ueberweg, *Logic*, p. 465). . . . A demonstration of this kind is, therefore, called . . . *Analytic*".

It is usual to draw a distinction between the two scientific methods: the *synthetic*, or that of the rational, deductive sciences; and the *analytic*, or that of the experimental, inductive sciences. There is reason for such a distinction: but only in this sense, that synthesis is the *predominant feature* of the former, and analysis of the latter; not in the sense that either feature belongs *exclusively* to either group. No such separation of analysis from synthesis is possible in actual thought. As a matter of fact, the self-evident, *a priori* axioms of the rational sciences necessarily *presuppose* the mental *analysis* of some few elementary observations, by which the mind is equipped with the concepts that form those rational principles. On the other hand, the general laws that are reached by the long and laborious analyses and inductions of the experimental scientist furnish us, in turn, with principles or starting points for synthetic or deductive reasoning processes.

In reality, therefore, there is one and *only one* scientific method: the *analytico-synthetic*, or *combined inductive and deductive* method.[1]

Whether analysis or synthesis will predominate in any particular science, or at any particular stage in the growth of a science, will depend on whether the subject-matter is best approached from the side of the abstract universal, or of the concrete particular. But the two methods are not essentially opposed; rather they "differ only as the road by which we ascend from a valley to a mountain does from that by which we descend from the mountain into the valley, which is no difference of road, but only a difference in the going".[2]

This, moreover, is what we should expect when we reflect on the unity of human nature; and it is confirmed by the findings of psychology. Man derives his abstract ideas from data furnished by his senses. Sense observation must, therefore, be the forerunner of all rational speculation. The formation of abstract concepts from the data of sense experience involves analysis of the latter. These abstract concepts are in turn combined in manifold ways by the activity of the intellect, and are being constantly reapplied to the facts of sense observation. Thus it is that rational speculation is ever returning to those same sense realities which first awake its activity. *All* science is "of the *universal* and *necessary*" (to use the language of Aristotle); but it is no less true that all science must aim at explaining the *contingent*, *individual* facts of our sense experience. It must not only ascend by analysis and abstraction from the particular to the universal, from fact to law, from effect to cause, but it

[1] *Cf.* MELLONE, *Introd. Text-Book of Logic*, pp. 383 *sqq.*

[2] *Port Royal Logic*, p. 314, quoted by Professor Welton, *Logic*, ii., p. 212.

pounding, or *building up* its conceptions, so to speak, into more and more complex wholes, *synthesizing* its gradually acquired truths into a logical, harmonious, and progressive system.

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[2] *Port Royal Logic*, p. 314, quoted by Professor Welton, *Logic*, ii., p. 212.

must also, by a regressive movement of thought, apply its abstract principles again to concrete facts, and by means of the former explain the latter. This combined and alternating use of analysis and synthesis will be more fully illustrated in connexion with the treatment of *Induction*, *Demonstration*, and *Scientific Explanation*. It is commonly employed in the physical sciences, and it is the only method by which a reliable philosophy of man and the universe can be constructed.[1]

There have been, in different ages, philosophers such as Descartes (1576-1650) and Spinoza (1632-1677), who have thought it possible to build up a philosophy by the purely synthetic or deductive method, on the basis of a few self-evident fundamental truths. Such projects are chimerical, for philosophy is expected to offer an intelligible interpretation of universal human experience, and must, therefore, set out from an analysis of this latter.

The method of philosophy, too, like the methods of the sciences, is largely influenced by the prevailing general views and standpoints of each successive period: synthesis predominating in one school or in one epoch of philosophic development, analysis in another: the former, for instance, in Plato and Neo-Platonism, in St. Augustine and the early Middle Ages; the latter in "scientific" and "inductive" philosophy since the Renaissance; and neither, perhaps, asserting undue supremacy in Aristotle, or in Scholasticism among its best accredited representatives, whether mediaeval or modern.

A system of philosophy aims at working out and establishing some definite world-view, some interpretation of human experience as a whole. The method or methods that may be involved in the elaboration of such a thought-system will themselves usually imply assent to certain fundamental judgments, whether these be put forward as axioms or as postulates (203, 231). And hence it is that systems of philosophy are to be judged not only by their explicit positive teaching or *contents*, but also by their *methods*, for these too imply doctrines.[2] Indeed, it has been said that metaphysical systems differ merely in the standpoints from which they approach the interpretation of experience. This is only an exaggeration of the undoubted truth that every such system is largely influenced and characterized by some predominating point of view. Thus, the idea of a process of *Development*, a tendency towards the realization of an ideal, as pervading not only thought but reality, has always exercised more or less influence on the trend of philosophical speculation. But the scientific discoveries of the last few centuries in regard to organic evolution among the forms of life, have led many to suspect the existence and operation of an all-pervading law of *Evolution*, and to adopt, in all departments, only methods of research directly based on this postulate. The wisdom of this procedure is questionable. If it is really unsound, results will in due time reveal its deficiencies.

203. GENERAL RULES OF METHOD.—Various rules or canons, of more or less practical utility, have been laid down for observance in the pursuit of truth, under the title of *General Rules of Method*. They are of the nature of counsels. A full

[1] *Cf.* MERCIER, *Logique*, p. 374: Pratique de l'analyse et de la synthèse en philosophie.

[2] *Cf.* DE WULF, *Scholasticism Old and New*, pp. 190-200.

discussion of their grounds and significance would not be convenient at the present stage of our investigations. For the purpose of enumeration we may conveniently reduce them to the following:—

I. *We should select as starting point the simplest, easiest, most familiar objects of thought.* What is simplest and easiest to understand, will, however, depend on the amount and kind of knowledge already possessed by the seeker; and will, therefore, be relative and variable. Each must determine, from his own knowledge, what element or elements of the particular subject-matter under investigation may be most easily grasped by him.

Whether what is simplest *in itself* is simplest *for us*, will be largely determined by the nature of the subject-matter in hand. Looked at in itself, the abstract, universal principle or law is simpler, less complex in content, than any concrete fact under it: *e.g.* the law of gravitation than the fall of an apple;[1] but it is not always this simple, abstract aspect of reality that comes first under our notice or is most familiar to us. Of some aspects of reality it is the widest and most general truths that are most easily grasped, as in the case of the axioms of the rational sciences; and then the method employed will be mainly synthetic. Oftener, however, it is the concrete, complex, many-sided fact of sense with which we are most familiar, as in the data of the inductive sciences; and then the method employed will be mainly analytic.

II. *We should proceed from the known to the unknown*, GRADUALLY, *step by step, in an orderly, logical sequence of thought, and not hastily, irregularly*, "PER SALTUM".

To secure this, we must observe carefully all the canons of definition, division, reasoning, demonstration, etc. Failure in the observance of these canons will usually expose us to error, and will inevitably involve inversion, repetition, and consequent confusion. Innumerable examples of those defects have been instanced from the order followed in Euclid's elements of geometry.[2] The importance of explicitly examining and testing every step of our progress cannot be exaggerated. In no other way can thoroughly *scientific* knowledge be either secured or retained in the mind: whereas, on the other hand, a clearly perceived, logical, organic connexion between truth and truth is necessarily a powerful aid to memory.

[1] WELTON, *op. cit.*, p. 216. [2] *ibid.*, p. 225.

Moreover, it is by careful separation of a problem or subject into its various parts and details that we are enabled to distinguish betweeen the accidental and the essential, and to avoid being misled by superficial resemblances and seeming connexions. *Habit, association, familiarity*, are apt to lead us astray. We very easily mistake invariable sequence for causality, and apparent reasons for real ones.[1] The principal sources and classes of such mistakes will be enumerated in the sections on *Fallacies*.

III. *While, on the one hand, we must never accept anything as true which we do not clearly know to be so, on the other hand, we must not expect the same degree of certitude, or the same cogency of evidence, in all the sciences.* Disregard of the second portion of this rule has led many, especially in modern times, into *scepticism*, *i.e.* doubt about the capacity of the human mind to attain to certitude about anything. Taking too narrow a view of "science," they expect *cogent* evidence in the concrete subject-matter of the human sciences—social, economic, and ethical—evidence which, of their very nature, these sciences cannot be expected to yield.[2] And when it is not forthcoming they drift into scepticism. One would imagine that St. Thomas Aquinas was writing for the twentieth century, rather than the thirteenth, when he penned these sentences: "There are some who will not receive anything that is told them unless it is mathematically proved. This is usual with those who have had a mathematical training, because custom is second nature. But it may be also due to the possession of a strong imagination, combined with an undeveloped judicial faculty. Others there are who will not receive anything unless there is put before them some illustration of it that can strike their senses. This, too, results either from habit, or from the predominance of the influence exerted over them by their senses, or from want of intellectual discrimination. . . . Others, however, there are who

[1] "An Englishman resident in some city in South America sees united in the inhabitants a profession of the Catholic religion, a great laxity of morals, and an absence of all energy, fortitude or perseverance. Neglecting our rule, he comes to the conclusion that there is a necessary connexion between Catholicism and the vices around him. . . . Or, again, we may have observed in the newspapers that a larger number of persons lose their lives by drowning on a Sunday than on any other day. On this fact the Scotch Presbyterian makes the remark that it can only be explained by the anger of God with all who take their pleasure on His Holy day; quite overlooking the circumstance that it is on Sunday that a great number of excursionists of the middle and lower classes, who are unskilled in the use of boats and can rarely swim, take their pleasure on the water."—CLARKE, *Logic*, pp. 469, 470.

[2] *Cf.* JOSEPH, *Logic*, p. 489.

wish that everything offered them should be based on certitude, that is, as the fruit of diligent rational inquiry. This is the attitude of a sound understanding in judging, and of sound reason in investigating : *provided always that* [such certitude] *be not sought in matters where it cannot possibly be found.*" [1]

IV. *We must keep before us, as clearly as we can, the end to be attained in our inquiry or argument, and suit our method to the attainment of this end.* Of course, when the end in view is the *discovery* of *new* knowledge, as distinct from the communication of knowledge already possessed, to others, we cannot have a clear or definite conception of what we are looking for : if we had, our inquiry would be superfluous. Still, we must have some general suspicion of it : otherwise we should not think of looking for it at all. Discoveries are, no doubt, sometimes made haphazard, by groping in the dark ; but this is the exception. As a rule, our progress in knowledge is guided by *hypotheses*, based on *analogies* with what we already know.

Besides those general canons of method, special rules are sometimes formulated for the synthetic method, and special rules for the analytic. In the chapters dealing with *Induction* we shall examine the latter method at some length, and we shall there see that although the process by which we rise from the perception of concrete, individual facts of sense, to the apprehension of general truths, is one of very great importance, yet it is scarcely possible to formulate any mechanical set of guiding rules for it.

It is the synthetic method that systematizes the truths discovered by analysis, and explains concrete reality by applying to the latter analytically discovered laws. The rules laid down by some logicians for its employment are almost too obvious to need special statement. For instance, we are reminded that we must start either from axioms that are indisputably self-evident, or from general truths already proved. The usual error here is by defect, by taking for granted what is neither sufficiently simple to be self-evident, nor has been clearly proved—the fallacy known as *Undue Assumption of Axioms.* But philosophers, nowadays, not unfrequently err by excess, by demanding proof for what is so clearly self-evident as to be indemonstrable. They call into question the claim of any principles, however self-evident, to our unconditional intellectual assent. They doubt or deny that such abstract, self-evident axioms give us any insight into the real nature of things, confining the validity of such axioms to the sphere of *subjective* mental appearances, and according them at most a merely provisional acceptance as "assumptions" or "postulates" which may perhaps be some day verified as *objectively* valid, or may perhaps be destined to remain as mere "directive" or "regulative" principles of our thought-processes. It is, of course, a grave mistake thus to confound self-evident truths about the data of our experience with those mere "working hypotheses" and "methodological

[1] Lect. V. in *Metaph.* 2.

assumptions "[1] which all investigators have sometimes to make, and which are perfectly legitimate in their proper sphere; but an inquiry into the grounds of this erroneous tendency in modern philosophy would not be opportune here.

Aristotle and the Scholastics examined in minute detail the requirements of the synthetic processes through which we advance by demonstrative reasoning from simple, self-evident first principles to more complex scientific conclusions. Their teaching will be outlined in the chapter on *Demonstration.*

The remainder of the present chapter will be devoted to the application of analysis and synthesis to the *teaching* or *exposition*, as distinct from the *discovery* and *proof*, of truth.

204. DIDACTICS: ANALYSIS AND SYNTHESIS IN TEACHING.—When our object is not to discover truth as yet unknown to us, but to communicate what we know already to others, our method will be no longer constructive or inventive, but *instructive* or *educative*; instructive if it aims merely at the communication of knowledge to the intellect; educative if it aims at the formation of right mental habits and character as well. The latter is the scope of the art of *Pedagogics*; the former alone, that of *Didactics*. This latter, therefore, is the sole concern of the logician.

What, then, is the proper method of teaching or exposition? Broadly speaking, it is laid down that while the analytic method is the great method of discovery the synthetic method is the great method of instruction. And in general terms this is correct. But the statement needs to be carefully limited and qualified.

The analytic method is not *exclusively* the method of discovery; as witness the many discoveries of pure and applied mathematics. Nor, similarly, is the synthetic method always the best method of exposition. It is, of course, obviously the best in teaching the pure deductive sciences; for in these the abstract principles, being simpler than their complex applications and conclusions, are more easily grasped by the beginner. But even here we need initial observation of concrete facts or instances as an aid to the *abstraction* of the simple notions, and to the intuition of the principles from which these sciences start. This initial stage is analytic in its character. The teacher familiarizes his pupils with concrete instances, facts, models, embodying the abstract principles he wishes them to grasp. In dealing with children especially, it is necessary to dwell at length on concrete things: these are more familiar: and the child's power of grasping even the simplest abstract principles, and reasoning from them, is comparatively undeveloped. The aim, at this early stage, will rather be to awaken the child's powers of observation and intuition, to arouse its curiosity and stimulate its interest by presenting to it simple but attractive facts, combined with judicious interrogations and suggestions, calculated to draw out the pupil's powers of observation, comparison, and inference.[2]

[1] *Cf.* JOSEPH, *op. cit.*, p. 523. *Cf. infra*, 237.

[2] Professor Willmann, in Germany, has published, under the title of *Didaktik als Bildungslehre*, a work of the highest merit on intellectual training. Habrich, a pupil of Willmann's, has supplied the teachers of intermediate education in Germany with a useful treatise on psychology, "*Paedagogische Psychologie*," in harmony with the principles of scholastic teaching. From another standpoint, *cf.* Herbert Spencer's works on *Education*.

Moreover, the pupil should be trained, as far as possible, to discover, himself, the reasons and causes of the things observed by him. This involves the use of the analytic method, and develops the *spirit of analysis* in the learner. Such initiation into the method of independent personal investigation constitutes the immense difference there is between *intellectual education proper* and *mere instruction.*

This method of teaching by suggestion, of drawing out the learner's powers by judicious questioning, is called the *Socratic* method, after the Grecian sage who made such a fruitful use of it. He, himself, appropriately called it the μαιευτικὴ τέχνη, the art of *intellectual obstetrics* or *mental midwifery.*[1]

This stage of analysis and observation is a necessary step towards *abstraction* of ideas and *intuition* of first principles. These notions and principles become in turn the *explanatory reasons* of the facts in which they are realized. The learner will next be taught, by an application of the *synthetic* method, to make use of those principles and laws for the understanding and *explanation* of concrete phenomena.

Thus he will be taught to make use both of observation and of abstraction, both of analysis and of synthesis. The former without the latter would lead to narrowness of view, to the shortsighted philosophy of Positivism ; the latter without the former, to barren, empty speculations, and to the substitution of mere verbal explanation for real science. The sciences of observation develop the spirit of specialized research ; the mathematical and metaphysical sciences, the deductive, speculative turn of mind.

It will be seen, therefore, that *as a rule the method employed in exposition is the same as that employed in discovery;* that the art of teaching must follow nature ; that the mind of the learner must follow *substantially* the same path, whether he discover truth on his own account or be guided into the knowledge of it by one who is already in possession of it.

Of course, when the exposition "is intended for well-prepared adults—as when one writes a text-book, the most appropriate method is, generally speaking, that of synthesis, as by that method the necessary relations of the parts of the subject to each other are most clearly shown."[2] But even here it is well to remember that the abstract, universal principle or law is not always the easiest to grasp at the starting-point. In an example from chemistry, given by Father Clarke in his *Logic*,[3] we are told that "in each of these opposite processes [analysis and synthesis], the rule . . . of commencing with what is more familiar, and thence proceeding to what is more remote and

[1] Socrates used to seek from others the knowledge they imagined they possessed, and which he himself pretended not to possess. His arguments took the form of dialogues, each in two parts. In the first, his "irony" confounded his interlocutor and convinced the latter of the weaknesses and drawbacks of his position. In the second, Socrates gradually drew from him a new and truer definition, a better understanding, of the matter in dispute. After silencing his opponent in the first or destructive stage of his discourse, he would begin by another series of questions to construct a new solution of the problem—to substitute for the exploded error, or "spurious offspring," the "veritable fruit" of a "new-born" truth. The conclusion of the dialogue thus became the "fruit of their personal reflection," the "child of their thought".—C. PIAT, *Socrate*, pp. 106-109, Paris, Alcan, 1900.

[2] WELTON, *Logic*, ii., p. 214. [3] pp. 471-74.

unfamiliar, is observed by the chemist. In his investigation he commences with that which is most familiar to ordinary mortals (*nobis notiora*), the water of the spring where thousands have drunk or bathed, and thence proceeds to the various chemical agents it contains which are to us a mystery, though in themselves they may be so simple as to admit of no further analysis. In imparting to others the results of his experiments he begins from what is simpler in itself and therefore more familiar to nature (*naturae notiora*), and thence proceeds to the complex results with which ordinary men are familiar, however complex they may in themselves be." But, if the audience is composed of "ordinary mortals" to whom the elements—however much simpler and more knowable they may be in themselves—are so many "mysteries," would not the lecturer be better advised to commence his exposition with the more familiar water, and to lead his audience along *substantially* the same path as he himself had followed in the first instance?

It seems rather a mistake, therefore, to apply the synthetic method exclusively, to the exposition of the subject-matter of those sciences in which analysis has been the main instrument of discovery. It is rightly used in the teaching of the pure deductive sciences such as mathematics; but the exposition—at least the early stages of the exposition—of those sciences in which analysis, observation, and experiment have played a conspicuous part, should be rather analytic than synthetic. For example, the method followed by Maher and Mercier in their well-known treatises on psychology—the analytic or empirical phase leading up to the synthetic or rational one—is very much superior to the exclusively synthetic method adopted by many Scholastic writers in their Latin treatises on the subject. In accordance with the Scholastic axiom, *Operari sequitur Esse*, we ought to *commence* by examining and analysing the data on which our scientific knowledge of man is based, *viz.* his *activities*, to arrive next at a knowledge of his *faculties*, and ultimately of his *nature*, *origin*, and *destiny*.

We are only following nature in adopting such a course of analytico-synthetic exposition. The manner of using analysis in teaching will, however, be slightly different from the manner of using it in discovery.[1] In the process of discovery, our analysis is necessarily slow, tedious, tentative, guided merely by analogy and hypothesis, often erratic owing to our being misled by false analogies and wrong hypotheses; our experiments are necessarily multiplied and often practically blind, though seldom quite aimless. But in the process of exposition it is manifest that, having traversed the way before, and being now in possession of the scientific knowledge which was our goal, our didactic analysis may be much more direct and definite. We may exclude all the gropings and deviations that occurred in the first search after the truth, the misleading analogies and wrong hypotheses; we may carefully select the most appropriate instances and experiments for disclosing the law in question to our pupils, and thus shorten the road for them: but we shall be travelling *substantially* the same road and employing the same method as previously.

205. SCHOLASTIC METHODS OF EXPOSITION AND DEBATE. The mediaeval Schoolmen followed the advice of the founder of the Lyceum: "Before you try to solve any problem," wrote Aristotle, "set forth clearly the reasons or difficulties that militate against the solution you are about to propose. In that

[1] *Cf.* WELTON, ii., p. 220.

way you will see better where is the heart or kernel of the question, the exact point in dispute; you will fix your attention on it, and you will retain a firmer conviction of what you have seen to stand successfully the shock of the debate."[1]

Open the *Summa Theologica* of St. Thomas, that monumental synthesis of mediaeval wisdom "*ad eruditionem incipientium*".[2] At the beginning of each Question (*Quaestio*) or Sub-question (*Articulus*) will be found a résumé of all the arguments, from reason and authority, that can be brought *against* the intended solution. They are introduced by the familiar "*Videtur quod non* . . .". Next comes the doctrinal affirmation of the thesis or solution, introduced by the words "*Sed contra* . . .," and usually illustrated rather than proved by some quotation from Scripture or from the Fathers. Then comes the body of the article (*Corpus Articuli*), introduced by the phrase "*Respondeo dicendum quod* . . .," and containing the principle on which the solution is based, together with its main proofs in the usual syllogistic form. Finally, we have the further application of this same principle to the solution of each of the various difficulties proposed against the thesis at the commencement: "*Ad primum dicendum quod* . . ." "*Ad secundum* . . . ," etc.

At the public debates that were held in the mediaeval universities at certain fixed intervals during the year, usually before Christmas and Easter ("*Disputationes Quodlibetales*," as they were called), the procedure was slightly different. Any auditor might raise a question and indicate in a general way the arguments in favour of the solution that had his preference. The "*respondens*," *i.e.* the candidate for degrees, or his master, formulated *their* view, and based it on some fundamental argument. This position was at once attacked by the objector, and so the debate was opened. On the morrow, or one of the following days, the master repeated, arranged, and "determined," or settled definitively, the various questions discussed. These "*Determinationes*" have come down to us in the copious volumes of mediaeval philosophy and theology known as "*Quodlibeta*".[3]

The method of carrying on academic debates in Scholastic philosophy and theology, still in use in schools, colleges, and universities, where these subjects are taught, is the same in principle as the above, if somewhat different in detail. The exercise is strictly syllogistic, and it undoubtedly gives the student

[1] *Metaphysics* iii., 1; *Nicomachaean Ethics*, vii., 1. Here is the comment of St. Thomas: "Postis his quae videntur probabilia circa praedicta, prius inducamus dubitationes, et sic ostendemus omnia quae sunt maxime probabilia circa praedicta . . . quia si in materia aliqua dissolvantur difficultates et relinquuntur ut vera illa quae sunt probabilia, sufficienter est determinatum."—*loc. cit.*, lect. 1.

[2] "Quia catholicae veritatis doctor non solum provectos debet instruere, sed ad eum pertinet etiam incipientes erudire, propositum nostrae intentionis in hoc opere est ea quae ad Christianam religionem pertinent, eo modo tradere, secundum quod congruit ad eruditionem incipientium." A few brief sentences next tell us why he undertook the work: to rid theology of many useless questions, and to give an *orderly* exposition of it for the benefit of learners; and in what spirit: "cum confidentia divini auxilii." Those few simple sentences form the whole preface or prologue to one of the greatest works that human genius has ever produced.

[3] *Cf.* De Wulf, *History of Medieval Philosophy*, p. 258, note from Mandonnet's *Siger de Brabant, etc.*

an invaluable training in exact reasoning. The following outline may be found helpful to academic disputants.

The professor fixes upon a thesis, appoints a pupil to "defend" it, and one or more others to "object" to it. At the appointed time the defender ("*defendens*," "*respondens*") enters the pulpit or bema, announces the thesis, adding, if desirable, a very brief exposition and proof. The objector ("*objiciens*") then asserts the contradictory of the thesis, proving his assertion by a syllogism. The defender resumes with the introductory phrase, "*Sic argumentaris, Domine*" ("This is your argument, Sir"), repeats the syllogism slowly and clearly, deliberating on the way in which he ought to deal with each premiss, the consequence, and the conclusion. Having repeated the syllogism, and also the introductory phrase, he again takes up and repeats once more the major, and now passes judgment on it: "I grant the major" ("*Concedo majorem*"), if he considers it true; "I distinguish the major" ("*Distinguo majorem*"), if he sees in it a true sense and a false sense, which two he will separate by the addition of some well-chosen technical phrase to show the true sense and the false one, qualifying the false by "I deny" ("*Nego*"), and the true by "I grant" ("*Concedo*"); "Please prove the major" ("*Faveas probare majorem*"), if he considers the major entirely false; "Let the major pass" ("*Transeat major*"), if he considers it irrelevant, or does not wish to pass definite judgment on it. In general, the objector should so construct his syllogisms that the major will not admit of total denial. Should the defender thus request his adversary to prove the major, the former need not proceed to the minor of the original syllogism, but listen to and deal with the proof brought forward for the major. If the defender has granted, or "distinguished" the major, he proceeds to repeat the minor, and either "denies" or "contra-distinguishes" it ("*Nego minorem*" or "*Contra-distinguo minorem*"). It is only when he "denies" either premiss, or "distinguishes" one and "contra-distinguishes" the other, that he has a right to "deny" the "consequence" or probative force (*consequentia*), and therefore also the conclusion (*consequens*), of the syllogism ("*Nego consequens et consequentiam*"). To "contra-distinguish" the minor is to introduce the same distinction into it as into the major, granting the member corresponding to that denied, and denying the member corresponding to that admitted, in the case of the major. It may sometimes be necessary to introduce a further distinction into either or both members of a distinction in order to sift fully the true from the false: this process is called "subdistinguishing" ("*Subdistinguo*").

The objector then continues the debate by proceeding to prove syllogistically the proposition denied by the defender, in the sense in which it was denied, commencing by the words, "I prove the major (or minor) denied" ["*Probo majorem* (or *minorem*) *negatam*"]; and the defender proceeds to deal with the new syllogism as before.

The objector may, at any stage, request or allow the defender to explain the precise force of the distinctions he has made in an answer: which the defender does as briefly and clearly as possible, introducing his explanation by the words, "I explain the distinction (distinctions) introduced" ["*Et explico distinctionem datam* (*distinctiones datas*)"]. Various courses may here present themselves to the objector.

He may, notwithstanding the explanations offered, urge some proposition in the sense in which it has been denied. "But . . . Therefore the difficulty remains". "*Atqui . . . Ergo stat difficultas*"). To which the defender replies, "I deny what you subsume" ["*Nego subsumtum*"]. The objector must then proceed to prove the proposition in the sense in which it has been denied. ["*Probo subsumptum.*"]

Or, again, the objector may urge the difficulty in a modified way, owing to some concessions made by the defender in his explanation; which he does by commencing, "But I insist . . .," or, "But I urge the difficulty from your own admissions" ("*Atqui insto . . .*," or, "*Atqui ex concessis urgeo difficultatem*".)

The real point of the difficulty ought to be kept in the *minors* as far as possible: the distinctions made ought to be real, not merely verbal, *i.e.* expressive of the same syllogism in different terms: quibbling and sophisms ought to be rigorously excluded: the questions selected ought to be the more serious ones, and the difficulties likewise: if the objector really feels the difficulty he is putting, so much the better: waste of time, vain display of acuteness in making distinctions, or syllogisms more subtle than solid, should not be tolerated: the number of syllogistic steps leading up to the full solution of any difficulty will, of course, depend on the nature of the latter, but need not usually exceed four or five, unless, indeed, a modified phase of the difficulty, or a practically new difficulty, arises in the course of its solution: exactness, lucidity, brevity in the formation of syllogisms and distinctions, ought to be insisted on: and therefore, also, the necessary means to this end, *viz. familiarity with the technical terminology* of the philosophical problems under discussion, and of philosophical terminology in general.

Such are the principal canons laid down for observance in those exercises.[1] There is no reason why they should not be conducted in the vernacular if necessary, rather than in Latin. The method is not wedded to any language; and philosophical thinking would be much less erratic and illogical than it is at the present day if such disciplines formed an essential part of philosophical training.

The *Scholastic* system of philosophy is identified with constructive and didactic methods which are nowadays eliciting a more accurate and sympathetic appreciation from scholars, after a long period of prejudice and misunderstanding. It took shape in the early mediaeval schools of Europe under the combined influence of St. Augustine, Plato, and a few of the logical writings of Aristotle. But the introduction of the latter's works into the Western schools towards the close of the twelfth century gave Scholasticism its predominantly Aristotelean character in the thirteenth.[2] To its preponderating use of synthesis as a constructive method we have already referred (201). Its elaborate system of teaching, too, has had a profound influence on the development of learning during many centuries. While recognizing its limitations, we are bound in the interests of historical truth to give it credit for many excellences. In general, we may say that the Scholastic method,

[1] *Cf.* ZIGLIARA, *Logica*, (46), *De methodo disputandi.*
[2] *Cf.* DE WULF, *History of Medieval Philosophy*, pp. 101-48; *Scholasticism Old and New*, pp. 19-88, 168-82.

whether constructive or didactic, trains the mind to careful reflection and develops the critical faculty.

In the first place, it certainly gives one the habit of disentangling and clearing up his ideas, of arranging them in order, of introducing rigorous logical sequence among them.

Then, secondly, it teaches us to distinguish certainty from probability, truth from appearances, science from plausible theorizing, and established conclusions from unverified hypotheses.

Thirdly, it inculcates a spirit of disinterested inquiry after the truth. In Scholastic philosophy truth is regarded in its native, unadorned beauty, so to speak ; it is sought for its own sake, and with a dispassionate calm : to the Scholastic, rhetoric makes no appeal : mere rhetoric excites the imagination and emotions, disturbs the balance of judgment, begets confusion of ideas, and hasty, ill-considered views. An inflammatory discourse that will arouse an untrained audience to the highest pitch of passion or enthusiasm may not be able to stand the test of a cold analysis, or the logic of the syllogism. The language of Scholasticism is the very antithesis of rhetorical. It "simply and solely expresses the intellectual concept, abstracting from all its relations to the other faculties of the soul, and from the reactions it may call forth in them. All possible obstacles between the mind and the objective truth are pitilessly set aside. Its style, stripped of all ornament, free from all feeling and sentiment and all the artifices of rhetoric, and hence so often accused of crudeness and barbarism, has all the exactness and precision of a mathematical formula or proposition ; it is pre-eminently truthful and clear. It was methodically and most successfully shaped into the aptest possible instrument for the systematization of thought : the instrument that was to build up the great *Summae*, whose materials lay scattered for generations through a whole world of literature. Reduced to the simple form and proportions of proposition and syllogism, those truths could be logically moulded into an organic whole in which each part received a prominence due to its relative importance."[1] We are often nowadays reminded of what Plato said : We ought to tend to the truth *with our whole soul*—*σὺν ὅλῃ τῇ ψυχῇ . . . εἰς τὸ ὄν καὶ τοῦ οὕτος το φανότοτον . . . τουτο δ' εἶναὶ φαμεν τὰγαθόν.*[2] The Scholastics receive those words with respect, but also with caution. When the truth *is known*, yes, by all means, let us love it, embrace it with all the ardour of our souls, act up to it, work for it, suffer for it if needs be, and if duty demands the sacrifice. But in *searching* for the truth our chance of finding it will be in proportion to the degree in which our intellect succeeds in laying aside all considerations foreign to the truth itself. At bottom, the truth is always good, always truly useful, therefore ; of that there can be no doubt. But this or that doctrine, which is subjectively judged to be useful, may not be so in reality ; and some other, judged to be dangerous, may be the only one truly useful in the long run, because it happens to be the one that is really true.

Fourthly and finally, the Scholastic method counteracts the narrowing influence exerted on the mind by a constant and exclusive contact with the

[1] P. RICHARD, *Étude critique sur le but et la nature de la scolastique* (*Revue Thomiste*, May and November, 1904).

[2] PLATO, *Republic*, vii.

concrete, positive facts of sense; it nourishes in the soul what we may call the craving for the universal, the desire to grasp the idea in the fact, the abiding law in the contingent phenomenon.

In a word: clearness, precision, severe logic, method; a sense or perception of the true; love of the truth simply for its own sake; elevation of thought and a fresh and speculative turn of mind: such are the qualities developed by the Scholastic method in those who are formed upon it. Many great philosophers have placed on record authoritative eulogiums of the syllogism.[1] Such a writer as Huxley, who is certainly free from the suspicion of partiality in this matter, pays a willing tribute of admiration to "Scholastic philosophy, that marvellous monument of patience and genius, constructed by the human mind to give a logically unified answer to the problems raised by the spectacle of the universe".[2]

But the Scholastic method is not without its limitations. A method, being a means to an end, becomes useless, or even injurious, when wrongly employed. The Scholastic method exercises mainly the *speculative* reason: it is primarily explicative, synthetic. It accustoms one to understand how a conclusion is connected with certain premisses, how conclusions follow from principles; but it develops very little, if at all, the *habit of observation;* it gives little or no stimulus to personal initiative in the discovery of new truth. A training in the positive sciences is, therefore, the necessary complement of a "Scholastic" formation or discipline of the mind. To round off and perfect this latter training, nothing is more efficacious than contact with facts; since the intellect must derive all its ideas from external or internal sense experience, no mere verbal descriptions of phenomena can equal the direct and immediate perception of these latter. By his example and by his works, Aristotle is no less the master of scientists than of philosophers. Not only Roger Bacon, but Albert the Great, St. Thomas, Duns Scotus, were faithful to his method. Pope Leo XIII. has recommended us expressly, in his encyclical *Aeterni Patris,* to "receive with a willing and grateful mind every word of wisdom, every useful thing by whomsoever it may have been discovered or planned". It must also be of very great utility to supplement a training in the Scholastic method by reviewing the *history* of scientific progress, so as to realize what provisional hypotheses and theories, what guesses and approximations, what deviations and errors even, the human mind has had to pass through in its journey towards the discovery of every new truth.

Again, the importance attached by Scholasticism to *certain* science inclines its disciple to depreciate the value of the merely *probable* and *provisional.* To the "Scholastic" mind, the slowness of experimental work is irksome: it easily becomes impatient of the problematic character of most historical, sociological, political, and economic inductions, and of the many reserves with which the materials of the special sciences must be employed.[3] But, while it is very right and proper to seek for certitude, and very praiseworthy to look for demonstrative reasons, it is wrong to expect the impossible; where certitude cannot be had it is unreasonable to demand it.[4]

[1] *Cf.* Leibniz, *Nouv. Ess.*, iv., 17, § 4.—*apud* WELTON, *op. cit.*, i., p. 411.
[2] HUXLEY, *Animal Automatism and Other Essays*, p. 41.
[3] *Cf.* MERCIER, *Origines de la psychologie contemporaine*, pp. 450 *sqq.*
[4] Sunt aliqui qui omnia volunt sibi dici per certitudinem. . . . Et hoc contingit

Furthermore, exclusive preoccupation with the *true*, exclusive attention to the relation of things to the *intellect* alone, may disturb the harmony that ought to regulate the development of our faculties. The Scholastic method interprets reality by referring the latter to intellect alone. Avowedly, and on principle, the standpoint of its research is above and beyond the domain of emotion and will; it brings into action the intellect alone. Now, no one may, with impunity, submit himself exclusively to any such purely intellectual regime.[1] It perfects and develops one side only of our being, the side that is fundamental and essential, no doubt, but which, nevertheless, is not the whole man. The mind that is excessively given to such a discipline develops an unduly abstract and speculative turn, and loses very largely all just appreciation of the great complexity of concrete, actual things.[2] All exclusive preoccupation with a special order of truths entails, of necessity, the inconvenience just referred to. All *specialists* are prone to contract a peculiar sort of "mentality" that tends to make them narrow, and suspicious of truths outside their own chosen circle.

Finally, there is hardly any need to point out that the excessive use—that is, the *abuse*—of the Scholastic method, may make one insensible to form, to elegance of expression. A literary culture alone will counterbalance this danger of an exclusively abstract, logical, and "intellectualist" mental discipline.

MERCIER, *Logique*, pp. 271, 294-96, 371-84. WELTON, *Logic*, vol. ii., bk. vi. MELLONE, *Introd. Text-Book of Logic*, pp. 291, 383 *sqq*. DE WULF, *History of Medieval Philosophy*, pp. 137, 254 *sqq*.; *Scholasticism Old and New*, pp. 19-31. ZIGLIARA, *Logica*, (44), (45), (46).

propter bonitatem intellectus judicantis, et rationis inquirentis; *dummodo non quæratur certitudo in his, in quibus certitudo esse non potest*.—ST. THOMAS, *In II. Metaph*., Lect. 5. *Cf. supra*, 203.

[1] *Cf.* RICHARD, *Revue Thomiste*, Nov.-Dec., p. 564.

[2] Compare what Newman says in his *Grammar of Assent* about *Inference*, and about what he calls the *Illative Sense;* also Pascal's striking passage (*Pensées*, section I, p. 318. Brunsch. edit.) on the *ésprit géométrique* and the *ésprit de finesse:* "The reason why certain practically shrewd people (*ésprits fins*) are not great geometricians is because they are utterly unable to give their minds to the principles of geometry. But the reason why geometricians are not shrewd (*fins*) is because they do not see what is under their eyes; accustomed to the clear truths of geometry, and to reasoning from well-grasped, tangible principles, they get lost in small things (*choses de finesse*) where the principles are not at all tangible. Here the principles are hardly seen at all, but rather felt; they can only with the greatest difficulty be impressed on those who do not happen to feel them themselves; and things of this sort are so delicate and so numerous that it requires an exceedingly keen and delicate faculty to feel them, and to judge them rightly according to this feeling, when, as happens oftenest, we cannot demonstrate them in geometrical order, seeing that we have not their principles in that way, and that it would be undertaking infinite labour to try to get at them so. We must, as it were, see the thing at a glance rather than by progressive reasoning, at least in a certain measure. And hence it is rarely we find shrewd geometricians . . . because they wish to treat complex things (*choses fins*) geometrically, and make themselves ridiculous by commencing with definitions and principles: which is not the way in that sort of reasoning. It is not that the mind does not reason; it does, but tacitly, naturally, without art, in a way which none may mechanically express, and with which few indeed are adequately endowed,"

CHAPTER II.

INDUCTION IN ITS VARIOUS SENSES. INTRODUCTORY AND HISTORICAL NOTIONS.

206. THE PROBLEM OF INDUCTION: ASCENT FROM THE PARTICULAR TO THE UNIVERSAL.—In the foregoing chapter we have gleaned some general notions about method, and about the processes of analysis and synthesis involved in method. We now purpose to deal with the analytic method and the doctrine of *Induction*. To the main problem of induction we have referred already (194, 198). How do we, from particular facts of sense experience, attain to a knowledge of necessary, universal truths? Such universal judgments we have seen to be essential not only to all deductive, but to all mediate, reasoning whatsoever (193, 195). We have called them abstract, general, universal, generic judgments (92). They are likewise called logical and scientific principles, axioms, laws of thought, laws of physical nature, etc. We have expressed them both categorically: "*M* as such is *P*"; "All *M's* are *P*"; "Whatever is *M* is *P*," etc.—and hypothetically: "If anything is *M* it is *P*"; "If *S* is *M* it is *P*," etc. And now we have to analyse the conscious processes by which, from the apprehension of particular facts, instances, cases (containing *S*, *M*, *P*, etc.), we reach a certain knowledge of such general truths or laws. Since, moreover, the essential merit and excellence of "*scientific*" knowledge lies in the fact that it is a knowledge *of the universal* truth, principle, law, etc.—and, through this, of the *particular* phenomena or instances under it, —the importance of clearly understanding the process by which, and the rational grounds on which, we give our assent to the universal truth, will be at once apparent.

We have distinguished three kinds of universal truths (195). There are, *firstly*, those absolutely necessary, self-evident axioms such as the laws of thought, metaphysical principles such as the principle of causality "*Whatever happens has a*

cause," ethical principles like "*Virtue is praiseworthy*," geometrical and mathematical axioms such as "*Two and two are four*"; and the whole vast body of truths that can be derived from such principles by *pure demonstration*. These truths are all *in materia necessaria;* they have to do with abstract essences, or objects of thought considered in a possible state, apart from the changing conditions of actual existence in time and space.

The process by which we come into possession of truths of this class presents no logical difficulty. It is simply a process of forming abstract and universal concepts; of analysing and comparing these with one another; of thus *seeing* intellectually self-evident, necessary relations between them; of generalizing these relations and formulating them in necessary or analytic propositions. The process embraces *conception* and *judgment*, but does not involve logical *inference* or reasoning proper. It is from sense observation of a few instances that we form the concepts: we need such observations in order to get, for example, the notions of "whole," and "part," and "greater". But having once *abstracted* these intellectual notions from sense experience, and compared them with one another, we have an immediate intellectual *intuition* of the *necessary* truth that "the whole is greater than its part": and this truth we see to apply to *every* whole, actual and possible, known and unknown: we assent to it not because we have examined all the instances—for we have not—but because we perceive the relation to be universal because it is *necessary*.[1]

Now, this simple process of *abstraction*, *intuition*, and *generalization*, by which we attain to a knowledge of self-evident, necessary principles, through the notions which we abstract from sense experience, is sometimes called *Induction*. But this is using the word in such a wide sense as to make it embrace *every mental process by which we ascend from or through the particular to the universal*. Aristotle used the equivalent Greek term in this wide sense: *Ἐπαγωγὴ ἡ ἀπὸ τῶν καθ' ἕκαστον ἐπὶ τὰ καθόλου ἔφοδος*.[2]

[1] *Cf.* JOSEPH, *Logic*, pp. 356, 363 (2), 508 *sqq*.

[2] *Top*. i. 12. Truths of the class with which we are dealing are described in Scholastic philosophy as *per se notae* (86), *i.e. knowable in themselves*, by a full analysis of the notions involved in them. In some of these truths the notions are so simple as to be within the reach of all who are endowed with ordinary intelligence. These are said to be *per se notae quoad omnes*. In other cases, however, the notions may be so complex—as, for instance, in the remoter mathematical conclusions—that although the truths embodying them are knowable in themselves

The processes we have just described as *abstraction, intuition*, and *generalization*, of simple, self-evident, necessary axioms, are usually described by modern logicians as "*geometrical*" or "*mathematical*" *induction*.[1] And the reason assigned for the superior cogency of the evidence and certitude we have in regard to these truths, as compared with those we reach by "*physical*" induction, is stated to be this: that in the former the objects compared, being abstract, *have their essential qualities fixed for certain* by ourselves, *in the definitions* we impose upon them; while in the latter the *essential* qualities of the objects—which are now concrete things and agencies existing and acting in physical nature—are not fixed by definitions which we impose on them, but "have to be discovered and proved". Thus Dr. Mellone writes:[2]

> "The universality of the result [that "the angles at the base of an isosceles triangle are equal"] depends upon our being absolutely certain of what are the essentials of the kind of triangle in question; and we can be certain of these because in geometry definitions have not to be *discovered*. The geometrician can frame his own definitions, and change them, if necessary . . . the mathematician makes his own definitions of what is essential and argues from them. But in Nature the essential conditions have to be discovered and proved. This is the great difference between mathematical and physical induction, and all the difficulties of physical induction result from it."

This explanation of the difference between *metaphysically* or *absolutely* necessary and universal truths on the one hand, and *physically* or *contingently* necessary and universal truths on the other, differs from the scholastic account of them only in one particular, but one which is all-important. Our definitions of the *abstract* objects of thought with which the mathematical sciences

(*quoad se*), yet we may not have grasped the intension of the notions sufficiently to see the necessity of the connexion between them: they may not be clear *to us* (*quoad nos*) (*cf.* MAHER, *Psychology*, p. 289, n. 33; JOYCE, *Logic*, p. 239). The truth of these we learn by *Demonstration*, *i.e.* by gradually tracing their rational, logical connexion with the former ones.

The demonstration of a remote geometrical conclusion is simply the process of showing *how* and *why* it is true, by revealing the rational connexions it has with simpler antecedent truths, and ultimately with first principles (195). This process is essentially *deductive*: a diagram may be necessary in order to help the imagination, and to serve as a concrete illustration or instance: but it is not *from* the diagram, *from* the *instance*, but, through it, from wider and simpler necessary principles that the conclusion is derived. It is only by an improper use of language that this process can be described as "Geometrical *Induction*". *Cf. Palaestra Logica*, pp. 103-104.

[1] It is in the mathematical sciences we find the simplest and most obvious examples of such axioms. *Cf.* MELLONE, *op. cit.*, pp. 265-70. On the nature of mathematical reasoning, *cf.* JOSEPH, *op. cit.*, chap. xxv.; *infra*, 258.

[2] *ibid.* pp. 267, 269.

are concerned are just as really and truly *discovered* by us in Nature, *i.e.* in the world revealed to our senses, as our "physical" definitions, our concepts of the nature and activities of physical agencies, are. They represent *reality* just as surely as the latter do. The "*abstract* objects" which they define are really embodied in the world that is revealed to our senses, *i.e.* in the physical universe. These objects and these definitions are not arbitrary creations of our minds, fictions which we may modify at will. If they were so, the pure deductive sciences would give us no knowledge about *reality*, no *real* knowledge: they would be a mere *dream* about the *unreal*.

Besides the analytic, absolutely necessary, and universal judgments we have just examined, there are, *secondly*, those that we have called physically necessary and universal, and *thirdly*, those that we have described as morally necessary and universal. The judgments of these two latter classes are synthetic; and it is the process by which we reach these—more especially the *physical* truths or laws (201)—that most properly deserves the name of "induction" or "physical induction". The discovery and proof of such laws is the aim of all the physical, natural, or positive sciences; for in these laws lies the *explanation* of the facts and phenomena of those vast domains of sense experience. To determine the *laws* according to which those phenomena happen; to get at the *nature* of the things of experience; to understand phenomena by the *laws* that govern them, and individual things by the *natures* which abide and act in them: such is the ambition of the physical scientist. He sets out from the observation of complex, varying phenomena, to extract from them their common principles and abiding laws: his work is mainly a work of *decomposing*, *dividing*, *analysing*: his method is called *analytic*; and his whole process of ascent from particular facts to general laws is called "*scientific*" or "*physical*" *Induction*.

The doctrine of induction has been developed from, and largely based upon, the remarkable growth of knowledge which the last few centuries have witnessed in the *physical sciences*. In these sciences, especially, it finds its application. From them, therefore, it naturally draws its aptest illustrations, and we need not be surprised to find treatises on inductive logic often read like pages from a handbook on some natural science (201). Nevertheless, it is important to remember that induction is equally applicable to the data of the social, anthropological, and philosophical sciences, as well as to physics: and, moreover, it is only in so far as it is thus universally applicable that it falls strictly within the scope of logic.

207. THE SO-CALLED "INDUCTIVE SYLLOGISM," OR "INDUCTION BY SIMPLE ENUMERATION OF INSTANCES"—"COMPLETE" AND "INCOMPLETE."—Since induction is an ascent from particular instances to general truths, from "some" to "all," it has been rightly described as a process of *generalization*. But we have already repeatedly distinguished between the mere concrete, collective, enumerative universal, and the really scientific universal which is an abstract judgment, embodying some more or less necessary principle or law (92, 195). It is this latter that *scientific* induction proper aims at establishing. Before dealing with this, however, it will be convenient to examine the process by which the collective judgment is reached. This process, too, has been called "induction": "induction *by complete enumeration*," "formal," "perfect": to distinguish it from the other or "scientific" induction, which has sometimes been described as "incomplete," "material," "imperfect". The induction of the collective judgment from a complete enumeration of its constituent instances is "formal" and "perfect" merely by reason of the absolute certitude which we necessarily possess about the sum-total when we have examined all the instances. But to call scientific induction, which attains to the general law by an analysis of *some* instances, "incomplete" and "imperfect," is singularly unfortunate and misleading; for it insinuates that this is a partial application of the former process, that it, too, attains to the universal *by enumeration*, and that its result is "imperfect" or uncertain, inasmuch as the enumeration is "incomplete". As a matter of fact, it does not reach the universal by enumeration at all. This we shall see later on.[1] Let us here examine the process by which the collective judgment is reached: the so-called "inductive syllogism". "Induction by complete enumeration" may be defined as *the process by which we predicate about a whole class or collection of things what we have already predicated of each thing separately*.[2]

[1] *Cf.* JOSEPH, *Logic*, pp. 467-68; *infra*, 209, 211.

[2] Father Joyce (*Principles of Logic*, p. 228) confines this "inductive syllogism" to the "logical parts" (*species*) of a "logical whole" (*genus*). It applies equally well to the "individuals" of a "lowest class," when these are limited in number and can be exhaustively enumerated. Its principle, "Whatever can be predicated of each of the parts successively can be similarly predicated of the whole," is not to be regarded as the reciprocal of the Aristotelean *Dictum de omni et nullo*: for this latter must be interpreted to refer to an *abstract*, not to a *concrete*, universal: and, in passing from the abstract "*M as such*" to the "*All M's*" of the *Dictum*, we postulate the principle to be discussed below (223-25) called the *Uniformity of Nature*. *Cf.* 253-54.

It is described by Aristotle in his *Prior Analytics.*[1] It is the simple summing up of separate instances into an actual collection. He speaks of it as in a certain sense the opposite of the syllogism; Καὶ τρόπον τινὰ ἀντίκειται ἡ ἐπαγωγὴ τῷ συλλογισμῷ, *inductio quodammodo opponitur syllogismo.* The syllogism essentially implies a comparison of two extreme terms (S, P) with a third *middle term* (M). Enumerative induction has no middle term *different from the minor extreme.*[2] The middle term (M) in the syllogism must be, at least once, strictly universal: the corresponding term, which stands as minor extreme in enumerative induction, is not a *strict* universal—applicable equally to an *indefinite* number of realizations—but an *actually* complete collection, a collective, actual whole; and the so-called minor extreme (S), which stands as middle term in enumerative induction, is a consecutive enumeration of the individual instances, equal in point of actual extension to the middle term. An example or two will make this clear:—

S is P	*Saul, David, Solomon were men of remarkable achievements;*
S is M	*Saul, David, Solomon were all kings of the whole of Palestine;*
∴ *M is P*	∴ *All the kings of the whole of Palestine were men of remarkable achievements.*

Or, again, to take Aristotle's own example:[3]

S is P	*Man, horse, mule, etc., are long-lived.*
S is M	*Man, horse, mule, etc., are bile-less.*[4]
∴ *M is P*	∴ *All bile-less animals are long-lived.*

From these examples we can understand Aristotle's definition of the "inductive syllogism" as "proving the major term of the middle by means of the minor," *i.e.* proving the *universal*, which stands as major of the deductive reasoning, "*M is P*"—proving that P can be predicated of the *whole* collection (M)—by predicating P of *each member individually* (S). The class or collection is

[1] *Anal. Prior*, ii., 23 (25). [2] *ibid.*

[3] Aristotle's "individuals are not particular *individual* things, but *species*, which he combines under a *genus*. . . . He regarded an exhaustive summation of the *species* which compose a genus as quite feasible."—Welton, *Logic*, vol. ii., p. 33. *Cf.* Joyce, *Logic*, p. 228.

[4] By bile-less animals Aristotle meant all those species of quadrupeds that have no excess of choleric humours—a list which he considered it quite possible to complete. *Cf.* Joseph, *Logic*, p. 351 n.

symbolized by M as being *greater* in point of *possible* extension than the individuals enumerated (S), though *actually* equal to them, and naturally *less* than the *genus* characterized by the attribute P.

What is to be said about the value of this process?

Firstly, it will not be valid unless the enumeration is complete. The enumeration must be διὰ πάντων, as Aristotle expresses it; else the argument will be fallacious: there will be an illicit process of the subject of the conclusion. St. Thomas likewise insists that as long as we base our conclusion on enumeration the latter must be complete.[1] *So long then as we concentrate our attention on the mere enumeration of instances, and disregard their nature, we can never be certain of our conclusion until we are certain that our enumeration is actually complete:* "opportet supponere quod accepta sint omnia". Now we can practically never be certain, in regard to the occurrence of natural phenomena, that our enumeration of instances is complete: and this is the first obvious limitation of the process as a means of reaching certain knowledge. *Mere "enumerative" induction, then, has only a provisional value.* It enables us to say that so far as our actual knowledge goes, such or such an enumeration may be regarded as complete; that it *is* complete we usually have no warrant to affirm categorically. There are, of course, cases in which an incomplete enumeration of instances may yield a *very high degree of probability* for a *universal* conclusion, *viz.* when we are dealing with phenomena such that if an instance contrary to those examined existed we should in all probability have encountered it. The truth of such a generalization cannot reasonably be doubted so long as no negative instance turns up.[2]

Secondly, even where the enumeration of instances *is* complete, the process does not lead to *scientific* knowledge, *i.e.* the knowledge of a *strictly universal conclusion* embodying what can be called a *law.* And the reason is manifest. The conclusion expresses a simple addition of instances, and is, therefore, simply a collective proposition whose subject is an actual whole; whereas the strict

[1] "Opportet supponere quod accepta sint omnia quae continentur sub aliquo communi; alioquin inducens non poterit ex singularibus acceptis concludere universale. . . . Patet quod inducens facta inductione quod Socrates currat et Plato et Cicero, non potest ex necessitate concludere, quod omnis homo currit, nisi detur sibi a respondente, quod nihil aliud contineatur sub homine, quam ista quae inducta sunt" (*In II. Anal. Post.*, lect. 4).

[2] *Cf.* JOSEPH, *Logic*, p. 491; MELLONE, *op. cit.*, p. 251, referring to Aristotle, *Top.*, viii., 8.

universal proposition, the *abstract* universal, can be reached only by *generalization* of the *abstract* judgment which establishes *some sort of necessary connexion of attributes* between subject and predicate. Adding parts to parts, to form a natural whole, gives us a *collective* idea. Considering an object in the abstract, apart from its individualizing characteristics, putting it into relation with its concrete realizations, actual or possible, indefinite in number, seeing that it is predicable of all, is to *universalize* and to make scientific progress. For "all science is of the universal and necessary";[1] *i.e.* it is expressive of *necessary*, and therefore *universal*, relations between the objects of our thought. The strict universal is no mere actual collection; it is applicable to an *indefinite* number of instances. Therefore, this kind of induction does not put us in possession of *scientific* or *necessary* truth.

It assumes, as we have seen, the external form of a syllogism in the third figure, but it is no more a true syllogistic process than is the apparent syllogism whose premisses contain no true universal, but only collective propositions. In fact it is just the reverse of the process which John Stuart Mill erroneously put forward as the true type of syllogistic reasoning (195).

To observe successively that each of the planets describes an elliptical orbit around the sun, and then to say that *all* the planets describe such an ellipse, is simply to group together isolated observations in a formula to aid the memory, but this is not *ascending from the particular to the universal.* Similarly, to conclude that, because the senses *a*, *b*, *c*, *d*, *e*, are each an occasion of error, therefore *all* the senses are an occasion of error, is certainly not to go through a *scientific reasoning* process: but rather through an arithmetical process which simply tells us that five times one are five.

Examples might be multiplied indefinitely. They all point to the same conclusion: that observation pure and simple puts us in possession of particular facts, and that the grouping together of those facts in a collective notion may help the memory and abbreviate the expression of thought, but will not lead to *scientific knowledge of any necessary truth or law.*

Aristotle distinguished clearly between the formation of an actual whole from its parts and the elaboration of a universal notion: "Even if we succeeded in showing separately," he writes,[2] "whether by the same or by separate proofs, that equilateral, isosceles, and scalene triangles have each their interior angles equal to two right angles, we should not yet have any right to assert the universal proposition: '*The* triangle, *as such*, has its interior angles equal to two right angles'." The separate proofs would not necessarily have given us a *universal* knowledge (καθόλου) of the triangle as such. Hence, we should not yet know whether the attribute, "*having their interior*

[1] Ἡ μὲν ἐπιστήμη καθόλου καὶ δι' ἀναγκαίων.—ARISTOTLE, *Post. Anal.*, i., 33.
[2] *Post. Anal.*, i., 5 (5-7).

angles equal to two right angles," belonged to the triangle *as such*, and, therefore, *to all possible triangles.*

Nor, even when we see that the three species, equilateral, isosceles, and scalene, are exhaustive of the *genus* triangle, can we be said to know scientifically that the latter *as such* has the sum of its interior angles equal to two right angles: unless we have proved this attribute to belong to each of the three species, not on different grounds peculiar to each case, but on *some common ground* inherent in their common nature *as triangles*: "εἰ ταὐτὸν ἦν τριγώνῳ εἶναι καὶ ἰσοπλεύρῳ ᾗ ἑκάστῳ ἢ πᾶσιν—si eadem sit τοῦ esse ratio triangulo et aequilatero, aut cuique trianguli speciei aut omnibus."[1] In order that such a conclusion be anything more than an *enumerative* judgment "it would be necessary to show that the reason for the inherence of *P* is the same in regard to all the parts of *M*".[2]

But mere enumeration of the individuals of species (or of the species of a genus) cannot of itself reveal to us anything in their common nature to serve as a sufficient and necessary ground for predicating any attributes found in all the examined individuals (or species), about the species (or genus) as such.

That Aristotle was acquainted with the true method of arriving at such a scientific or necessary knowledge of the nature of things we shall presently show (208). That he realized the inability of an incomplete enumeration as such to *prove* a really *general* principle, is manifest from what he says of the so-called "inductive syllogism" described above. When he speaks of it as a way of "*proving* the major term of the middle by means of the minor"[3] *i.e.* of proving the universal principle "*M is P*," "If anything is *M* it is *P*," which stands as major in the demonstrative syllogism in the first figure, he does not mean "proving" in the strict sense of *demonstration* (*ἀπόδειξις*), for strict demonstration is always by syllogisms in the first figure. He only means that the inductive syllogism is a way of *illustrating*, making clearer by instances or examples (δηλοῦν; πιθανώτερον, σαφέστε-

[1] *ibid.*, (6).

[2] JOYCE, *Logic*, p. 229. The author observes that Euclid is usually able to do this in cases where he proves successively that something is true of each of all the possible instances of a logical whole. *Cf.* JOSEPH (*op. cit.*, p. 503): "The peculiar nature of our subject-matter [here] enables us to see that no other alternatives are possible within the genus than those which we have considered; and therefore we can be sure that our induction is 'perfect'. The nature of our subject-matter further assures us that it can be by no accident that every species of the genus exhibits the same property; and therefore our conclusion is a genuinely universal judgment about the genus, and not a mere enumerative judgment about its species. We are sure that a general ground exists, although we have not found a proof by it." No doubt, if we are assured that the species exhibit the same property, "by no accident," our conclusion is universal; but, even then, we only know *that* it is so, not *why* it is so: until we can "show that the reason for the" property "is the same in regard to all" triangles.

[3] *Anal. Prior*, ii., 23, (25).

ρον, ποιεῖν),[1] the general principle. "It is a mode of arranging a deductive argument so as to enable us to realize psychologically, the truth of the general principle (ἀρχή) which is the real major premise—a mode of illustrating the principle by bringing forward instances. Of course we cannot get 'all' the instances, except where the number is limited; but this fact does not vitiate an illustrative 'induction' such as Aristotle had in view (*cf. Anal. Post*, i., 4, 73*b* 33)."[2]

If, therefore, Aristotle regarded the conclusion of any enumerative induction as a strict, generic universal, he regarded the knowledge of this as reached not by enumeration, but by analysis.[3]

As long as we have any doubt about the completeness of our enumeration—which is nearly always,—and still rely *on it alone* for our conclusion, we can only have *provisional* and *probable*, not *absolute* and *certain*, knowledge, of the truth of the latter as a really general proposition. But both the process and the conclusion have in such cases this amount of utility, that *they suggest* to us, more or less forcibly, the existence *of some natural law, i.e. some necessary natural connexion* between the attribute predicated and the class of things in question. When we find that *a, b, c, d, e,* are *P*; and know already that *a, b, c, d, e,* are *S* (whether all *S* or only some *S*, does not matter much), the surmise inevitably suggests itself that there may be something (say *M*) *in the nature of S* (and therefore in *all S's*, whether examined or not) which is the natural ground for P. In other words, the conclusion "*Every S may, in virtue of the M that is in it, be P*" suggests itself *as an hypothesis* worthy of investigation. Thus, our attention is drawn away from the *number* of *S's*; and the tendency asserts itself not to aim at completing the enumeration—which is usually impossible,—but to examine the *nature* of the phenomena in question, (the *S's*), and to seek in them for *some natural attribute or property* (*M*) that will be the ground or reason for our predicating *P* of them. This marks the passage to *scientific induction*, whereby we are able, *without a complete enumeration of instances*, to rise from particular facts to the conception and discovery of some universal natural law.

208. Scientific Induction as Treated by Aristotle and the Mediaeval Scholastics.—We have seen that the *general* conclusion, when derived from an *incomplete* enumeration

[1] *Cf. Anal. Prior*, ii., 23; *Top.*, i., 12; *Anal. Post*, i., 31.

[2] Mellone, *op. cit.*, p. 247.

[3] *Cf.* Joseph, *Logic*, pp. 356-57.

of instances, is never certain, and that such induction is called "imperfect". We find it sometimes stated by modern logicians that the only way of ascending from the particular to the general, explicitly treated by Aristotle, and the only way known to the mediaeval Scholastic logicians, was that of *enumerative* induction, "complete" and "incomplete"; that we find in these authors no trace of the method of *modern scientific induction*, the method of attaining to the universal by *analysing* a limited number of instances and seeking therein a connexion of *content*, of *attributes*, a *causal* connexion, in the *nature* of the phenomena considered. Thus, Professor Welton writes:[1] "The scholastic logicians . . . made the essence of induction to consist in enumeration"; and Dr. Mellone:[2] "With the mediaeval logicians induction became simply a process of counting particular things". And these authors merely give expression to a traditional misconception, the origin and growth of which are clearly and succinctly accounted for by Father Joyce, in his *Logic* (p. 233):

> "The error seems to have arisen from the fact that the most famous of the Scholastics (St. Thomas, Albert the Great, Scotus) do not employ the term induction as the distinctive name of the inference by which we establish universal laws of nature. Following the terminology of Aristotle . . . they called it proof from experience (*ἐμπειρία, experimentum, experientia*). The significance of the term induction was somewhat vague. It covered all argument from the particular to the general [*cf.* 206]. Hence (as *e.g.* in Scotus, *Anal. Prior.*, ii., q. 8) it might include this meaning among others. But it was more usually employed to denote the formal process of perfect induction [207] arranged as an inductive syllogism. Moreover, it was sometimes pointed out, that our argument might be thrown into the form of an inductive syllogism: for, though the enumeration was incomplete, yet in these few instances we have equivalently seen all [*cf. infra*, 209]. It was by a later generation that the term induction was restricted to its present signification. Incautious readers, finding in certain passages the inductive syllogism described as the formula of inductive argument, jumped too hastily to the conclusion that the mediaeval philosophers rested their knowledge of the laws of nature on no basis but enumeration."

Now, from the very fact that Aristotle and the Scholastics considered it possible to reach a truth about "*all*," actual and possible, known and unknown, by an acquaintance with "*some*," they must have recognized a method of ascent to the "*all*," other than enumeration. And so they did: *viz.* the method nowadays known as *Physical or Scientific Induction.*

When, therefore, we hear it stated that *Scientific Induction* is

[1] *op. cit.*, p. 33.

[2] *op. cit.*, p. 247.

an achievement of the modern mind, we must not infer that it was entirely unknown to the ancients. That to modern thought the honour was reserved of seizing upon the full significance of the method, and of applying it with such marked success, even the most ardent defenders of Aristotle and the Scholastics need not deny.[1] But that the principle of this method was known to the latter, their works give unmistakable evidence.

And, firstly, let us turn to Aristotle himself:—

"Repeated sensations," he writes, "leave impressions in the memory, and these engender experience (ἐμπειρία); experience suggests abstraction, which separates from the particular instances *the one in relation with the many* (τὸ ἓν παρὰ τὰ πολλά), that is to say, the *universal*. But the abstract put in relation with an indefinite number of individuals, is a principle of science and of art".[2]

Turning, now, to St. Thomas's full and lucid commentary[3] on the passage just quoted, it would be difficult to find a plainer illustration of the modern inductive *Method of Agreement:* A physician has learned by repeated *experiences* that a certain herb has cured several patients of fever. From these experiences he ascends to the apprehension of the *universal principle* that "*this kind of herb* cures *patients afflicted with this kind of fever*". St. Thomas does not explicitly state the principle, or examine the process by which the ascent is made; obviously, however, it is *not* made by enumeration of instances, complete or incomplete.

[1] Ueberweg rightly remarks that "The recognition of the full significance of the inductive method in the sciences was reserved for modern times" (*System der Logik*, § 127).

[2] *Post. Anal.*, ii., 19, (5).

[3] "Ex memoria multoties facta circa eamdem rem in diversis tamen singularibus, fit *experimentum:* quia experimentum [ἐμπειρία] nihil aliud videtur, quam accipere aliquid ex multis in memoria retentis. Sed, tamen, experimentum indiget aliqua ratiocinatione circa particularia, per quam confertur unum ad aliud, quod est proprium rationis. Puta, cum talis recordatur quod talis herba multoties sanavit multos a febre, dicitur esse experimentum *quod talis herba sit sanativa febris.* Ratio autem non sistit in experimento particularium; sed ex *multis* particularibus in quibus expertus sit, accipit *unum commune quod firmatur in anima*, et considerat illud absque consideratione alicujus singularium, et *hoc accipit ut principium artis et scientiae.* Puta, diu medicus consideravit hanc herbam sanasse Socratem febrientem, et Platonem, et multos alios singulares homines; cum autem sua consideratio ad hoc ascendit quod *talis species herbae sanat febrientem simpliciter*, hoc accipitur ut quaedam regula artis medicinae" (St. Thomas, in *loc. cit.*). It will be observed that there is no mention here of "*Inductio*" but only of "*Experimentum*". It is significant, too, that these passages from Aristotle and St. Thomas are from the *Posterior Analytics, i.e.* from that part of the *Organon* which treats of *Certain Science*, while the passages quoted above in reference to enumerative induction—complete and incomplete—are taken from the *Prior Analytics* and the *Topics, i.e.* the parts that refer, the one to the *formal side* of reasoning, the other to *probable* arguments.

But another leading Scholastic, Duns Scotus, has analysed with a good deal of precision the procedure by which the generalization is effected. When a phenomenon occurs repeatedly under the influence of a cause that is not free, we must conclude, he teaches, that the effect in question has a "natural" connexion with the cause. . . . For it is impossible that a *necessary* cause produce the same effect *regularly*, unless it is *determined* by its *natural tendency*—its *directive* principle or *form*, as he calls it—to produce this effect. The effect must spring from the *nature* of that cause and not from any accidental, concomitant agencies; for accidental agencies do not produce regular effects. And that any such regular series of effects is due to the *nature* of a certain cause, we know from *experience*: because we have seen this cause followed by these effects, when acting now in one set of conditions, again in a different set, and altogether in many varieties of circumstances.[1] Thus, Scotus points out as the rational, self-evident basis of induction, the judgment that *what* REGULARLY *results from the action of* NON-FREE *causes cannot be the result of mere* CHANCE, *but must have a necessary connexion with the* NATURE *of those causes*; and he furthermore points to the necessity of *varying our experiences*, in order to separate, from the changing and accidental circumstances that accompany the appearance of the phenomenon in question, the one agency or group of agencies on which it is really dependent, which forms its *real cause*: a plain application of the modern Method of Agreement.

Why, then, it may be asked, did the Scholastics of the Middle Ages, if they knew the theory of scientific induction, and the principle underlying it, not proceed to apply the method, and so anticipate by centuries the wonderful

[1] "De cognitis per experientam dico, quod licet experientia non habeatur de omnibus singularibus, sed de pluribus, nec quod semper, sed quod pluries; tamen expertus infallibiliter novit, quod ita est, et quod semper et in omnibus; et hoc per istam propositionem quiescentem in anima: QUIDQUID EVENIT UT IN PLURIBUS AB ALIQUA CAUSA NON LIBERA, EST EFFECTUS NATURALIS ILLIUS CAUSAE. Quae propositio nota est intellectui, licet accepisset terminos ejus a sensu errante, quia *causa non libera non potest producere ut in pluribus effectum*, ad cujus oppositum ordinatur, *vel ad quem ex forma sua non ordinatur* . . . sed causa *casualis* ordinatur ad producendum oppositum effectus naturalis, vel non ad istum producendum, ergo *nihil est causa casualis respectu effectus frequenter producti ab eo*, et ita si non est libera, est *naturalis*. . . . Quod autem iste effectus evenit a tali causa producente ut in pluribus, hoc acceptum est *per experientiam*; quia inveniendo *nunc* talem naturam *cum tali accidente*, *nunc cum tali*, inventum est, quod, quantumcumque esset *diversitas accidentium talium*, semper istam naturam sequebatur talis effectus. Ergo non per aliquod accidens, per accidens illius naturae, sed *per naturam ipsam in se* consequitur tallis effectus" (*In. I. Sent. dist.* iii., *Q.* iv, 9).

strides which physical science has made since the Renaissance? Many good reasons may be assigned.

One is that in those ages philosophers were more preoccupied with the philosophy of mind than with that of external nature, with the application of reason to principles accepted on authority, with the explanation of revealed religion and the unfolding of the contents of the Divine deposit of Revelation by means of philosophical principles and methods (203). And, as the full meaning and proper understanding of those great truths and principles are arrived at by the application of the deductive or synthetic method, the attention of those philosophers was not arrested by the possibilities of knowledge that might have been opened up through a more careful analysis of the complex phenomena of external nature.[1]

But another, and more important, consideration is that they had not the means of prosecuting such an analysis. They knew the method theoretically, but this knowledge in itself was of little use. When there is question of establishing a *law of Physical Nature*—such as the laws of the planetary motions, or of the refraction of light—it is not enough to know that "a *non-free* cause cannot *regularly* produce an effect that is opposed to its *natural tendency*, an effect it is not determined *by its nature* to produce,"—"causa *non libera* non potest producere *ut in pluribus* effectum, ad cujus oppositum ordinatur, vel *ad quem ex forma sua non ordinatur*". This abstract, hypothetical principle merely asserts that *if* "necessary" or "non-free" causes exist, causes predisposed by an internal tendency ("*forma*") to produce definite effects, the latter will occur with the regularity of a "law"; but it does not of itself authorize us to assert *categorically* that there *are* such internal tendencies or principles of finality in nature, that there *are* causes predisposed to manifest such fixed, unchanging activities (*cf.* 223); and still less to affirm with *certainty* that *this or that oft-observed combination of particular phenomena is the expression of some one of those causal tendencies* existing in nature.

Such a categorical conclusion as the latter can be justified only by a diligent observation of the natural phenomena to which it refers. And nature is infinitely complex: so that the establishment of a certain conclusion that *this* series of phenomena reveals *this* universal physical *law*, necessarily presupposes a detailed and accurate weighing, reasoning, analysing, and comparing of all the elements that enter into the phenomena in question. The phenomena of physical nature exist in space and time: accurate quantitative measurement is, therefore, at the basis of all experimental research: and hence, *the discovery of instruments for delicate measurement* was an indispensable condition for the progress of the physical sciences. But Aristotle and the mediaeval Scholastics had neither the clock for the accurate measurement of time, nor the balance for the exact estimation of weight, nor the thermometer for measuring temperature, nor the barometer for measuring atmospheric pressure, nor the telescope to observe the heavens, nor the microscope to reveal the mysteries of the minute structure and composition of organic tissues. It is true, indeed, that the sagacity of great genius, the patience of long reflection, and disinterested zeal in the pursuit of truth, can contribute much, even with the aid of mere ordinary observation, to the development of scientific speculation: witness the wonderful perfection of the Ptolemaic

[1] *Cf.* CLARKE, *Logic*, p. 480.

astronomy. Indeed the superior powers of Aristotle, and of his mediaeval Christian commentators, in the domain of ordinary, unaided observation, are undisputed at the present day. But it would be wrong to arrogate to them an honour they would be themselves the first to disclaim,—the honour of creating sciences which could not possibly have arisen without the invention of the special instruments of observation and measurement just referred to.

Accurate experimentation was impossible in the Middle Ages, in the absence of those delicate means of weighing and measuring that are the invention of a more modern era. The thirteenth century, however,—the golden age of Scholasticism—produced at least one exceptional and extraordinary man, whose name cannot be passed over in connexion with the rise of scientific induction. Roger Bacon, a Franciscan monk, who lived through the greater part of that century, dying at Oxford in 1294, rose far above the commonplaces of his time in his advocacy of the experimental method. His life was one impassioned and even fanatical plea for the positive sciences. Nor did he content himself with pleading: he set an example by devoting his great genius to conducting scientific experiments and inventing instruments for that purpose.

He distinguished four possible ways of gaining a knowledge of nature: authority, (*a priori*) reasoning, observation, and experiment. And he tells us that of these four the first ranks lowest in worth: "*auctoritas debilior est ratione*"; the second, dialectic reasoning, does not satisfy the mind: "*non certificat*"; nor the third, which is ordinary, superficial observation. The fourth alone—"internal" or "intrinsic" experience—is convincing, and that owing to the aid it receives from mathematics and geometry. He anticipated more renowned and more modern philosophers in an attempt to establish one general science that would submit to mathematical principles all the varied interactions of the bodies that make up the physical universe.[1]

209. LORD BACON'S "NOVUM ORGANON": THE TWO IDEALS OF GENERALIZATION.—The English monk of the thirteenth century understood the nature and method of experimental science as well as, if not better than, his namesake of the sixteenth. Francis Bacon, Lord Verulam (1561-1626), is commonly regarded as the "founder of the inductive method". Wrongly, however; because, in the first place, his method of "interpreting nature" has never been adopted: "The value of this method," writes Jevons,[2] "may be estimated historically by the fact that it has not been followed by any of the great masters of science."

Bacon blamed his predecessors, the "deductive" philosophers, for "anticipating" nature instead of "interpreting" it. After enumerating four great sources of such fallacious "anticipations"—the "*Idola*" or *Phantoms*: (*a*) *of the Tribe* (common to all men), (*b*) *of the Cave* (due to personal idiosyncrasies), (*c*) *of the Market-*

[1] *Op. maj.*, p. iv., dist. i., c. iii., dist. ii.-iv.; *Opus tertium*, c. 29-37, etc.; *cf.* DELORME, *Dictionnaire de théologie catholique*, *s.v.* *Bacon*.

[2] *Principles of Science*, p. 507.

Place (due to public catch-cries, shibboleths, etc.), (*d*) *of the Theatre* (due to fashion)—he goes on to expound his own "method". He appears to have regarded all physical phenomena as collections and combinations of sensible properties of matter, each of the latter being a simple thing, a "simple nature," and each due to some "form," *i.e.* to some essential constitutive principle[1] of the material agencies in which such sensible properties are revealed. This is merely a statement of the scholastic principle that the properties of an agent reveal its specific nature or "formal cause". But Bacon conceived it to be the duty of the scientist to draw up a complete catalogue of all the sensible properties exhibited throughout all nature, and of all the "forms" to which these could be due: an utterly impracticable undertaking. Next, in order to facilitate the process of tracing each property to its "form," or cause, tables or catalogues were to be drawn up, exhibiting the relations of conjunction or concomitance, separation, and variation, between the forms and the properties: a still more arduous and unpromising task. Bacon never attempted to carry out these schemes himself. The first grave defect of his "method" is, therefore, its inutility.

Next, assuming the possibility of compiling such data, he pointed out that the cause, or "form," of a given sensible property could be best detected by a process that would successively *eliminate* all the other rival "forms," and thus bring to light the proper one. *Every "form" which is present when the property in question is absent, or absent when the latter is present, or which does not increase and decrease concomitantly with the latter, is to be rejected as not being the "form" causally connected with the latter.* Such is the principle on which the method proceeds, the principle of *elimination*, or *exclusion* of the non-causal or casual concomitants of a phenomenon. It is theoretically sound: "where you cannot (as in mathematics) see that a proposition *must* universally be true, but have to rely for the proof of it on the facts of your experience, there is no other way of establishing it than by showing that facts disprove its rivals".[2]

[1] The tendency of the science of Bacon's time to substitute for the *qualitative* conceptions of the Scholastics, *quantitative, picturable, measurable* conceptions, is revealed in his changing and uncertain ways of conceiving "form". He appears to have finally fixed upon the notion of something measurable in terms of "spatial and temporal relations of bodies" (WELTON, *op. cit.*, ii., p. 36), something which has been described in present-day scientific language as a "principle of corpuscular structure" (JOSEPH, *Logic*, p. 364).

[2] JOSEPH, *op. cit.*, p. 366.

We shall see this principle applied later on by J. S. Mill, and universally adopted. But, for its safe application we do not need, as Bacon taught, to have antecedently elaborated a completely exhaustive catalogue of all the "forms" and "sense-qualities" in the universe. It will suffice to eliminate all the possible *pertinent* alternatives, suggested by a careful analysis of the matter under investigation.

But, owing to the enumerative character of the process as conceived by Bacon, and to the practical impossibility of a complete enumeration of the alternative factors involved, the process, so applied, could never reach a *necessary* and *universal* law: for (to use Bacon's own words) when "the axiom being established is more extensive and broader" than "those particulars out of which it is extracted" (*Novum Organon*, i., 105, 106)—and this is what happens as long as his impossible "catalogues" are not complete and absolutely reliable—he fails to indicate any principle (other than enumeration) which might justify him in drawing a *universal* conclusion from such a defective enumeration of alternatives. This, then, is a second serious defect of the "method".

Next, it must be pointed out that although he says "Induction which proceeds by simple enumeration is a puerile thing and concludes uncertainly" (i., 105), and that "the syllogism is not applied at all to the principles of science" (i., 13), yet, as a matter of fact, his whole process is a simple application of the categorical syllogism in the second figure, combined with the *modus tollendo ponens* of the mixed disjunctive syllogism; in which latter, moreover, the disjunctive major is *assumed* to be complete, *even though his catalogues of forms and qualities remained incomplete throughout.* Bacon's own example will illustrate this.

Let f be the "form" of heat (the "form" we are endeavouring to detect or select from among all the known "forms"). Let h represent the sensible quality of heat. Let A, B, . . . Y, Z, represent the whole collection of "forms" or "natures" in the universe. Then:

f is either A or B or C or . . . X or Y or Z;

(1) But A is not present with h,
And f is present with h,
Therefore f is not A;

(2) And B is present in the absence of h,
While f is not present in the absence of h,
Therefore f is not B;

(3) And C does not vary concomitantly with *h*,
While *f* does vary concomitantly with *h*,
Therefore *f* is not C ;

and so on, until all the "forms" except one, say Z, are thus eliminated by syllogisms in *Cesare* or *Camestres*. Then we have, finally, this mixed disjunctive argument in the *modus tollendo ponens*, verifying the form of heat as Z :—

f is either A or B or C or . . . X or Y or Z ;
But *f* is neither A nor B nor . . . X nor Y ;
Therefore *f* is Z.

This latter form of argument is regarded by many as the typically "inductive" inference.[1] The various arguments, (1), (2), and (3), by which we apply various methods of elimination, suggest that instances (of the circumstances accompanying heat) are no longer being merely enumerated, but that the nature of their connexion (with heat) is being sifted by experiment. This marks the transition from enumerative to scientific induction.

As was pointed out already, two possible tendencies may arise from an incomplete enumeration of instances. The *first*, with which many of the Scholastics, and Bacon himself, seem to have been preoccupied, is to realize, somehow or other, the ideal of a complete enumeration. To realize it *actually* is, for the most part, chimerical, and moreover, it does not lead us to the true universal. To realize it *virtually*, *i.e.* by falling back on some rational principle which might justify us in saying : "and so on of the unexamined instances"—"*et sic de ceteris*"—is to yield in reality to the *second* tendency, while under the traditional sway of the first : the second tendency being to abandon the mere *enumeration* of the instances, to concentrate attention on their *material* side, on the *quality*, the *nature* of the facts we are dealing with, and to ask ourselves : Is it not possible and permissible to rise to the conception and enunciation of a strictly *universal* physical law from an examination of *some* instances only ? It is possible to do so ; and the difficulty of the process of physical induction, by which we accomplish this ascent, is not a difficulty of *principle* or *method*, but rather of *application* : it is a difficulty that belongs not to the *logical*, but to the *practical*, order.[2]

The ancient Greek philosophers, and the Scholastics of the Middle Ages, were quite as well aware as any modern exponent

[1] *Cf.* 197 ; JOSEPH, *op. cit.*, p. 405. [2] *Cf.* JOYCE, *op. cit.*, p. 217.

of the inductive method that (1) *Explanation* of the phenomena of physical nature consists in a thorough knowledge of their *connexion with their respective causes*; (2) that physical causes act *regularly, uniformly*; (3) that, therefore, if we could be sure of having discovered and fixed upon the *natural cause* of a given phenomenon, amid all its complex surroundings (by a process of *abstraction*), we could at once (by *generalization*) formulate the *physical law* that *always and everywhere this cause will act in the same way and produce this same phenomenon.* But the difficulties that beset the work of bringing to light with certitude the causal connexion—the work of observing, analysing, experimenting, etc.—were so great that neither the ancient nor the mediaeval nature-philosophers had the courage and perseverance to grapple with them. Hence, they made little or no serious effort to test the worth of the probable conclusions which they based upon an incomplete enumeration of superficially observed instances. The scientists of the seventeenth and eighteenth centuries, Galileo, Torricelli, Pascal, Descartes, Newton, etc., were making *practical* efforts in many directions to scrutinize and question physical nature more closely, long before logicians attempted to formulate and interpret the *theory* of these researches. Lord Bacon's attempt at the conscious formulation of a theory was a failure. Sir Isaac Newton (1642-1727) was conspicuously successful both in theory and in practice. Since the latter's day, many workers, both in the natural and in the mental sciences, have sought to formulate the *Theory* of inductive research. But those sciences are so progressive in their methods, and views so fundamentally divergent as to the nature of knowledge are propounded by philosophers, that there is still comparatively little uniformity of treatment in the domain of inductive logic.[1]

210. MODERN CONCEPTIONS OF INDUCTION: NEWTON, WHEWELL, J. S. MILL, JEVONS.—Newton taught that in the pursuit of knowledge we must start with an *analysis* of *observed* facts: that we must *suppose* and formulate some *general law* suggested by these facts: that we must, by *synthetic* or *deductive* reasoning, derive consequences from this law, thus to determine whether the law coincides with all observed facts or not.

Indeed, most writers on induction agree in recognizing certain well-defined steps or stages in our progress from particular facts

[1] For varieties of treatment *cf.* VENN, *Empirical Logic*, pp. 353 *sqq.*; WELTON, *op. cit.*, ii., bk. v., chap. ii.

to general laws: the formulation of an hypothesis suggested somehow or other by an initial observation of phenomena; the moulding, remodelling, generalizing of this hypothesis, by successive eliminations and exclusions—under the guidance of certain canons of more or less practical utility, commonly known (after J. S. Mill) as the "experimental methods" or "inductive methods"; the final "verification" or "establishment" of the hypothesis as a "law"; the attempted "explanation" of this law by wider laws; the commencement of the synthetic or deductive stage by the application of the established law to particular facts for the "explanation" of these latter.

Not all writers, however, attach the same importance to the various stages. Whewell,[1] for instance, lays great stress on the invention of hypotheses, or, in his own language, the "colligation of facts by means of an exact and appropriate conception,"[2] as the most important step in the *discovery* of scientific truths. To the subsequent process of generalizing the abstract hypothesis, of remoulding and remodelling and verifying it by the application of fixed canons, he devotes much less attention. Its *verification* or *proof* he holds to consist in *deducing* consequences from it, and ascertaining whether it thus *foretells* phenomena, at least those of the same kind as the phenomena for the explanation of which it was invented. Should an hypothesis, invented to explain "*one class* of facts," be also found "to explain *another class* of a different nature," it is more firmly established than by any other means: this Whewell calls *Consilience of Inductions*.[3]

J. S. Mill, on the other hand, almost entirely ignored the theory of the initial step of conceiving an hypothesis. He addressed himself to the process of generalizing directly from particulars—a process quite impossible apart from the abstract conception of some guiding hypothesis,—and to the establishment of rules or canons for the correct carrying out of this process. No doubt, this latter stage lends itself to methodical treatment, while the former stage does not; and Mill tried to justify his mode of treatment by the plea that as a logician he was concerned only with the *proof* of general truths, not with their *discovery*. There does not seem to be much force in such a plea.

[1] Flourished 1794-1866; among his writings are the *History of the Inductive Sciences* (3 vols., 1837; 2nd edit. 1847; 3rd, 1857) and a *Philosophy of the Inductive Sciences* (2 vols., 1840; 2nd edit. 1847; 3rd, in three vols., bearing separate titles, of which one was called *Novum Organon Renovatum*, 1858-60).

[2] *apud* WELTON, *op. cit.*, ii., p. 50. [3] *ibid.*, p. 51.

A truth cannot be said to be discovered in the full and complete sense of the word until it is thoroughly verified, or proved to be a truth. It may be formulated and held as true with more or less probability—as the result of an enumerative induction or of an analogical argument, as an hypothesis or as an "empirical generalization,"—but it cannot be fairly said to be fully "discovered" until we have both "verified" it, or proved *that* it is true, and "explained" it, or shown *why* and *wherefore* it is true, by connecting it necessarily with already known and established truths. In any case, Mill *de facto* regarded his "inductive methods" as methods of *discovery* as well as of *proof*, and described induction itself as "the operation of discovering and proving general propositions".[1]

Finally, we may mention Jevons,[2] as an author who takes a thoroughly enumerative view of induction, making the whole process consist in a successive enumeration and determination of all the mathematically possible hypotheses that might account for a given result or phenomenon. Obviously, in this view *certitude* about our inductive conclusions is practically never attainable, for the ideal of a perfect enumeration of instances is beyond our reach.[3] The method is open to the same objections as Bacon's method, which Jevons himself criticizes. In the "infinite ballot-box" of nature, the determination of the chances of an invariable sequence of any two "balls" is a problem in the mathematical theory of probability, the solution of which cannot of its nature give us certitude (267). Nor can the result of this "inverse problem" be made any more certain or definite by *arbitrarily* limiting the elements in the antecedent to those contained in the consequent, nor, indeed, by *arbitrarily* limiting them in any way. "If, for instance, we . . . say: Given that certain combinations of A, B, and C, are the existent ones, find a solution in terms of A, B, C, and nothing else, from which this result shall follow, no complaint can be made. The problem is a very limited one, but it may be useful. . . . But to make the same restriction when the problem is, Given that dew is copious on a cold, clear night, or given that a magnetic needle is deflected by an electric current, find a solution which shall introduce no fresh terms into the statement of the phenomena, would be a mere parody of physical investigation."[4] Indeed, the only way of reaching certitude is that precluded by Jevons's view of induction; namely, by abandoning the *enumerative* ideal and addressing ourselves to an *analysis of the nature* of the phenomena in question, on the rational assumption that constant coexistences or sequences of phenomena have their explanation in the existence of *fixed natures*, *of stable tendencies* and lines of action, in the phenomena themselves, and with the conviction that these fixed

[1] *Logic*, iii., I., § 2.

[2] Flourished 1835-1882; Logical writings: *Principles of Science* (2 vols. 1874; 2nd edit. 1 vol. 1877); *Elementary Lessons in Logic* (1870); *Primer of Logic* (1876) *Studies in Deductive Logic* (1880).

[3] *Cf.* JOSEPH, *Logic*, p. 487.

[4] VENN, *Empirical Logic*, pp. 360, 361.

natures, these stable tendencies, have their ultimate explanation in the omnipotent will of an all-wise ruler of the universe (224).

The views of an individual author on these latter *ultimate presuppositions and foundations* of induction are pretty sure to influence and colour his conception of the various steps in the mental process itself by which the mind moves inductively from particular to general, from fact to law. Some such views, propounded by recent writers, will be examined in due course.

211. ANALYSIS AND ILLUSTRATION OF THE PROCESS OF SCIENTIFIC INDUCTION.—From the preceding paragraphs of the present chapter we can gather what the main problem of induction is, and what its method of procedure ought to be. It seeks a *scientific* knowledge of the concrete, particular phenomena of our experience, *i.e.* a knowledge of them through their *causes* and *laws*, a knowledge, which, bringing to light their nature, their origin, the purpose of their existence or occurrence, will lay hold of what is universal, permanent, abiding, in them. Amid the changing and chaotic elements that make up our world of unanalysed and unexplored sense experience, induction will try to trace the permanent connexions of cause and effect, to eliminate the variable conditions and surroundings of each phenomenon, and to lay bare its connexion with its real ground or cause. Now, in order to do this, we must not merely *observe* with accuracy the phenomenon we wish to explain; but next, and necessarily, we must *suppose* that amid all its immediate conditions and surroundings some element or elements constitute its determining cause, and yield the law of its occurrence; and then we must proceed to *test* or *verify* our supposition by *deducing* consequences from the latter, and comparing our conclusions with actual facts, analysed by further observation and experiment. This process of testing we must prosecute until we reach a full conviction that the supposed cause of the phenomenon is the necessitating and indispensable, and therefore the true or real, cause, of the facts examined. When we have thus established an isolated law, we may on the one hand endeavour to *explain* this law itself by seeking its connexions with other already known laws, and on the other hand apply the law itself to the *explanation* of all facts that come under it.

(1) Preliminary observation of facts; (2) supposition as to their cause; (3) verification of our supposition; (4) explanation, and (5) application, of the established law: such are the essential steps in the inductive discovery and proof of scientific truths. The deductive application of the general law to the facts is the

final step, by which we reach a *scientific* knowledge of those facts in the observation of which the whole process had its origin.[1]

The method here outlined is recognized by Mill:[2] he calls it "deductive," admitting its application only to the more complex phenomena due to a combination of causes; yet he is forced to allow that it is to this method "the human mind is indebted for its most conspicuous triumphs in the investigation of nature".[3] The advocacy of this method by many of our more recent inductive logicians, Bosanquet, Sigwart, Welton, Joseph, etc., is a wholesome reaction against the Empiricism of the school of Mill.

The various steps indicated above will form the subject-matter of subsequent chapters. The whole process, however, is based upon certain fundamental principles and postulates which call for explanation and justification at the outset (Chaps. III. and IV.). With an example[4] to illustrate the inductive process, and a comparison of the latter with deductive inference, we may conclude the present chapter.

"Let a chemist take some *hydrogen*, a gas without colour, taste, or smell; which burns with an intensely hot bluish flame; which is 14·4 times lighter than air, 23·326 litres weighing 2 grammes. Let him take another and very different sort of gas, *chlorine*; of a yellowish colour and an unpleasant, suffocating smell; density 2·44, weighing 35·5 times more than hydrogen, 22·326 litres weighing 71 grammes.

"Let the chemist mix those two gases in a glass vessel, and place it in the sunlight: a violent combination will suddenly take place, disengaging 22 thermal units or calories of heat; after which the chemist finds in the vessel a new body, whose distinctive properties have acquired for it the name of *hydrochloric acid.* This new body will attack most of the metals and combine with them to form various salts; it will combine with the aqueous vapour of the atmosphere to form a colourless, acid solution, etc.

"So far he has *observed a fact* [first step]. Next, how is it to be explained? Why did it happen? What is its cause? He *supposes* that it is due to some *law* of nature [second step]; he supposes the formation of hydrochloric acid to be due to some *property* inherent in those two gases, acting in certain conditions, still to be determined. This suspicion of his is an *hypothesis*, which he must now proceed to verify.

"For this latter purpose [third step] he will multiply and vary his experiments. For example, he will let the sunshine act on a mixture of chlorine and oxygen; supposing *a priori* that they too will combine; but he finds that they will not. It is not every two gases, therefore, that will combine under the action of the sunlight. But, perhaps, at least any quantities whatever of hydrogen and chlorine will combine? *A priori*, again, the supposition is permissible; but again it is negatived by the facts. For repeated experiments

[1] *Cf.* 252: *Regressive Demonstration.*
[2] *Logic*, iii., xi. and xiv.
[3] *ibid.* xi., § 3.
[4] From Mercier's *Logique*, pp. 300 *sqq.*

establish that they will combine only in the proportion of 1 to 35·5 by weight, or—which is the same—of 1 to 1 by volume. When those proportions are brought together under the influence of sunlight—no matter how little or great the absolute quantities may be, milligrammes, centigrammes, decigrammes—the combination will take place. On the other hand, when those proportions are not maintained, the quantity of the one which is in excess of its due proportion to the total quantity of the other, will remain over, unaffected by the combination.

"Here, then, are other facts in presence of which the observer finds himself: Two definite gases, mixed in definite proportions of 1 to 1 by volume or 1 to 35·5 by weight, combine under the action of sunlight—the absolute quantities of each being indifferent to the result and indefinitely variable. Neither of these gases, mixed with any other gases, combine with the latter in the same conditions and proportions; if mixed with each other in any other proportions than those indicated, they will not combine completely, but will leave the surplus above the proportion unmolested. Further, the chemist remarks that, after the combination, *one* volume of hydrogen and *one* volume of chlorine, combining under definite conditions of temperature and pressure, yield *two* volumes of hydrochloric acid gas.

"Are all those facts—which recur repeatedly in similar circumstances—the result of mere chance coincidences of disconnected and indifferent causes? They are not: they cannot be. Reason will not admit that any such complex, harmonious, stable series of facts could be due to chance. They must be the expression of a *law*; they must find their sufficient reason in the *nature* of the combining bodies.

"The chemist finds this sufficient reason in what he calls the '*affinities*' of the reacting bodies; the metaphysician, in '*properties* inherent in the *nature*' of those bodies, and indicative of the energies of those natures. The language is different, but at bottom the idea is the same: *There are in the world such complex, harmonious, stable series of facts as cannot be due to chance activities, but must be the result and expression of natural laws*; and the formation of hydrochloric acid from hydrogen and chlorine is a manifestation of such a law.

"Thus it is that, from the total complex groups of circumstances in which he has witnessed the formation of hydrochloric acid, the chemist abstracts or gathers by induction the truth that hydrogen and chlorine have the *property* of combining in the proportions indicated, with a disengagement of 22 calories of heat for the formation of each molecule-gramme of hydrochloric acid. The combination being, moreover, found to be independent of the particular place and time, and of the absolute quantities of the bodies used, he can foretell with certainty that *always* and *everywhere* those gases will combine in those definite proportions to form the compound body, when submitted to the action of the sunlight under the same general conditions.

"In a word, the *law* of hydrogen and chlorine is to combine, *always* and *everywhere*, under the above-mentioned conditions. The chemist who has observed all the facts and extracted that law from them has *made an induction*."

. . . .

"Thus, the chemist has verified his hypothesis that the two gases, hydrogen and chlorine, have the *natural property* of combining in the definite

proportions of 1 to 35·5 by weight. They have an innate, inherent, intrinsic tendency to do so, in certain recognized conditions and under the influence of certain known natural agencies. Such is *the law of their nature*; for, as St. Thomas profoundly remarks, the law of a being is the 'natural inclination which carries it towards the end it has to realize in the universe'.[1]

"Of course, the knowledge of that law does not exhaust all that is knowable about the nature of the bodies in question. By no means. It merely lifts a corner of the veil. The chemical property discovered simply shows us the natures of the bodies that possess it, *under one of their aspects*."

Writers on induction do not usually emphasize the next step, which is the *deductive* application of the verified law to facts. Yet, if we regard induction as the *total* process by which we reach a scientific knowledge *of the individual phenomena* of nature, this deductive step is essential. The utility of our abstract knowledge of law will always lie in its applicability to concrete facts. "It is a *natural property* of hydrogen and chlorine to combine in certain proportions under certain conditions to form hydrochloric acid". Such is our abstract law. "Here are quantities of those gases in the due proportions; therefore if submitted to the action of sunlight, they will form a certain quantity of hydrochloric acid with a disengagement of a certain quantity of heat." Such is our deductive application. It will be seen at once, therefore, how induction contributes to that "knowledge of things by their causes" which is the only knowledge dignified by Aristotle with the title of "scientific". It will be easy, likewise, to see wherein lies the difference between the method of the rational or deductive sciences and that of the inductive sciences, and what is their point of contact (202). In the former, deduction is immediately possible after the conception of a few definitions or first principles, seen intuitively on a simple analysis and comparison of a few very simple concepts (206). In the latter, on the contrary, deduction from the general law cannot commence until the law has been established by a process that is often tedious and difficult.[2]

Hence, if we give the name of *Induction* to that whole method of procedure by which we establish the *conclusions* of the *positive* sciences, we must distinguish two phases in it (202): one *deductive*, which gives us *science*, in the Aristotelean sense of the word, *i.e.* the *explanation of observed facts by their causes*; the other, *preliminary* to this explanation, the stage in which the *general law* is reached, and which alone modern logicians call *Induction*, in the special and restricted meaning of this term.[3]

Whether this strictly inductive phase of the whole procedure—the side by which we ascend from concrete, particular facts to abstract, universal laws—involves any *reasoning* process which is not syllogistic or *deductive* (192), a comparison of induction with deduction will now enable us to determine.

[1] *Summa Theologica*, 1a, 2ae, *q*. 93, *a*. 7 *sqq*. *Cf. infra*, 217.

[2] "In the former [mathematical], generalization is unnoticed because it is all-pervading; for the relevant conditions are distinguished from the first. In the latter, generalization comes to an end and attracts attention as the result of a long effort; for all our task is to distinguish the relevant from the irrelevant conditions."—JOSEPH, *Logic*, p. 509.

[3] *Cf*. MELLONE, *op. cit.*, 382-86.

212. SCIENTIFIC INDUCTION AND DEDUCTIVE INFERENCE.—We have already compared *Induction* with *Deduction*, understanding these terms as descriptive of *Method* (202; *cf.* 187). They are sometimes contrasted with each other as *forms of logical inference*. But, since induction is not a form of logical inference at all,[1] such a comparison is misleading. The so-called "Induction by Complete Enumeration" we have shown to be as unworthy of the name of a *reasoning process* as is the so-called "syllogism" understood in the sense attached to this term by John Stuart Mill (207). Both processes deal with mere collective propositions, and are simply additions of actual parts to form an actual whole ("induction"), or redistribution of that whole into its parts ("syllogism"). The one process is the reverse of the other, but neither is a *reasoning* process: the one is summation of individuals into a group; the other, distribution of the group into its members: neither reaches the abstract, universal judgment. *Complete enumerative induction*, therefore, cannot be compared with the *genuine syllogism* in any figure. *Incomplete enumerative induction*, however, in so far as it shows a connexion between two objects (M and P) to be possible, and suggests that the connexion may be necessary and universal, is naturally formulated, as we have seen, in the third figure of syllogism (172, 207); but it is, in a certain sense, the opposite of the *scientific* syllogism (in the first figure): inasmuch, namely, as it seeks to establish a general principle from instances, while the latter applies an already established principle to instances.

Does *scientific* induction, however, admit of any comparison with deduction, or deductive inference, or the syllogism? With deduction as a method, yes. If we take both as methods, and understand by induction the method whereby we ascend from the consideration of particular facts to the establishment of some general truth, in this meaning it is, of course, the reverse of the whole deductive method, by which—in the mathematical sciences, for example—we descend from the conception of some simple and general truth to the understanding of some less simple and less general one in the light of the former (202). The two processes move in opposite directions: they view things from opposite standpoints: they lead the mind along reality and into the understanding of it by presenting opposite aspects of it: in

[1] *Cf. supra*, 197. JOYCE, *Logic*, p. 217.

the one case the concrete and particular aspect comes first, in the other the abstract and universal aspect.

In the positive sciences, or sciences of observation as they are called, the ascent to the general is difficult; the descent to the explanation of familiar facts by applying the principle is comparatively easy; in the abstract, rational sciences, the ascent from particular to general is easy, the descent from the general to its applications is difficult (202). Here, however, the contrast between induction and deduction ceases. As mental processes they are both essential to the attainment of science; for this is the knowledge of fact by law, of effect by cause, of particular by general.

But, apparently owing to Aristotle's conception and treatment of enumerative induction as a sort of syllogism without a middle term, and to the fact that induction aims at generalizing from particular experiences, some logicians have sought to represent induction as a special form of logical inference, distinct from, and in contrast with, those forms of inference which they conceive as deductive. Now, to represent induction as simply a form of inference is rather a misleading simplification of what is in reality a whole series or combination of processes, some of which "are not processes of *reasoning* at all".[1] Others of them, no doubt, *are* logical inferences; *but not of any new form*, distinct from the various forms of mediate inference, categorical, hypothetical and disjunctive, that were known and analysed in logic prior to the modern development of induction. These are the only forms of inference known to induction; and a glance at the various steps in the inductive process (211) will show how far they enter into it. The preliminary observation of the facts to be investigated, and of all their surrounding circumstances, is, of course, not a logical inference of any sort, though it may indeed involve inferences, both immediate and mediate (238, 263). The conception of an hypothesis as to the general law connecting the facts with their causes, is not itself an inference either. But the verification of an hypothesis may be expressed in a series of inferences, each taking the form of a *mixed hypothetical syllogism*: "If this hypothesis is true, certain events ought to follow from a certain combination of agencies; but (by observation or experiment we proceed to find that) they do follow (or do not, as the

[1] JOSEPH, *op. cit.*, p. 482.

case may be); therefore the hypothesis is probably true (or certainly false, as the case may be)." From this we see that the *modus tollens* efficaciously *disproves*, and so *eliminates*, all hypotheses that are unable to account for the facts under investigation. No single application of the *modus ponens*, however, can *verify* the hypothesis with certitude. Nor, indeed, strictly speaking, could any accumulation of them give us certitude about the antecedent, regarding the matter from the point of view of *formal* inference; though often, as we shall see, hypotheses are sufficiently verified in this manner (230). We usually try to verify our hypothesis by showing not only that it will account for the facts, but that *no other hypothesis will* account for them. We may sometimes be able to do this by showing the facts to be such that they necessarily involve the cause supposed in our hypothesis: "If this hypothesis were not true, the facts could not be such and such; but they are such and such; therefore, the hypothesis is true."[1] It is rarely, however, we can directly show the facts to be such that of their nature they necessarily involve the supposed cause: for the most part we have to be content with showing that they must involve it for the reason that no other conceivable supposition can account for them. That is to say, we verify our hypothesis by eliminating all competing alternatives. Now, this process naturally assumes the form of a *mixed* disjunctive syllogism in the *modus tollendo ponens*:—

"The cause of x is either a or b or c or d . . . or z

[1] In his admirably clear exposition of Newton's researches on gravitation, Mr. Joseph (*Logic*, pp. 477-82), illustrating the various stages of the inductive process, says that "the final argument, in which the agreement of the facts with the results of this hypothesis and of no other is shown to require the acceptance of this hypothesis, is inductive" (p. 482). The term "inductive" cannot here be used in a sense opposed to syllogistic, for the argument to which he applies it is a mixed hypothetical syllogism. It is as follows: "Assuming that the continual deflexion of the planets from a rectilinear path is due to an attractive form [force?], their actual motions, if my statement of the law of attraction is true, would be thus and thus; if it is false, they would be otherwise: but they are thus and thus, and therefore my statement is true" (*ibid.*). It may be symbolized thus: "If A then C, and if not A then not C; but C; therefore A". The standard syllogism, embodying the axiom applied in this reasoning, and analogous to the standard syllogisms applying the *Dictum de omni*, etc., in 192, might be expressed thus: "If a supposed cause accounts adequately for any real fact, and is the only cause which can account for it, then that supposed cause is real; but (by analysis of the facts, through observation and experiment) we see that this supposed cause, and it alone, can adequately account for this real fact; therefore this supposed cause is real". The author applies the term "inductive" to reasonings which are not *explanatory*, which merely convince us *that* a judgment must be true, without giving us any insight into the reason *why* it is true.

It is not *b* or *c* or *d* . . . or *z*
∴ It is *a*"[1]

—where *a*, *b*, *c*, . . . *z* are supposed alternative causes of the phenomenon *x*. This reasoning is, as Mr. Joseph observes,[2] "in form very simple; but the discovery of proper premisses is very hard". How is the investigator to determine the extent of the *major* premiss, the field of pertinent alternative hypotheses?—since he cannot realize Bacon's ideal of cataloguing all the causes in the universe (209). Obviously, it is to be defined by prudence rather than by inference. Then he must verify the *minor* premiss "piecemeal by hypothetical arguments that rest upon one or other of the [usual] grounds of elimination".[3]

Finally, when, having verified our hypothesis, we apply it to the *explanation* of facts, or when we explain itself by the application to it of wider laws, our reasoning is obviously syllogistic and deductive.

Is any of the forms of inference outlined above, so characteristic of the inductive method as to merit the title of "*inductive reasoning*"? It matters little whether we so describe any of them, provided we bear in mind that "Induction" is much more than any of them: that it involves many processes other than mere inference. And, indeed, the same may be said of the title "Deduction" as applied to forms of inference rather than to method. We have already seen (192) that there is no uniformity of usage in the application of the titles "deductive" and "syllogistic" to forms of inference. A "deductive" inference is perhaps most commonly understood to signify an inference in which it is sought to subordinate some special cases (or classes of cases) under some wider principle or law. This would be chiefly characteristic of syllogisms in the first figure, whether categorical or hypothetical. But then, syllogisms in the second or third figures would not be deductive in this sense; for in them there is not usually any subordination of instances to a rule. Mathematical reasoning, too, proceeds in large part from known principles not to *subordinate* cases, but to other *co-ordinate* and *coextensive* principles: in these sciences the cognate truths are so related that very often either of a pair, *a* and *b*, can be used equally well to prove the other: the related truths are reciprocal; and yet mathematical reasoning is universally regarded as deductive.[4] Hence, the subordination or subsumption of a case under a rule is hardly a satisfactory criterion of "deductive" inference.

Mr. Joseph's treatment of the contrast between deduction and induction is instructive. "Inductive Logic," he rightly remarks, "has not really laid bare any new forms of reasoning; we have already seen that Bacon's Induction is a disjunctive argument . . . Or if anyone likes . . . to call inference deductive when it proceeds from conditions to their consequences, and inductive when it proceeds from facts to the conditions that account for them, he will find

[1] JOSEPH, *op. cit.*, p. 406. [2] *ibid.* [3] *ibid.*
[4] *Cf.* JOSEPH, *op. cit.*, p. 368, 369 n. 2: also pp. 503 *sqq.*

"*a.* that the two processes cannot be kept rigidly apart. Whoever infers from the facts of experience the conditions which account for them must at the same time in thought deduce those facts from those conditions.

"*b.* that what *has been called* Deductive Logic, what Inductive Logic has been contrasted with, analyses forms of inference which, if the antithesis between Induction and Deduction be thus understood, must be called inductive." [1]

He himself regards the mixed disjunctive argument as a typically inductive form of inference, owing to the use that is made of it in verifying an hypothesis by the exclusion or elimination of alternative hypotheses. But he suggests a deeper distinction between deductive and inductive inference: "The true antithesis is, as Aristotle saw, the antithesis between Dialectic and Demonstration; or in more modern phrase, between Induction and Explanation ".[2] Inductive inference, then, would be the inference which *convinces* us *that* a proposition is true (because certain facts are incompatible with any other alternative), without, however, *explaining why* it is true, without *demonstrating* it; while deductive inference would not only prove *that* a proposition is true, but would also *explain* or *demonstrate* it, or, in other words, show us *why* it is true. This is an intelligible and useful distinction; but, obviously, it is based on the *matter* of our inferences: it cannot be regarded as a distinction between *forms* of inference, except in so far as some of the recognized forms of logical inference are found to be more naturally applicable to matter in which we can *demonstrate* our conclusions, and others to matter in which we can only set up our conclusions as *de facto* true, without seeing *why* they are so. Now, the disjunctive form of reasoning is, as we have seen, the form into which the inductive verification of an hypothesis naturally falls. And to verify an hypothesis in this way is merely to show *that* it is true, without further explaining it or showing *why* it is true: "the essence of inductive reasoning lies in the use of . . . facts to disprove erroneous theories of causal connexion. It is . . . a process of *elimination*. The facts will never show directly that *a* is the cause of *x*; you can only draw that conclusion, if you show that *nothing else is*." [3]

"You establish a particular hypothesis about the cause of a phenomenon, by showing that, consistently with the relation of cause and effect, the facts do not permit you to regard it as the effect of anything else (and *mutatis mutandis* if you are inquiring into the effect of anything). It is this which makes the reasoning merely inductive. If you could show in accordance with known or accepted scientific principles that the alleged cause was of a nature to produce the effect ascribed to it, your reasoning would be deductive; . . . you would be applying them to produce a conclusion which you see to be involved in their truth; and if we suppose the principles to be of such a nature that we can see they must be true, then the conclusion will appear necessary, and a thing that could not conceivably be otherwise." [4]

[1] *Cf.* JOSEPH, *op. cit.*, p. 369.

[2] *ibid.*, p. 369. "The two antitheses," he adds, "are not quite identical, because some dialectical arguments are not inductive, and explanation is not demonstrative unless the premisses from which it proceeds are known to be true. The reasoning from those premisses is however the same, whether the premisses are known or only believed to be true."—*ibid.* n. 1.

[3] *ibid.*, p. 395.

[4] *ibid.*, p. 399.

"There is an enormous number of general propositions, which we accept for no better reason than that the facts are inconsistent with our denying them, and not because in themselves they have anything which could have led us to suppose them true, antecedently to our experience. When it is said that we ought always to follow experience, it is meant that we ought not to trust our notions of what seems antecedently fit to be true, or mere guesses as to the connexions that subsist in nature, but accept only those connexions which our experience forces us to accept because it is inconsistent with any alternative. Such reasoning is called *a posteriori*, because it starts from the facts, which are conceived as logically dependent on, or posterior to, their principles, and thence infers the principles on which they are dependent. Conversely, deductive reasoning is often called *a priori*, because it starts from the principles or conditions, which are conceived as logically prior to the consequences that follow from them [1]. . . . But it is an error to suppose that all general principles are arrived at *a posteriori* or by process of merely showing that facts are not consistent with any other. . . . Still it is true that in the inductive sciences the vast majority of our generalizations are reached either in this *a posteriori* manner, or by the help of deductions from other generalizations so reached. And it may be well to show by one or two examples how generalizations that rest merely on induction present as it were a blank wall to our intelligence, as something at which we cannot help arriving, but which we can in no way see through or make intrinsically plausible." [2] The author goes on to cite examples "to illustrate . . . what Bacon would call the 'surd and positive' character of conclusions resting only on induction ". [3] One of these examples will be sufficient here: "Facts show that the excision of the thyroid gland dulls the intelligence: could any one see that this must be so? Explanation may show that on a contribution which the gland, when properly functioning, makes to the circulating blood depends the health of the brain; but that comes later than the discovery of the effects of excision; and even so can we understand the connexion, which facts establish, between the state of the mind and the health of the brain?" [4]

These extracts will show a clear and intelligible distinction between two ideals of the knowledge we aim at by inference: the knowledge *that* certain things are so, and the further knowledge *why* they are so; and, by way of consequence, between "inductive" forms of inference, which naturally subserve the former ideal, and "deductive," "demonstrative," "explanatory" forms of inference, which subserve the latter ideal. Some of these points will receive further notice later, in the chapter on *Explanation*.

213. Relation of Antecedent to Consequent in Deduction and Induction: the Latter Considered as an "Inverse Process".—We have seen that the logical inferences involved in the inductive process assume one or other of the forms commonly recognized in "formal" or "deductive" logic; and that

[1] "Or, in another sense, illustrated in most mathematical reasoning, because the premisses, without being more general than the conclusion, or giving the cause why it is true, are not based upon an appeal to facts which might conceivably have been otherwise."

[2] Joseph, *op. cit.*, pp. 400, 401. [3] *ibid.*, p. 402. [4] *ibid.*, p. 401.

the whole ascent from particular to general cannot be intelligibly described as an "inductive syllogism," or as the opposite of the "deductive syllogism". We may, however, regard both processes in a light which will admit of their being compared and contrasted. The relation of premisses to conclusion in the syllogism is identical with the relation of antecedent to consequent in a hypothetical proposition (134, 148, 165); the premisses or antecedent being regarded as *a* "ground" or "reason" whose affirmation gives us a right to affirm the conclusion or consequent, though not as *the sole, exclusive, only possible* ground for affirming the latter. Hence, given the antecedent, we may infer the consequent, though we cannot, conversely, affirm the antecedent if we are given the consequent (140). In other words, a given logical antecedent is regarded as *necessitating* some *definite* consequent, while this same logical consequent is not regarded as *definitely necessitating* that antecedent. Now, if we regard deduction as the passage of thought from *logical antecedent* to *logical consequent* (understanding these terms in the sense just indicated), deduction may be described as a *direct* or *definite* process, reaching a definite result. And if we regard induction as the passage of thought from the *real consequent* or effect, regarded as logical consequent, to the *real antecedent* or cause, regarded as logical antecedent, induction will appear to be an *inverse* or *indefinite* process, reaching only indefinite results: since, for any given effect, considered as logical consequent, there may be a plurality of causes, considered as logical antecedents. This leads us to the consideration of induction as an "inverse problem," or "inverse process," in comparison with deduction regarded as a "direct problem," or "direct process".

In what sense, then, may induction be fairly described as an "inverse process," the inverse of deduction? It has been sometimes so described by logicians. The term is borrowed from mathematics, and there it has a quite intelligible meaning. A direct process is one by which, given certain data and laws of inference, we arrive at a *definite* conclusion: the inverse process is that by which, given the conclusion, we try to get back to the data. While the former always gives a definite result, the latter may yield very indefinite ones. For example, given 4×4, what is the product? Answer (definitely): 16. Given the product 16, what number multiplied by itself yields this product? Answer (indefinitely): *plus* 4, or *minus* 4. Or again, of what factors is

16 the product? Answer (indefinitely): 2×8, or 4×4 (inverse processes).

Transferred to logic, this character of indefiniteness in the inverse process is further emphasized. Given the conclusion of a syllogism, find the premisses. An entirely indefinite problem, this, since *any one* out of an immense number of middle terms may conceivably mediate the conclusion: and the *inventio medii*, the finding of a *real* or *true*, as opposed to an *imaginary*, middle term (167), like the invention and verification of an hypothesis, is amenable to no law or method. The *specifying* of a middle term would remove some of the indefiniteness, leaving only the possible moods of the syllogism to be determined; the assigning of one whole premiss would leave the other premiss (definitely) to be determined.[1] Something like this Professor Welton must have in mind[2] when he agrees, with Jevons, that "induction is . . . an inverse process; it is the finding *major* premisses when the conclusions are given". But why *major* premisses? The inductive problem seems rather that of finding the *whole* (proper and correct) *antecedent* (*major* and *minor*), given the *consequent* or conclusion. Given certain *facts* or *effects*, construct and verify an *hypothesis* as to their *cause*. And from what we have already said about the indefiniteness of the passage from a given *effect* or *consequent* to a *definite cause* or *antecedent*, as compared with the direct process of arguing from cause to effect, from antecedent to consequent, it will easily be understood why the former process has been described as *inverse*, and the latter as *direct*. Yet, by describing induction as an inverse process, the impression may be conveyed that it reaches, *de facto*, only indefinite results. Such an impression would be erroneous; for the aim of induction is precisely to eliminate this indefiniteness by proving some one of the conceivable alternative antecedents to be the real antecedent: which it does, as we have seen, by the indirect method of disproving the other alternatives.

JOSEPH, *Logic*, chaps. xviii., xx., xxiii. MELLONE, *op. cit.*, pp. 244 *sqq.*, 265 *sqq.* JOYCE, *Logic*, chap. xiv. WELTON, *op. cit.*, bk. v., chap. ii. VENN, *Empirical Logic*, chap. xiv. MILL, *Logic* III., ii., iii. MERCIER, *Logique*, pp. 298-307.

[1] *Cf.* VENN, *Empirical Logic*, pp. 359 *sqq.*
[2] *Logic*, vol. ii., p. 59 (italics ours).

CHAPTER III.

PRESUPPOSITIONS OF INDUCTION: CONCEPTS OF "REASON" AND "CAUSE".

214. JUSTIFICATION OF CHAPTERS III. AND IV.—Since the days of Lord Bacon, many conflicting theories on the *nature* and *grounds* of induction have been advocated by logicians. Seeing that induction is the method by which we attain to a knowledge of universal truths about the nature and activities of the things of the visible universe, animate and inanimate, it is easy to understand the importance of the whole subject. Before, therefore, we proceed to analyse more closely the process of ascent from phenomena to their laws, we must examine the rational principles that underli- the process. Modern logicians have analysed these principles in great detail, thus importing into the logic of induction long disquisitions which would, perhaps, find a more appropriate place in psychology, criteriology, and cosmology.

No doubt, the logic of induction cannot be understood without a *statement* of the principles which underlie the process. But a treatise on logic is hardly the proper place for their full exposition and *vindication;* nor is it our intention to go into them here at any great length. We shall endeavour to confine ourselves to a brief explanation of the rational foundations of the inductive process, *i.e.* the principles that justify us in rising from *particular facts* to the conception of a *general law;* and to a brief criticism of some current views that seem more or less erroneous, regarding those principles. In doing this, however, we cannot avoid some reference to numerous notions which must be left for fuller treatment to more suitable branches of philosophy. What these notions are, will be at once suggested by any ordinary example of a natural phenomenon that calls for interpretation from us. Let us take an instance.[1]

This morning, for example, at half-past eight o'clock, the ash-

[1] Adapted from Mercier's *Logique*, pp. 298 *sqq.*

tree on the hill in front of my window was struck by lightning, its foliage withered and burned, and its trunk rent asunder. This *individual event* has *happened once*. With its own peculiar train of *circumstances*, it had never happened before and it will never happen again. But, at other times and places, other trees—and houses and people—have been *similarly* struck by lightning. We examine the *effects produced* by lightning in such cases. We try to find out under what *conditions* exactly they have been produced. *How* is it that the brilliant flash, which dazzles our eyes in the storm, is accompanied by such effects? If our search be successful, we shall learn the *nature* of the lightning, the *law* (that is, the *how?* the *quomodo?*) of its action, and we shall then *understand*, "*by their causes*," the effects produced.

To *understand things by* a knowledge of *their causes* is the aim of all science. Now, even the most superficial observation of the *phenomena* of nature convinces us that their *variability* is bounded and ruled by a certain general sameness, or fixity, or uniformity. The things of nature differ, no doubt, in many ways from one another; yet each of them belongs to a certain *class*, in virtue of some common *attributes*—else how or why would they have common class names? Each belongs to some *specific type*, inorganic, vegetable, animal, human, whose fundamental *uniformity*, and relative *fixity*, are ever conspicuous throughout the incessant evolution and *change of circumstance* to which the transient individuals of the class are subject. And what is true of "things" is equally true of "events".

It will be observed, from the expressions italicized in the foregoing paragraphs, that in the inductive process by which we rise from *facts* to *laws* we are seeking for *reasons*, or *explanations*, for the *how* and the *why* of some phenomenon: we regard this latter as an *effect*, and look for its *cause*: we observe *similarity* amid *variety*: we study the *conditions* and *circumstances* in which the phenomenon takes place: we analyse the *causes* that lead up to it, and try to find out the *nature* of these causes and the *law* according to which they act. Obviously, therefore, our understanding of the inductive process will depend on our manner of conceiving *cause*, *reason*, *law*, *uniformity*, *identity*, etc. Hence, some explanation of the principles of "Sufficient Reason," and "Causality," and "Uniformity of Nature," and of their bearing on the inductive process, is evidently called for at the present stage. And first as to the *Principle of Sufficient Reason*.

215. "REALITY" AND THE "PRINCIPLE OF SUFFICIENT REASON".[1]—Among the presuppositions of induction, many authors set down in the first place the Principle of Sufficient Reason: that "whatever is *judged* to be true must have a reason in our *thought* for being so judged; and whatever *is* or *happens* in the *real* order must have a sufficient reason or cause for so being or happening". As a matter of fact, this principle is a presupposition not of induction alone, but of all search whatever after truth. It simply postulates that reality is intelligible, and its explanation attainable—at least to some extent. Unless we assume that we can discover truth, it is idle to seek for truth. All actual search after truth presupposes that some truth can be found, and the gradual discovery of truth justifies the assumption. The postulate is, therefore, reasonable and necessary. But at the outset its meaning is essentially vague, and it is only by progress in the discovery of truth that we can gradually attach definite meanings to the terms "*sufficient* reason," "intelligible," "explanation," etc.

The principle refers to two orders, the logical and the real. In the logical order, the order of thought, the premisses are the sufficient reason of the conclusion, the antecedent of the consequent, until finally we come to some antecedent which, being self-evident, has the sufficient reason of its truth *in itself;* that is, *in the reality* which the judgment in question interprets by means of two concepts carrying in them the evident ground for the relation which the mind sets up between them. So, too, in the real order, in the order of things, everything that exists, every fact that happens, must have a sufficient reason, either beyond itself—in that its happening or inception is *the effect of a cause distinct from it*,—or in itself—in that it *exists necessarily* and *of itself*, and is itself the explanation of its own existence.

But the two orders—of thought and reality—are not mutually isolated and independent: the "objective evidence" which is the *logical* ground or reason of first principles, is simply REAL BEING *put into relation with the* MIND.[2] "*Objectum Intellectus est Ens*": "*The object of Intellect is Reality*". This maxim of scholastic philosophy is the assertion, against subjective

[1] *Cf.* brochure entitled "*The Inductive Sciences: An Inquiry into some of their Methods and Postulates*," by the present writer (Dublin: Browne & Nolan Ltd., 1910), pp. 10 *sqq.*

[2] *i.e. ontological* truth. *Cf.* 248.

phenomenism, of the power of the human mind to reach *real, objective truth.*

The Principle of Sufficient Reason is, therefore, not merely *formal* but *real* (16). Not only can we not judge of a thing without a sufficient reason in the thing itself for that judgment; but also, the thing itself, the reality itself, which is the object of our thought, cannot be what it is, and as it is, unless it have a sufficient reason in itself, or connected with itself, why it is, or why it is so and not otherwise.

But when we ask *what* shall we be obliged to regard as the *ultimate* "sufficient reason" or "explanation" of our experience as a whole, the answer will obviously depend upon the view which careful and prolonged reflection on that experience will lead us to form about the nature and meaning of all reality. And the conclusions we may reach on this fundamental question will, of course, determine our interpretation of the exact scope and significance of the principle under consideration. Two erroneous interpretations of the principle, based on erroneous views about the nature of reality, may be noticed here, in contrast with the scholastic interpretation which is based on the philosophy of theism.

One is variously known as *Empiricism, Sensism, Positivism, Agnosticism.* It is a mistake in method, no less than an error in fact, to assume, even in regard to the inorganic universe, that no judgment about the latter can be accepted as true without the same sort of cogent evidence which compels intellectual assent in the mathematical science of abstract mechanics; that a fact is "intelligible" or "knowable" only in so far forth as it illustrates the laws of mechanical motion and inertia; that the introduction of "purpose," "design," "intelligence," "final causes," as factors to help in explaining the processes of physical nature, is "unscientific" inasmuch as these factors cannot be "computed" in terms of the laws and principles of mechanics, and are, therefore, themselves not scientifically "intelligible". Any such narrowing of the concept of what is "knowable" or "intelligible" is entirely gratuitous and unwarranted. Yet the *Positivist* philosophy, which has been popular in modern scientific circles, insinuates this misleading interpretation of what is in itself a true and reasonable principle. For this philosophy would have us believe that what is beyond the range of sense experience is "unknowable" (*Agnosticism*); and that the phenomena of sense experience are "knowable" or "intelligible" only in so far as their uniform coexistences and sequences throughout space and time exemplify and constitute the "laws" of mechanics (*Mechanical Atomism*). Reality *may* surely be "knowable," even though not amenable to any such laws; and there *may* be some reality within the reach of our intellect, even though it be beyond the reach of our senses.[1] It was the misfortune of English philosophy, under the influence of such men as Hume and Mill, to sink into this *Sensism*. By declaring all reality to be, in ultimate analysis, *a flow of sensations in the individual consciousness*, they really declared all "knowledge," all "rational explanation" of

[1] *Cf.* JOSEPH, *Logic*, pp. 382 *sqq.*

human experience, to be impossible. We shall see an illustration of this in their futile attempts to explain and justify men's belief in the *Uniformity of Physical Causation*. The Positivism which endeavours to interpret all human experience as a process ruled by mechanical necessity has for its obverse side Agnosticism—that is, a declaration of inability to assign any ultimate rational ground that will explain human experience as a whole. This, like all scepticism, is really an abandonment of the principle of sufficient reason; for it says, in effect, "We believe certain things, but, ultimately, we do not and cannot know why we believe them". The empiricist so interprets experience as to end in agnosticism about the suprasensible, and in inconsistency about what he calls the "scientific *laws* or *generalizations*" of sense phenomena. In his attempt to account for the reliability of those generalizations, their stability, their characteristic of necessity, he really explains this away, and leaves no rational ground for human certitude (*cf.* 219, 224).

Another extreme and erroneous interpretation of the principle of sufficient reason, another narrow view about the "intelligibility" of experience, and the possibility of "explaining" the latter, is that of *Hegelian Idealism*. This is the very antithesis to Empiricism. The latter fails to account for the "*must*," the element of necessity, which characterizes, in varying degrees, our judgments about reality; the former errs by attributing the same absolute intellectual necessity to *all* our judgments about reality. In a word, it claims that the world as a whole is "intelligible" and capable of "rational explanation" only on the assumption that it is one vast self-contained and self-explaining system of ideas, or thought-relations, which reveal themselves to individual minds as endowed with the same metaphysical necessity which characterizes our abstract judgments about the possible essences of things. This, too, is an assumption which unduly narrows the scope of "explanation" and the sphere of what is "knowable" or "intelligible". If we reduce the reality of things, in this fashion, to a mental fabrication of thought-relations, if we make reality a '*theory*,' a mere mental '*construction*,' a "determination" of things "by each other as constituents of one order, a determination which only exists for thought"; if we say: "It is not that there is first the reality of things,[1] and then a theory about it. The reality *is* a theory";[2] if, we contend that "mere feelings . . . except as related to each other through relation to thought, are not facts at all,"[3] that it is only by *thinking* them we make them *real* and give them a *nature*:[4] are we not setting up a mental creation, a system of abstract thought-relations, in the place of reality, and ignoring the claims of that sense experience which certainly puts us into contact with reality?

This *Idealistic Monism* misinterprets reality. Nor is the postulate involved in it a necessary one: the postulate that all reality is one great mental or intellectual system, one great thought or idea, unfolding itself in individual minds according to necessary laws of logical thought; for, surely, we can make

[1] But surely there is some *reality* in the "things" themselves? Surely reality does not lie *exclusively* in any "determination" of relations established by thought between those "things"?

[2] GREEN, *Philosophical Works*, vol. ii., p. 269;—*apud* WELTON, ii., p. 2.

[3] *ibid.*, pp. 385-86.

[4] *Cf.* HERSCHEL, *Nat. Phil.*, 109;—*apud* WELTON, *ibid.*

progress in knowledge without the aid of such an assumption; and, besides, there is at least this other alternative postulate—the postulate of *Theism*—that the sense world reveals itself to the human intellect not as the self-evolution, at once logical and real, of a Sole Being that is at once thought and reality, but, rather, as a distinct system, dependent no less on a Supreme Will for its actuality than on a Supreme Intelligence for its intelligibility.

This, then, is another possible alternative, and it is the true one: that the whole world of human experience (including the human mind itself) is the creation of a Supreme Free Will, governed by laws laid down by a Supreme Intelligence (*Philosophy of Theism*); and not the logically necessary unfolding or evolution of one Sole Idea-Being (*Idealistic Pantheism*); or the obverse and unknowable background of the transient panorama which constitutes the individual man's sense consciousness (*Empiricism*, *Agnosticism*).

These are three alternative points of view—there are others also—from which individual writers on inductive logic may proceed to lay down principles for the guidance of the student in his search after truth, whether in science, in philosophy, or in theology. The differences between them are revealed in the respective ways in which writers of each of these schools treat of causality, hypothesis, and generalization (based on the law of Nature's Uniformity), as well as in the different ideals they set up regarding Scientific Explanation and Physical Certitude. In the chapters that follow, we shall, therefore, have occasion to recur repeatedly to the views we have just mentioned; and the principle itself of sufficient reason will arise again explicitly, in connexion with the theory of Demonstration and Scientific Explanation.

216. THE PRINCIPLE OF CAUSALITY IN INDUCTION: ARISTOTLE'S CLASSIFICATION OF CAUSES.—When applied to contingent things, to the phenomena of nature around us, the principle of sufficient reason resolves itself into the *Principle of Causality*. This latter is an *a priori*, self-evident principle, like the former. Cause and effect being correlative, to say that "Every effect has a cause" is to state a truism. The principle is usually stated thus: "Whatever happens (occurs, takes place, begins to be) has a cause". The axiom '*Ex nihilo nihil fit*,' is a negative statement of the same principle. And another statement of it, "Whatever is *contingent* (*i.e.* whatever does not contain in itself, in its own essence, the sufficient reason of its actual existence) has a cause," shows the connexion of the principle of causality with the principle of sufficient reason. Being that is *necessary* and *self-existent* has no cause. It is itself the reason of its own existence; whereas all contingent being is caused. The principle of causality is evidently a *necessary* principle in regard to contingent being, *i.e.* it is essentially involved in our very concept of contingent being. Nothing can happen without a cause: whatever happens has *necessarily* a cause, *i.e.* something which

brings it about, which makes it happen, whether this cause be free (*i.e.* self-determining) or not, in its mode of action.

Now, induction being mainly concerned with the discovery of the *causes* which bring about the phenomena that constitute our experience, it is important to have a clear understanding in the first place about what scientists, logicians, and philosophers mean by a "cause".

What we may call the common and traditional notion of "cause" is that of "anything which contributes in any positive way to the existence or happening of something else". Aristotle distinguished between two *intrinsic* causes, the "formal" and the "material," which constituted that "something," and two other causes, the "efficient" and the "final," *extrinsic* to the "something," and in regard to which the latter is properly called an "effect". The notions of "formal" and "final" causes are closely connected with the Aristotelean view of nature as revealing *Purpose* and *Design* (217). We shall see that modern science and philosophy are not the better for discarding these notions.[1] Inductive logicians confine their attention almost exclusively to the study of *efficient* causality; and of this many have perverted, or rather abandoned, the traditional notion.

The popular idea of "efficient cause" is that of an *agent* or *agency*—something which by means of perceptible action, motion, or change, produces some new state or condition of things. Thus understood, a cause is clearly distinguished from a *condition;* the latter being anything which, though *necessary* for the happening of the effect, does not contribute *positively* thereto: windows, for instance, being the condition, not the cause, of the daylight in a room. Most logicians of induction, however, ignore this distinction[2]: the reason being that, so far as *physical science* is concerned, it is of no importance. Nor indeed is it, provided we assume that the duty of the physical scientist as such is merely to discover all the antecedents, positive and negative, of whatsoever sort, which are *sufficient* and *indispenable* for the happening of any given phenomenon, without troubling himself about the manner in which they contribute thereto.[3] But not all are willing to set such limits to the scope of physical science; though, of course,

[1] *Cf.* VENN, *Empirical Logic*, pp. 47-62, as an example of the attitude of the empirical school of logicians towards them.

[2] *Cf.* WELTON, *op. cit.*, ii., p. 19, where "cause" is defined as the "totality of conditions" requisite to the happening of a phenomenon.

[3] *Cf.* JOYCE, *op. cit.*, p. 221.

philosophers of the positivist school claim that when physical science has discovered the invariable antecedents of a phenomenon nothing further remains for investigation. Moreover, the logician of induction should not confine his investigations to the data of the physical sciences alone; he must investigate our thinking processes about all conceivable data. The distinction, therefore, between *condition* and *cause*, need not be altogether ignored.

We must next consider that the same thing or reality, the same phenomenon or agency, may evidently be the effect of certain efficient causes, and itself the efficient cause of other effects: looked at in one connexion, from one point of view, it is an effect; in another connexion and from another point of view it is an efficient cause. Thus we see, in physical nature, innumerable series or chains of efficient causes, wherein each link has others depending on it while itself in turn depends on others: whether each follows the other in time or all exist simultaneously. Every event in nature is the result of a long series of causal antecedents stretching indefinitely backward and outward in time and space, or rather of the convergence and co-operation of many such series. It is precisely owing to this fact that inductive research for the "efficient causes" of the phenomena of nature constitutes such a difficult problem. When can we be said to have discovered inductively the group of agencies which are to be regarded as the "total" efficient cause of any phenomenon? What portion of all the converging series of influences are we supposed to bring to light explicitly, and to designate as the total efficient cause of an event? How far backward and outward from the event, and how far forward and inward towards the event, are we to proceed in our analysis of its concomitant and antecedent circumstances?

Let us take, as an instance of natural causation, the formation of water from oxygen and hydrogen. If we draw through the process a line of demarcation (220) at the moment the water begins to appear, and regard the appearance of the latter as the effect to be explained, we shall evidently need to examine the antecedents *down to this very line itself*, lest any indispensable factor escape our notice. And in the backward direction, in enumerating remoter antecedents which were indispensable steps leading up to the final result, where are we to stop? Are we to include not only the necessary heat, but the source of the latter—the electric machine? and its maker? Are we to include not

only the gases, but the vessels containing them? and the agencies that produced them? and those that brought them together in the proper proportions? Evidently not. We must, clearly, draw the line limiting our backward search somewhere. And from the region between it and the line bordering on the effect we must set aside all the *individual* modes and circumstances which may indeed be essential for *this individual instance* of the effect, but which are *indifferent* and *irrelevant* so far as the production of *this kind of effect* is concerned:[1] for it is not with the *individual* effect, but with the *kind* of effect, the production of *water as such* (*i.e.* with the *abstract* and *universal*), that science is concerned. We must, therefore, confine our attention to *a limited group* of antecedents in close proximity to the effect, analyse this group, and set forth, as the "total *proximate* cause" of the phenomenon, only that collection of factors which we regard as the *sufficient* (or *necessitating*) and *indispensable* PROXIMATE factors in its production. Moreover, the scientist, in enumerating these, will omit, and usually does omit, such factors as are *obviously* necessary for the result: he assumes it to be universally understood that they are present. His concern rather is to detect *some one factor* (or collection and collocation of factors) which *immediately* precedes the effect, and which actually determines the effective co-operation of the existing forces in the production of this specific kind of effect.[2] This factor he usually calls the "*determining cause* (or *condition*)" of the phenomenon. *All* the factors in this proximate collection—all the *proximate* factors *sufficient* and *indispensable* for the effect—constitute what the scientist regards as the "*proximate* (efficient) *cause*" of the phenomenon in question.

Although the principle of causality refers primarily to *efficient* causes, *i.e.* causes which, by means of *action*, of real *change* or *motion*, produce their effects; and although modern writers on induction have concentrated their attention almost exclusively on *physical* efficient causes—*i.e.* those which are, in themselves or their action, *perceptible to the senses*—as being the more capable of exact *physical* investigation, and even of *mechanical measurement:* still, the scope of inductive logic must necessarily embrace the investigation of *material*, *formal*, and *final* causes, no less than of efficient causes.[3] To know a thing scientifically, we must know

[1] *Cf.* JOYCE, *op. cit.*, p. 221. [2] *Cf.* JOSEPH, *op. cit.*, p. 393.

[3] On this point we may refer the reader to a suggestive article by W. J. ROBERTS in *Mind* (N.S., no. 32, October, 1909), connecting the theory of induction with Aristotle's doctrine on formal cause.—*Cf.* JOSEPH, *Logic*, p. 457.

not merely its efficient cause or causes, but its inner nature, its *formal* and *material* causes. But its *specific* nature, or formal cause, is revealed by its sensible energies and properties: it is only by studying these—which are *nearer* and more familiar to us, whose knowledge comes through the senses—that we can know that which is *more remote* from us, though prior and simpler in itself, *viz.* the specific nature, the ground of the *universal law.* The principle underlying this progress of thought is expressed in the Scholastic aphorism: *Operari sequitur esse*, or, *Qualis est operatio, talis est natura: As a thing is so it acts: Action is the index of essence.* And a knowledge of the formal cause or specific nature of a thing helps to bring to light its purpose or function in the universe, *i.e.* its *final* cause, the reason why it exists and acts, the design it accomplishes in nature. Without a knowledge of the "*why and wherefore*" of a thing, our knowledge of it is incomplete.

When, for example, Pasteur's experiments demonstrated that fermentation is due to the action of a living microbe, or that every living cell has its origin from some other living cell, he discovered in the microbe the *efficient* cause of fermentation, in the parent-cell the *efficient* cause of the younger cell.

Again, the chemical elements combine to form compounds in certain definite proportions by weight. The components, hydrogen and chlorine, enter into the formation of hydrochloric acid in the proportion of 1 part of hydrogen to 35.5 parts of chlorine by weight. Those definite quantities are *material* causes of the various combinations into which these elements enter, for the material mass or quantity is independent of specific change, *i.e.* change of substance or substantial "form," and attaches to the material cause or principle in corporeal substances and agents.

Again, the combination of hydrogen and chlorine to form hydrochloric acid is seen to depend on the *chemical affinities* of the two reacting bodies for each other. These affinities being themselves *specific properties* of the respective components, the formation of the acid is the result of the *specific natures* of these components. But the specific nature and the specific properties of a body depend on, and are determined by, its specific, essential or substantial "*form*": its "*formal*" *cause.* So that when we determine the *law* of a chemical combination, *i.e.* the fixed, uniform mode of action of the substances involved—elements or compounds,—we are bringing to light the *formal causes*, the specific constituent principles, the specific *natures*, of those substances.

Again, the peculiar *affinities* of the chemical elements determine the combinations which their respective natures incline them to realize. Hence, to detect the affinities of these elements, and to characterize the latter by those affinities, is to discover the *innate tendencies* which incline these elements to form certain combinations. In other words, it is to discover the *internal purpose* of the reacting bodies. The establishment of the laws of a chemical combination is, therefore, the discovery not merely of the formal, but

also of the *final* causes, of such combination. That is to say, it shows the realization or formation of the compound as the *internal reason* why the bodies in question combine in that way; it reveals the *finis operis*, the intrinsic aim of that activity which is the product and complement of their respective natures or formal causes.

In some cases, the final cause is even the *primary* object of inductive research. This occurs very commonly in physiological investigations into the functions of living organs—as, for example, in the efforts of physiologists to discover the functions of the thyroid gland, or of the vermiform appendix.

217. "PURPOSE" OR "DESIGN": "FINAL CAUSES" AND "LAW" IN PHYSICAL NATURE.—It is intelligible that when inquiring into the causes and reasons of phenomena that may be due to human activity we should seek for their *final* causes, *i.e.* the motive, the design, the end in view in their production; but the meaning of seeking for final causes in the natural agencies even of the inorganic world may not be at first apparent: for, how can an agent devoid of perception and appetite, nay, even of life, act "for an end"? Such agents cannot, of course, act "for an end" in the same way as men ("*electivé*"), who, by free will, *elect* or *choose* the ends for which they act; or in the same way as animals ("*apprehensivé*"), which are *conscious* of the ends towards which their *appetites* move *instinctively*; but the inanimate agencies of the physical universe can and do act, each "for an end," *effectively* or *equivalently* ("*executivé*"), *i.e.* in such a way as to make it clear that they are *directed* or *oriented* in their action by a ruling intelligence, and for a purpose. That this is so in fact, we are convinced by the manifest order, harmony, regularity of natural phenomena. We know by experience that the causes at work in the whole vast physical universe act each in its own fixed way, uniformly, regularly: whence we conclude that every such agency must have impressed on its very essence, on its inner constitution, by the Creator, a definite "bent" or "tendency" or "inclination" ("appetitus *naturalis*") to act along certain definite lines, and so to discharge its appointed function in the universe. This inner principle of uniform action (which is its substantial or specific *formal cause*) we call the *nature* of the agent in question: and when it acts in the ordinary, normal way, we speak of its acting "according to its *nature*," or "according to *the law of its nature*". By the *nature* of a cause or agent we therefore *primarily* mean the *inner directive principle of its activity*: its essence regarded as a source or principle of orderly, intelligible

activity.[1] When we speak of *Physical Nature*, or the *Order of Nature*, or the *Course of Nature*, or *External* or *Visible Nature*, we use the term "Nature," in a collective sense, to signify the sum-total of all the created agencies that make up the visible (*i.e.* sensible, perceptible) material universe.

Hence, the extension of the concept of *purpose* from the domain of rational or human agents to the animal, vegetable, and inorganic kingdoms of nature, by Aristotelean and Scholastic philosophers, is no mere verbal metaphor. There is a true and proper sense, as these philosophers contend, in which *all* created agencies act in fulfilment of purpose, in which "ALL agents act for an end": "OMNE *agens agit propter finem*".

The conviction is gradually forced in upon us by our experience of natural phenomena, that every agency in nature must have some fixed, intrinsic principle of activity, in virtue of which it acts uniformly, and concurs with other physical agencies, not *capriciously* or *indifferently*, but along certain prearranged and predetermined lines; so that "exceptions" to this uniformity must be due in reality to the influence of some unknown natural causes, or, possibly, to the intervention of the First Cause. Here, at all events, is the great fact we gather from sense experience: that very complex combinations of numerous natural agents repeatedly concur to produce uniform series of effects. Of this great fact there can apparently be one, and only one, rational interpretation: that which conceives the proximate causes of such uniform series of phenomena as endowed each with a fixed *natural inclination* or *tendency* to act steadily and consistently along definite lines, as having each an internal *law* which dominates it, and in conformity with which it will act always and everywhere. This innate, stable tendency is what the mind grasps when it apprehends the law of the Uniformity of Nature. *The great fact of experience revealed in the regular, constant, harmonious concurrence of numerous and varied forces and agencies to produce uniform series of results, finds its sufficient reason and explanation only in a*

[1] The *Essence* of a thing ("*Essentia*" or "*Quidditas*") is that which makes the thing what it is ("*id quo res est id quod est*": the answer to the question "*Quid est illa res?*"). The *Nature* ("*Natura*") is the essence itself looked on as the directive principle of the thing's activities (the "principium *operationis*"). It was conceived by the Scholastics as the impression of a divine directive plan or design on the inner constitution of the created agency: "*Stabilis inclinatio vel appetitus finis, rebus a Deo inditus*," or again, "*Ars quaedam Divina indita rebus, per quam ad fines proprios non solum ducuntur sed quodammodo vadunt.*"—St. Thomas, QQ. DD. *De Veritate*, Q. xxii, a. i.

fixed NATURAL INCLINATION *or tendency of the agents which produce such results.* The expression "*natural inclination*" embodies a fundamental doctrine of Aristotelean philosophy; it implies that the agencies whose effects or manifestations we observe in the world around us are not, as the advocates of mechanical determinism (215) would have them, mere *efficient* agents capable of producing any or every result indifferently, but that each of them is endowed with an internal tendency in virtue of which it manifests a manner of being and acting *proper* to itself; which manner is called a *property* of the substance, and reveals the *specific nature* of this latter.

This view of the universe, as the expression of a divine plan,—hence called *teleological*,—renders intelligible the use of a term that is constantly recurring in the logic of induction: the term *Law* (*Lex*). Law means primarily an order, mandate, precept, emanating from the will of a superior (the legislator), and imposed upon a community subject to him.[1] The law, *as abiding in their minds and hearts*, by their knowledge of it and submission to it, secures a certain *uniformity in their conduct:* it becomes the immediate source and principle, in them, of a series of similar acts. Next, the term *Law* came to be applied to what was really its effect, to this *uniform series of similar acts.* It was then extended, in this latter sense, from the domain of human activity to the domain of physical, even inanimate, nature; and here it is now used, as, for example, in all the "physical" and "natural"[2] sciences, to denote any uniform series of connected phenomena, whether the connected elements exist simultaneously ("coexistences") or successively ("sequences"). The general propositions or statements which formulate such connexions are commonly referred to as "laws of physical nature": *e.g.* "Water seeks its own level," "All bodies fall with the same acceleration in a vacuum," "At a given temperature the volume of a given quantity of gas varies inversely as the pressure it sustains," "Heat can produce mechanical work, and *vice versa*, in definite, measurable proportions," "The strength of an electric current varies directly as the electromotive force and inversely as the resistance," "Every living cell has its origin from some other living cell," "Fermentation is due to the action of microbes".

[1] *Cf. The Inductive Sciences, etc.*, pp. 70 *sqq.*

[2] These terms are commonly regarded as synonymous; when they are distinguished, the former refers to the sciences of inorganic, inanimate nature, the latter to those of the living universe.

Most of these "laws" are merely formulæ *descriptive* of constant connexions which have been discovered to exist between phenomena, and of the conditions and circumstances in which such connexions are found to obtain. But those uniform happenings must, after the analogy of the uniform conduct of a community subject to the law of a superior, be themselves due to fixed principles of action inherent in the constitution of the natural agencies which manifest those uniform activities. Now, if we bring to light the agencies which are operative—and co-operative—in producing those regular coexistences and sequences, the mode of action and interaction of the *efficient* causes that are at work, the inner constitution or nature of those agencies, *i.e.* their *material* and *formal* causes, and the scope or purpose of those activities, we can formulate *explanatory* or *causal* laws, *i.e.* laws which will not merely express the existence of uniformities, but which, furthermore, will give us an insight into the "*how*" and the "*why*" of such uniformities (222).

The Aristotelean conception and classification of causes, and the Scholastic view of physical nature as a "cosmos," revealing purpose, design, intelligence, and subject to "law" in the sense just explained, have been almost completely ignored by modern exponents of the logic of induction.[1] Some of these latter have substituted a purely *mechanical* view of the universe, eliminating the notions of "design" and "efficiency" as superfluous, and retaining merely the notions of "invariable" or "necessary" "sequences" and "coexistences" of material phenomena, as the ultimate factors of a rational explanation of the universe. These writers have been induced by a rather superficial materialism to abandon, and even to ridicule, the rôle of *final* causes in philosophical research. Yielding too hastily to the natural craving for a *simple* solution of the problems raised by the universe, they have thought to satisfy themselves and others by proclaiming the sufficiency of *physical* efficient causality, *i.e.* of *invariable connexions* between *masses of matter in motion*, for the adequate explanation of all things. The attempt was necessarily futile, and is nowadays generally recognized as such. "The mechanical theory of the universe," writes Professor Welton,[2] "is simple, but inadequate *even in inorganic nature*; in organic nature it must be supplemented by the principle of development, and finally *by the conception of rational purpose*." To which we may add Mr. Joseph's testimony,[3] that "to a physical theory of the world consciousness remains unaccountable; such a theory therefore cannot be complete or final".

We shall see later (219, 224) that the "necessity" of those "uniform connexions" or "laws" can have no rational basis in the "mechanical" view of nature. Neither, however, does it receive a satisfactory explanation on the Hegelian, idealist view, that nature is merely a system of *thought-relations*, and that its "necessities" are identical with the necessities of thought (215).

[1] *Cf.* VENN, *Empirical Logic*, pp. 47-52.
[2] *Logic*, ii., pp. 206, 210; *cf.* p. 30 (italics ours).
[3] *op. cit.*, p. 384.

Writers who support this latter view bring out very clearly the shortcomings of Empiricism;[1] but, though they rightly include the concept of "final cause" *i.e.* of purpose, design, in their philosophical explanation of natural phenomena, they still fail to recognize explicitly the "formal" and "material" causes, as distinct from the "efficient causes," of phenomena, and are thus led to identify the *cause* with the *process*, and this latter with the *effect*.

In most of the physical sciences we are mainly concerned with the discovery and explanation of *processes*, *changes*, *motions*, *activities*, *actions* and *interactions* between material agencies: and our main concern here must be to find out what are the *proximate agencies* at work in a given process, to separate these from irrelevant and accidental surroundings, to detect the total (proximate) *agens* and the total (proximate) *patiens* in question, and to discover and understand the connexion of *physical efficient causality* between these.

But the discovery of a connexion of *efficient* causality, of action and interaction, between physical agents, is the discovery of active and passive *powers* or *properties* in these agents: and the discovery of such properties or powers leads to a knowledge of the intrinsic constitution, the *nature*, of these agents. As a thing acts, so it is: *Qualis operatio, talis natura.* All the insight we have into the inner nature and constitution of things is got by inference from their observed activities: *Operari sequitur esse.* And our knowledge of the inner nature and constitution of a physical cause, of the manner and conditions of its activities, will help us to understand its *raison d'être*, its function or rôle in the universe, the purpose it serves, the end it is designed to fulfil, the *final cause* of the processes in which it plays a part.

Thus we see that the search for any one class of cause is by no means incompatible with a search for the others. When one line of inquiry cannot be prosecuted, another may; and each often helps the others.

218. CONTRAST BETWEEN TRADITIONAL AND EMPIRICAL CONCEPTIONS OF EFFICIENT CAUSALITY.—When the word "cause" is used without qualification "*efficient* cause" is meant. Used in this sense, the term "cause" has almost completely changed its traditional signification; and with very confusing results. We must, therefore, note these changes of meaning carefully.

The traditional notion of efficient cause is that of "anything which positively contributes by way of action or change or motion to the production or happening or existence of anything else". *Positive influence by way of action* is what we mean by the "efficiency" of a cause. This traditional conception of *efficiency*, or *efficient causality*, we can find no reason or justification for abandoning. We shall therefore retain it. Furthermore, we must distinguish between the individual, substantial cause or agent itself ("the *agens*," the "principium *quod* agit"); the power, faculty, force, potential energy, of that cause (the "principium *quo* agens agit"); and the action ("actio") by which it produces its effect.

[1] *Cf.* Professor WELTON's criticisms of Mill, *op. cit.*, *passim*.

And, of efficient causes themselves, we may distinguish several kinds: the *First* or *Uncreated* Cause, and *second* or *created* causes; the *free* cause—which has the power of choice to act or not to act, which can determine itself to act or not, which has "dominion" or control of its act,—and the *non-free* or *necessary* or "*natural*" cause,—which, when placed in a definite set of circumstances, *does always act*, because it *must* act, because it has no power or control over its own act, but is by its very nature so constituted (by the First Cause) that (unless the First Cause miraculously interferes) it will, by a necessity of its nature, always act in those circumstances.

Now, most modern writers on induction have come to use the terms *cause*, and *efficient cause*, in the sense of a *non-free* or *necessary*[1] *cause*. This in itself is not surprising, seeing that they have mainly, if not exclusively, in view the physical universe, inorganic and organic, *exclusive of man*; and they may, perhaps, regard the adjective "physical," applied to "cause," as a sufficient indication that they are dealing only with causes understood to be connected "naturally" or "by a necessity of their nature" with their effects.[2] With this usage, then, we will not quarrel, provided it be distinctly understood that there are, or may be, in existence, *free* causes. Where ambiguity would be likely to arise, we should use the adjectives "free" or "necessary".

An unfortunate result, however, of identifying *efficient causality* with the uniform causality of *necessary* causes, calls for notice here. It is the confusion of two quite distinct principles, the "principle of causality" (216) and the "principle of the uniformity of nature" (223), under the common title of the "law of universal causation".[3] But it is one assertion that "The same causes, acting in similar circumstances, will always produce the same effect"; it is another and quite a different assertion that "Whatever happens has a cause". The former, which is known as the "principle of the uniformity of physical nature," is not *universally* true, of *all* causes: as we shall see later (223), it applies, strictly speaking, *only to "necessary" or "non-free"* causes; though it is often stated by modern writers in such a way as to insinuate that it is of universal application: which, of course, is tantamount to a denial of human free will. Similarly,

[1] The term "necessitating" would convey the idea better than "necessary". The latter term, however, can claim universal usage in this context.

[2] *Cf.* JOSEPH, *op. cit.*, p. 373.

[3] *Cf.* MELLONE, *op. cit.*, p. 281.

the latter assertion—the self-evident principle of causality: "Every event *necessarily* has a cause"—is not to be confounded with this other altogether different assertion, that "Every event has a *necessary* cause". This latter statement is not evident; nay, it is not even true. Effects produced in the universe by the *free* activity of man have, manifestly, not necessary but free causes.

Nevertheless, there are many modern writers on inductive logic, who insinuate—perhaps unconsciously—in their whole doctrine of causality that the only concept of cause which is at all intelligible or amenable to scientific treatment is the concept of a *necessary* or *necessitating* cause. Thus, Dr. Mellone refers to the self-evident principle of causality under the title of the *Law of Universal Causation*, and rightly remarks that it refers to "cause" in the widest sense: Every event must have some sort of cause, either a "necessary" (or "uniformly acting") cause, or a "capricious" cause, or—he might add—a cause which, though free, is not "capricious," and about the operation of which we can consequently generalize with some degree of safety.

"This principle [he writes] may be shown to be implied in all thinking. Even children and the lower races of men, though they do not think *of* it, think *according to* it. If the savage were content to leave any event unexplained, he would not imagine that all events are controlled by spirits, malevolent or benevolent. It is in fact IMPOSSIBLE TO THINK OF AN EVENT WITHOUT REFERRING IT TO A CAUSE, known or unknown. Even if we had a state of affairs where the past gave scarcely any assurance as to the future, our way of conceiving it would not be contrary to the principle of the universality of causation. We should think that some capricious power had added itself to the conditions, turning them now this way and now that." [1]

All that is quite true; for the word "cause" is clearly taken to include conditions, agencies, influences, and powers, of whatsoever kind, capricious and free no less than regular and necessary: on no other supposition indeed would the statement that "every event has a cause" be a self-evident axiom.

But Dr. Mellone goes on immediately to say that the principle of the *Uniformity of Nature*, or "*Uniformity of Causation*," as he prefers to call it—that "the same cause must have the same effect"—a principle which will be shown to refer properly only to *necessitating* causes (223)—is *included in* the previous principle of the universality of causation, that "every event has a cause". Surely this is not so. The *universality* of the law of causation throughout all contingent being, does not in itself imply that this causality is necessarily uniform. The self-evident principle of causality—that nothing can happen without a cause, *ex nihilo nihil fit*—understands "cause" in the widest conceivable sense of any real principle, whether free or mechanical, capricious or regular, which brings about the event: it has nothing to do with the question of repetition or regularity at all. Whereas *Uniformity of Causation*, even understood in the hypothetical sense in which Dr. Mellone takes it,[2] bears exclusively upon *regularity of repetition*, and is self-evident *only in regard to non-free*, or *necessary* causes, which are by nature so constituted

[1] MELLONE, *Introductory Text-book of Logic*, pp. 280-1.
[2] *ibid.*, p. 282.

and so endowed with one fixed tendency that in similar circumstances they must always produce similar effects. And yet Dr. Mellone continues:—

"The student will see on reflection that this principle is *included in* the principle of universal causation; for by cause is at least meant a condition on which the effect *always* follows. If it sometimes followed and sometimes did not, there would be no object in trying to discover it; you would simply not have a cause at all." [1]

No doubt we are free to define a cause as "a condition [or group of conditions, agencies, influences] on which the effect *always* follows," and indeed this is the narrower sense in which the term is usually understood when we speak of the non-free causes that operate in external nature, the causes to which the principle of uniformity properly applies. But it is certainly not identical with the wider sense in which Dr. Mellone had rightly used the word when formulating the self-evident law of universal causation, that "every event has a cause," for in this latter context the term "cause" included free and even "capricious" causes.

His final statement, that if the effect "sometimes followed and sometimes did not, . . . you would simply not have a cause at all," is quite too sweeping. What is true, of course, is this, that we can infer or generalize about the operation, beyond experience, of any cause, only in so far as we are warranted in assuming its operation to be regular, not capricious. But if Dr. Mellone's statement were true, it would follow that man is not the *cause* of what he does *freely*, and that no *science* of human conduct is possible. This is one unsatisfactory result of discarding the traditional notion of physical efficient causes as agencies or powers inherent in physical phenomena and productive of physical change, for the empiricist notion of such causes as "invariable and unconditional antecedents," *i.e.* phenomena or groups of phenomena "sufficient [or *necessitating*] and indispensable" for the appearance of other [consequent] phenomena.

The "efficiency" of causation is quite a distinct concept from the "necessity" of causation. Yet these are sometimes confounded. Professor Welton, for example, criticizing Mill's account of causality, writes [2] that the latter "finds cause in a set of conditions whose existence necessitates that of the effect," and he adds immediately that "greater efficiency than this no one would wish to establish". But efficiency is not necessity. A cause may be *efficient* and yet not be *necessitating*, but *free*. In fact it is from our con-

[1] MELLONE, *ibid.*, p. 281. Mr. JOSEPH (*op. cit.*, pp. 370 *sqq.*) adopts the same view: "There is no need then to distinguish the law of causation from the uniformity of nature; for—bating the possible exception of the causality of the human will—a cause which does not act uniformly is no cause at all; and if we are looking for the presuppositions of inductive inference it is plain that the only connexions whose existence would justify such inference are uniform connexions" (p. 376).

[2] *Logic*, ii., p. 19. Mr. Joseph makes a similar mistake about the verb *produce*: "to say that anything may produce anything is to empty the verb 'produce' of all its meaning. For the causal relation is a necessary relation, such that if you have one thing you *must* have another. To add that it does not matter what the other is, destroys the force of the *must*" (p. 374). No doubt it destroys the force of the "must"; but surely "must" is different from "produce," and this again from "*necessarily* produce".

sciousness of our own free volitional activity that we derive the notion of efficiency in the first instance. Efficiency we conceive as positive influence in the production of changes or effects by the exertion of power or force. The earliest efficient causality of which we become aware is our own *free* efficient causality. Then we come to conceive external nature as also endowed with powers or forces, as *efficient* in the production of changes or effects. Of course there have been and are philosophers who maintain that belief in real efficiency in nature is an illusion; and some have extended their denial even to the domain of mind as well. Occasionalists take up this attitude on the ground that efficient causality is essentially an attribute of the Creator, incommunicable to the creature. This view does not concern us here. Our present purpose is merely to emphasize an obvious distinction between the notion of *efficient* cause—whether *free* or *necessary*—and the notion of uniform connexion—whether of coexistence or sequence—between phenomena; and to point out that it is exclusively to the latter concept Empiricists apply the terms "causation" or "causality".

For the rest, as far as the theory of physical induction is concerned, this later usage is not without its conveniences; for in the first place, it is *regularity*, *uniformity*, *invariability* of the connexion between physical causes and their effects, that forms the real objective ground of our generalizations and inferences about them:[1] not the inner nature of that connexion itself. And secondly, if the physical scientist sets up as his ideal the discovery of the *perceptible* antecedents, or groups of antecedents, which are *sufficient* and *indispensable* for the production of certain phenomena—so as to be able to apply this knowledge in bringing such phenomena about—as in engineering and the other applied sciences—the ideal is a perfectly legitimate one.[2] Only, if he is himself thus content with the discovery of *proximate*, *perceptible* antecedents, he must not deny the possibility of prosecuting the search for *remoter*, *non-perceptible* agencies.[3] And if he bestows on those visible groups of "invariable and unconditional antecedents" the title of "causes," we need not object to this—for they are *de facto* "causes,"—though we have, perhaps, some reason to complain that he is changing the traditional meaning of an important term without sufficient justification: even were there no such things in existence as causes *in the traditional sense of the term*, he would not refute the traditional belief by merely changing the definition of the name.

We agree with Mr. Joseph that the freedom of the human will is a difficult problem, not to be argued here.[4] And the same may be said of "efficiency". But to deny to free will the title of "cause" does not solve the problem or prove free will to be a fiction. At all events men generally believe that they are free agents, and that they *freely* "*cause*" or "*produce*" effects. In the face of this fact we see no adequate reason to justify the logician in asserting absolutely and without qualification that "*causal* connexions are *necessary* and universal," or that "to assert causation is to assert uniformity of connexion".[5] The logician must take into account not merely the domain of physical nature,

[1] *Cf.* VENN, *Empirical Logic*, pp. 50, 51, 93; MILL, *Logic*, iii., v., § 2.

[2] *Cf.* JOYCE, *op. cit.*, p. 223.

[3] "I premise, then, that when . . . I speak of the cause of any phenomenon, I do not mean a cause which is not itself a phenomenon."—MILL, *Logic*, iii., v., § 2.

[4] *op. cit.*, p. 373.

[5] *ibid.*, p. 375 (italics ours).

but likewise the domain of human activity (201). Nor should he identify *physical science* with *science simply*: "If the non-mechanical conditions upon which physical changes depend (supposing that such there are) cannot be ascertained and formulated in a way which enables *physical science* to take account of them, it will treat them as non-existent. It is of no use to regard a factor, whose mode of action is unascertainable. It must remain for *science*—what the will is upon one theory of human freedom—a source of purely incalculable and to it irrational interference. But irrational interference is just what cannot be supposed to occur. No doubt an interference which admits an explanation according to law is not irrational; but if the law is unascertainable, it is as good as irrational. And this attitude of physical science has the practical justification, that if events are once admitted to occur in the material order whose conditions are unascertainable within that order, there is no point at which we can draw the line. Only by assuming that it can explain everything is it possible to find out how much it can explain in physical terms."[1] For the credit of physical science itself we should be sorry to find any of its students claim such pretensions for it as the author here attempts to justify (201). The physical scientist may, of course, legitimately *abstract from* the existence of "non-mechanical conditions," but he may not gratuitously *deny* their existence. Surely, too, a "factor" may be "unascertainable," in the sense of being "incalculable" *in terms of material atoms and motion*, may not admit of "explanation according to" *mechanical* "law," and yet need not be, or be called, "irrational," or "as good as irrational". Is that alone "rational" which is "mechanical"? or is there no "law," no "explanation," for what is not mechanical? Surely this is a vast and gratuitous assumption for anyone to make, whether in the name of physical science or on any other pretext. We fail to see why the physical scientist must *deny* that "events" *may* and *do* "occur in the material order whose conditions are unascertainable within that order". The man who ventures on such a denial would evince more daring than science. Finally, can science "find out how much it can explain" only "by assuming that it can explain everything"? Again we must confess our failure to see the necessity of any such extraordinary assumption.

Bearing those few cautions in mind, we may now glance at the growth of some prevalent views about physical causality.

219. THE SENSIST OR EMPIRICAL VIEW OF CAUSALITY: MILL'S TEACHING.—John Locke (1632-1704) had taught that causality, or the power to produce change, was not "contained in the real existence of things, but . . . extraneous and superinduced"—*i.e.* by the consideration of the mind. But causality in things seems to be as real as their substantiality, nay, as their very existence. Accordingly, either causality is real or all reality is simply a subjectively fabricated idea. The latter alternative was practically accepted by David Hume (1711-1776), who reduced all reality to a series of subjective feelings or states

[1] *op. cit.*, p. 386.

of consciousness; causality thus becoming a mere feeling of expectation of invariable succession in certain of those states—a conviction or belief resulting from the association of repeated experiences.

Though some of Hume's followers renounced the *subjectivity* of this view, all alike clung to the notion of *invariable sequence of phenomena in time* as constituting the essence of causality. "The cause" [of any physical fact, event, phenomenon], writes John Stuart Mill[1] (1806-1873) "is the sum total of the conditions, positive and negative taken together . . . which being realized, the consequent invariably follows. . . . The negative conditions . . . may be all summed up under one head, namely, the absence of preventing or counteracting causes." Here we have the *cause of a phenomenon* described as *that total group of antecedents which, whenever it is realized, is always followed by that phenomenon.*

But day is invariably followed by night; yet we do not call it the "cause" of night. Nor do we call night the "cause" of day, though it has been invariably followed by day. This is so, Mill goes on to say, because the sequence here is not *absolutely* or *unconditionally* invariable: it is invariable only *conditionally* upon the conduct or activity of other things—upon the rising and setting of the sun in the case contemplated. But invariable sequence is not causality, he tells us, unless the invariability arises wholly and entirely from the nature of the phenomena themselves, is unconditioned by anything extrinsic to the latter, is altogether independent of "whatever supposition we may make with regard to other things," and will therefore obtain "under all imaginable circumstances": that is, of course, "as long as the present constitution of things"[2] endures. Such kind of invariable sequence he terms "unconditional";[3] and he then goes on to give his final and scientifically exact definition of "the cause of a phenomenon" as "the antecedent or concurrence of antecedents on which it [the phenomenon] is invariably and *unconditionally* consequent".

By thus defining causation as sequence which is invariable, not

[1] *Logic*, III., v., § 3. Mill properly points out that popular usage generally fixes on *some* prominent *one* among those antecedents and applies to it exclusively the title of "cause".

[2] By this expression Mill tells us that he means "the ultimate laws of nature (whatever they may be) as distinguished from the derivative laws and from the collocations".—*op. cit.*, III., v., § 6.

[3] Notwithstanding the *condition* just set down about the permanence of the "present constitution of things".

by reason of any extrinsic conditions, but *unconditionally* and *by reason of the nature of the phenomena themselves* which constitute the sequence, Mill evidently intended to convey that an antecedent, in order to be a "cause," must have a "necessary" connexion with its consequent. This conception of the "cause" of a phenomenon—as something which, of itself, of its own nature, and not by reason of any extrinsic conditions, is *invariably* followed by that phenomenon—is precisely what had suggested the traditional notion of a *necessary* as opposed to a *free* cause: we are prompted to regard a cause as *necessarily* productive of an effect by observing it to be *always* and *in all circumstances* followed by that effect. By making the invariability of the connexion *independent of all other conditions*, and thus, as the only alternative, dependent on the nature of the connected phenomena themselves, Mill believed that he was giving intelligible expression to

"what writers mean when they say that the notion of cause involves the idea of necessity. If there be any meaning which confessedly belongs to the term necessity, it is *unconditionalness*. That which is necessary, that which *must* be, means that which will be, whatever supposition we may make in regard to all other things. The succession of day and night evidently is not necessary in this sense. It is conditional on the occurrence of other antecedents. That which will be followed by a given consequent when, and only when, some third circumstance also exists, is not the cause, even though no case should ever have occurred in which the phenomenon took place without it. . . . Let me add that the antecedent which is only conditionally invariable, is not the invariable antecedent [in the full sense of *absolutely* invariable—in the future as in the past ?]. Though a fact may in experience have always been followed by another fact, yet if the remainder of our experience teaches us that it might not always be so followed, or if the experience itself is such as leaves room for a possibility that the known cases may not correctly represent all possible cases, the hitherto invariable antecedent is not accounted the cause; but why? Because we are not sure that it *is* the [really, absolutely, unconditionally] invariable antecedent".[1]

But the idea of "necessity" is not the idea of *actually uniform and unvaried* sequence, though it is derived from our experience of the latter; or of *invariable* sequence, except we take *invariable* to mean not only that which has not varied and does not and will not vary, but that which *cannot* vary. And invariability in this latter sense need not be at all *unconditional* in order to be described as "necessity," for the "necessity" itself may conceivably be *conditional:* we can quite conceive a sequence

[1] *ibid.*, 6.

which *must* remain unaltered *so long as certain conditions are fulfilled.* This, in fact, is the only necessity experience warrants us in attributing to the sequences of nature; and, as we shall presently see, Mill's so-called "unconditional" invariability remained always ultimately "conditional".

How, then, it may be asked, from actual, limited experience of *unvaried* sequence in the past, can we get the idea of a sequence that is *invariable—i.e.* which "*cannot*" vary, which is "*necessary*"? Let us first recall the traditional Scholastic account of "physical necessity," which is simple and intelligible. "Necessity" may be either *intellectual, hypothetical, abstract*, connecting possible essences together in thought; or it may be *volitional, categorical, concrete*, connecting actual things or occurrences together in reality. With this latter we are concerned here. When we speak of "that which *must* be" in reference to *things* or *events*, when we say that one thing *must* follow another, we can only mean that they are so constituted and circumstanced that by their very nature or constitution, and collocation, they *cannot help* following each other.[1] Whatever Mill may say, no mere addition or multiplication of "*was*" and "*is*" and "*will be*" can ever generate an *absolute* or *unconditional* "*must be*". What is there, then, in the observed *unvaried* uniformity of nature in the past, to warrant us in thinking not only that it will, but that it *must*, continue unvaried? There is this: there is abundant evidence (of which the fact of *observed uniformity* itself forms a part) to warrant us in concluding with certitude that nature is the work of an All-Wise Creator and Ruler, who has so constituted and arranged its agencies that they will and *must* continue to exist and act, each in its own fixed, uniform way, *as long as He chooses in His wisdom to preserve and sustain it in existence.* Such is the uniformity, invariability, necessity, we ascribe to physical agencies: *conditional* on the Will of an All-Wise, All-Ruling Providence.[2] *There* is the ultimate rational basis which analysis of human experience reveals for our *physical, conditional* certitude about the general laws of physical science.

Now let us observe and contrast the alternative offered by the empiricist philosophy of Mill and his school. The physical cause of a phenomenon, he writes, is that which has always "been followed by" that phenomenon in past experience, provided this experience does not leave any "room for a possibility that the known cases may not correctly represent all possible cases". But if, as Mill's own philosophy teaches, we have no faculties of knowledge beyond our external and internal senses, whose only objects are *sense-phenomena*, associated, compounded, and otherwise modified in consciousness, how can the combination of past and present sense experience give us any

[1] We abstract here, for the sake of simplicity, from the conditional necessity, called *moral obligation*, to which the conduct of free, responsible agents is subject. To these the Creator has given the power to act as they choose; but He imposes on them a *necessity* by which they *must* (freely choose to) act in a certain way *if they are to attain their end.* But every *non-free* cause in animal, vegetable, and inorganic creation, He has endowed with such a nature and constitution as *categorically* directs it to attain the end He has freely intended it to reach.

[2] For a fuller development of these views on the *necessity* of physical laws and causes, *cf. infra*, Chap. IV., 224.

degree or kind of certitude, anything beyond a mere *feeling of expectation* that "all possible cases" will resemble the observed cases? And even for this feeling of expectation Mill can offer us no rational ground whatsoever. He states that our actual experience enables us somehow to make up our minds that a given observed antecedent of a phenomenon *will continue to be* the antecedent of the latter *unconditionally*: which means "whatever supposition we may make about other things," or "under all imaginable circumstances". But this he immediately limits and qualifies by saying that *the antecedent will continue so only* "*as long as the present constitution of things endures*," or *as long as*, and *on condition that*, "*the ultimate laws of nature* (whatever they may be)," *do not vary*. So, after all, the "*unconditional*" *invariability* turns out to be *conditional*. Our sense experience of an unvaried sequence only enables us, therefore, to believe that the latter will continue unvaried, *if* and *as long as* "the present constitution of things," "the ultimate laws of nature," will remain unaltered. And *what rational ground* have we, according to Mill, for believing that "the present constitution of things" *will* continue stable, and their "ultimate laws" unaltered? He does not tell us; and for a good reason: *his philosophy affords none*. It limits our knowledge to phenomena of sense; it is *agnostic*: it informs us that we can know facts, but nothing about the inner nature and ultimate causes of those facts. And thus the empirical theory of induction, as of knowledge generally, destroys all certitude by rearing the whole edifice of physical science on the basis of an underlying confession of helpless and hopeless ignorance.

In giving his final description of "cause" as the "*unconditionally*" *invariable antecedent*, Mill explained that the "cause" must be not merely the hitherto invariable antecedent, but something which is the antecedent in "all possible cases". By this he has been commonly interpreted to mean that the "cause" in in the strict sense is the antecedent (or group of antecedents) which is not merely "*sufficient*"[1] but "*indispensable*" for the appearance of the consequent: not only the antecedent which is invariably followed by the consequent in question, but *by which alone* the consequent is *invariably preceded*: so that the invariability is on both sides, the relation is a reciprocal one, and inference can proceed from effect to cause as well as from cause to effect.

Thus we may distinguish, in Mill's account of causality, (1) the looser scientific concept of "cause" as the antecedent (or group of antecedents) which, when present, is *always* followed by a certain consequent; (2) the still looser popular concept of "cause" as denoting some *one prominent* element of that group (abstracting from the others); (3) the stricter and more exact

[1] In the sense of "necessitating". This is the meaning commonly attached to the word in regard to physical causes. A free cause may be "sufficient" (in the ordinary sense of the word) to produce an effect, and yet not *necessitate* that effect.

scientific concept of "cause" as the antecedent which is not only *always* followed by the consequent in question, but which is the *only* antecedent so followed: not only the *sufficient*, but the *indispensable*, antecedent of that consequent.[1]

220. CAUSALITY, SEQUENCE IN TIME AND CONTIGUITY IN SPACE.—Mill's account is intelligible so far as it goes. He has, of course, never succeeded in assigning any ultimate rational explanation of the *fact* that natural causes and their effects *are* connected in the uniform, unchanging, "invariable" manner indicated by him. Apart from this defect, however, which is due to his empiricist theory of knowledge, there is the erroneous implication that *time sequence* is essential to causality, that two phenomena cannot be related as cause and effect unless they succeed each other in time.

Now, efficient physical causality does not necessarily imply that the cause must totally *precede* the effect *in time*. Even popular thought, which seizes on one prominent, partial element in the total cause—often a remote element—and on a similar element in the effect, does not regard "cause" and "effect" as *separate*, successive events, but only as distinct: the *immediate* cause and the *immediate* effect are always thought of as connected. The link connecting them—the *causation*, *action*, *change*, or *process*, as it is variously called—goes on in time and occupies time. The *immediate* cause, therefore, cannot entirely precede, but must also coexist with, the *immediate* effect. The producing cause and the produced effect must be simultaneous, for they are

[1] *Cf.* JOSEPH, *op. cit.*, pp. 64, 65—"When we call one thing [*i.e.* kind of thing] the cause of another, the real relation between them is not always the same . . . we say that molecular motion is the cause of heat, that the heat of the sun is the cause of growth, that starvation is sometimes the cause of death, that jealousy is a frequent cause of crime. We should in the first case maintain that the cause and effect are reciprocally necessary; no heat without molecular motion and no molecular motion without heat. In the second the effect cannot exist without the cause, but the cause may exist without the effect; for the sun shines on the moon but nothing grows there. In the third, the cause cannot exist without the effect, for starvation must produce death, but the effect may exist without the cause, since death need not have been produced by starvation. In the fourth case we can have the cause without the effect, and also the effect without the cause; for jealousy may exist without producing crime, and crime may occur without the motive of jealousy. It is plain, then, that we do not always mean the same thing by our words, when we say that two things are related as cause and effect; and any one who would classify and name the various modes in which two things may be causally related would do a great service to clear thinking." And the author adds: "that is the sort of service that Aristotle attempted in distinguishing the heads of predicables". *Cf.* also *op. cit.*, c. xxii.

correlative. If the cause *ceases to act*, then the effect *ceases to be produced*; for the "action" (*actio, facere*) of the cause and the "production" (*passio, fieri*) of the effect are *one and the same* process of *real change*. Hence the Scholastic axiom "*Cessante causa, cessat effectus*".

The act of "taking poison" may have ceased long before "death" occurs; but the poison, once introduced into the system, continues to exist and to operate, effecting changes which in turn cause other changes, until finally a condition of the organism is reached, which is so striking, familiar, and significant that it has received a special title to indicate it, *viz. death*. The first "act," and the final "state" or effect, are, therefore, connected by a continuous process of natural causation, each stage of which is both an effect (of the preceding one) and a cause (of the subsequent one); and wherever we draw a line of distinction in this process of change, the state of things on the one side of the line is the *immediate* cause, of which the contiguous state on the other side is the *immediate* effect.

"Cause and effect," writes Dr. Mellone,[1] "are divided by a simple mathematical line—a line destitute of breadth—which is thrown by our thought across the current of events; on one side we have the cause, on the other the effect. There is no pause in reality; the whole process is continuous; the immediate cause comes into full action only at the very moment when the effect begins to be produced. The point to be borne in mind is the *continuity* of cause and effect."

The whole process of change in the occurrence of any physical phenomenon is, therefore, continuous: there is one continued "*motus*" or motion throughout: this *motion* may be regarded either from the point of view of its origin, or from that of its termination: it will be called *action* ("*actio*") when looked at from the side of the cause or *agens* from which it originates, and "*passio*" when looked at from the side of the effect or *patiens* in which it terminates. The Scholastics marked and emphasized their appreciation of the unity and continuity of the whole process by crystallizing their view in the dictum "*Actio et passio sunt idem numero motus*": "*Acting*" and "*being acted on*" are one and the same real "*motion*," looked at from different standpoints.

But the Scholastics were at the same time careful not to confound the actual process of change ("*fieri*") either with the efficient causes themselves on the one hand, or with the stable

[1] *op. cit.*, p. 273.

result of the change (the effect "*in facto esse*") on the other. They rightly distinguished in every such process the (material) *substances* or agents at work (*substantial* causes), the *forces* or *powers* (*proximate* principles of action) through which those agents or causes act, and the *action* or *process of change* itself (218). They distinguished, furthermore, between the extrinsic causes (*efficient* and *final*), which they called "causes of the *actual change*" (*i.e.* of the "*fieri*" or *production* of the effect), and the intrinsic causes (*formal* and *material*) which they called (constitutive) causes of the *produced result* or effect, in its completed state ("*in facto esse*".)[1]

Ignoring those distinctions, modern writers have fallen into the error of actually *identifying* the "efficient cause" with its "effect," by regarding each as *a mere aspect of the process of change* itself, and this latter, apparently, as the sole reality. For example, Professor Welton[2] arrives at this conclusion: "Cause and effect are not two but one. That they are inseparable is indeed recognized by the relativity of the very terms themselves. A cause without an effect or an effect without a cause is a contradiction in terms and unthinkable. But we must go farther and say that in *content* they are absolutely identical. It is only in form that they can be distinguished and then we may speak of the one as determining and of the other as determined. Thus the combination of hydrogen and oxygen in the quantitative ratio of two to one determines that the effect shall be water, and the character of that effect is determined by the character of the elements which are combined, but the combined elements and the water are one and the same identical substance, and this substance is the content both of the cause and of the effect."

This is indeed going very far; much too far. To *identify* the efficient cause with its effect, the "producer" with the "produced," is not only setting popular thought and belief at defiance, but even espousing the implicit contradiction that an effect can produce itself.

When, therefore, we come to reflect on the *immediateness* of the cause to the effect, we see that while the scientist must indeed aim at grasping the former as closely as possible to the latter, in order to be sure of including every indispensable factor in the former, and so attaining as closely as he may to the ideal of a *reciprocal causal relation*, he must guard equally against *identifying* the cause with the effect, under pain of making all experimental search for "causes" meaningless and impossible. For, if the *effect* is *identical* with the *cause*, then when we know the effect we know the cause, and there can be no meaning in searching for the latter. Our "reciprocal relation" appears to have become a mere tautology; "The statement that cause and effect are 'identical' . . . becomes an extravagant paradox if taken seriously and applied to any particular case of causation determined by scientific experiment".[3]

[1] *Cf.* St. Thomas, *Summa Theol.*, i., q. 101, Art. 1;—*apud* Joyce, *op. cit.*, p. 248.

[2] *Op. cit.* ii., p. 25.

[3] Mellone, *op. cit.*, p. 274.

This confusion of cause with effect arises from losing sight of the category of *substance*, and of the all-important Scholastic distinctions between *agent* and *action*, and between extrinsic (efficient) and intrinsic (formal and material) causes[1] (216, 218). These distinctions are *real*; they are *in the reality*; they are not merely *mental* or *logical*, different ways of regarding one and the same reality. In all processes of physical change the formal and material causes are intrinsic to, and identical with, the interacting agents, because they *constitute* these latter. In the change by which oxygen and hydrogen produce water, the two former are *materially* identical with the latter, but then they differ *formally* (in their "formal" or "specifying" causes) from the latter and from each other. "The combined elements and the water are one and the same identical substance"; but if they are, they are *really different* from the *separate* elements, for these *on combining*, on *becoming* water, on *assuming* the "form" or "specifying principle" of water, lost the "forms" of oxygen and hydrogen respectively. If the water were *really identical* with the oxygen and hydrogen, the change would not have been *real* but merely *mental*: that is to say, the processes of external nature would not be *real* but *illusory*: the only real change taking place would be the change involved in the logical process of the thinking mind. And this, in fact, is what the advocates of Hegelian idealism profess to believe (215).

Similarly, physical causes occupy space and act in space, but that contiguity in space, direct or indirect contact, is essential to their activity, is not clearly evident. That there is and must be a connexion of some sort, *in reality* as well as *in thought*, between cause and effect, is undeniable. But is *actio in distans, i.e.* across *empty* space or *vacuum*, metaphysically or physically impossible? We know too little about the nature of matter, space, and material action or motion, to give a categorical and decided answer to either part of the question. "How can a body act where it is not?" Professor Welton repeats the old puzzling query,[2] and hazards an answer. But would it not be as well honestly to confess our ignorance of the "how"—remembering that this does not prove impossibility—as to say that the *body is there*, where its influence is felt, "in one very true and important sense of its reality"—in the sense of exerting influence there—while it *is not present there*, but *absent* from there, and *present in another place*, "in another sense of its reality — the sense in which reality is identified with visible and tangible form and tangible resistance"? What then is *space?*—if different "senses" or "aspects" of a body's reality may be in different parts of it? The author does not inform us; though, a few pages further on,[3] he seems to reduce all physical efficient activity to local motion, and this latter to change of "spatial relations". This reduction of even *qualitative* and *substantial* changes in physical nature to *mechanical* or *local change* has only its simplicity to

[1] *Cf.* JOSEPH, *Logic*, p. 451. [2] *op. cit.*, pp. 20, 21.

[3] *ibid.*, p. 24: "When it is said in this connexion [in mechanics, regarding the conservation of energy] that 'the cause equals the effect,' the 'cause' spoken of is not a thing but the efficient action of a thing, and this action reduces itself to its permanent attributes in a certain spatial relation to the object on which it acts". The efficient action of a physical cause is thus analysed into certain permanent attributes of that cause, *plus* certain spatial relations between it and the "object" upon which it is conceived to "act".

recommend it (217). It *explains* nothing adequately; and it is in fact rejected as inadequate by the author himself in a subsequent chapter of his logic.[1]

221. "PLURALITY OF CAUSES": "RECIPROCAL" AND "NON-RECIPROCAL" CAUSAL RELATIONS.—We have seen, so far, that the term "cause" has a multiplicity of kindred meanings: that besides the "formal" and "material" causes, or intrinsic constitutive principles of the visible, material agencies in nature, and besides the "final" causes, "ends" or "purposes" for which these act, there are also these agents themselves, which we have called "efficient" causes. We have distinguished between these efficient causes and the "action" or "motion" or "process" by which they produce their effects. We have also distinguished between efficient causes that are "free" or "self-determining" and efficient causes that are "necessary" or "necessitating"; and we have seen that we can lay down general propositions about the mode of operation of efficient causes throughout space and time only in so far as we are convinced that those causes act uniformly beyond the range of our own actual sense experience (218); observing that this uniformity, though not absent from the domain of free causes, is much more prevalent and reliable in the domain of "necessary" causes—that is, in the physical sciences, with which induction is mainly concerned (218, *cf.* 223). We have seen too that, generally speaking, every class or kind of phenomenon in nature results from the convergence and combination of numerous influences, agencies, and conditions, which are collectively "sufficient" (or "necessitating") and severally "indispensable" for the production of that special kind of phenomenon (216). The multiplicity and variety of these conditions, and their inseparable connexion with conditions *not* needed for the production of this kind of phenomenon, render it difficult for science to sift out and group together as the "cause" of the phenomenon, just those influences and *those only* which are sufficient and indispensable for its production. Combined with this difficulty of bringing to light the "cause" in this narrower and stricter sense of *reciprocal* cause (*cf.* 213), we have the consideration that from the *practical* point of view—*i.e.* of producing or preventing effects—acquaintance with a *plurality* of alternative "causes," in the wider sense of "sufficient" though not "indispensable" modes of producing that sort of effect, is more important and more desirable than an exact knowledge of

[1] *op. cit.*, pp. 209, 210. *Cf.* JOSEPH, *op. cit.*, p. 382.

the one "reciprocal" or "commensurate" cause of that effect. Hence the question arises, whether *Science* ought to aim at the discovery of *reciprocating* causal relations, or merely of causal relations such that although the "cause" will necessitate the "effect," this latter will not necessitate the former, but admit of a "*Plurality of Causes*" (*cf.* 138).

If we take a "physical" or "necessary" "cause" in the *popular* sense of some prominent or striking event which, when it happens, is always followed by another remarkable event (the "effect"), it will be evident that though the same natural cause, acting in similar circumstances (*e.g.* administering deadly poison), always produces the same effect (*e.g.* death), nevertheless the same effect (death) need not be always produced by the same cause (poison): that although "effect" can be inferred from "cause"—"*posita causa, ponitur effectus*"—still the converse, "cause" from "effect"—"*posito effectu, ponitur causa*"—cannot be lawfully inferred. And the reason is that in this sense of the term "cause," *the same "effect" may be produced by different "causes"*: that one and the same effect—death, for example—may be due to any one or more of an indefinite multitude of "causes". We speak popularly of an agency as the "natural cause" of a given result or effect, provided that this agency be *sufficient* (or *necessitating*)—even though it be not *indispensable*, in the sense of being the *only possible* agency—for the *production* of such an effect. And formal logic, recognizing this mode of thought and expression, and applying the Law of Parsimony (94), prohibits the simple conversion of the conditional proposition, which connects cause with effect as logical antecedent with consequent, as reciprocal (140). If, therefore, we take the terms "physical cause" and "effect" in this *practical* meaning, the former as that which always and necessarily produces the latter, and the latter as that which is produced, no doubt, by the former, *but which is or may be produced otherwise as well*, then it will be true to say that one and the same "effect" may have a *plurality of "causes,"* though one and the same (natural or necessary) "cause" cannot have a plurality of "effects".[1]

[1] Evidently, the co-operation or "composition" of many *partial* causes may contribute to the production of one single effect: *e.g.* a person's death may be due to a complication of diseases no single one of which would separately have proved fatal (*cf.* 244). But "composition" of (partial) causes is quite a different thing from "plurality of causes". The latter means that one and the same effect (*death*) may, in different instances, be produced by entirely different total causes (*poison, shooting, smallpox, old age*, etc.).

On the other hand, were we to understand by the "cause" of a given kind of event not any and every factor (or group of factors) capable of producing it, but that precise factor (or group), and that only, which (being present in all modes of producing it) *is itself capable*, and *alone* capable, of producing the event, then this kind of event can be produced by that one "cause," *and by it alone.* In other words, no event can have a "plurality of causes" in this stricter sense of the term "cause".[1] The doctrine that the same effect can have a "plurality of causes" holds good "as long as the 'cause' is understood in the popular way. *The plurality disappears before any exact scientific investigation*".[2] The subtraction of any factor from the "total cause" in this strict scientific sense, or the addition of any new factor to it, must necessarily modify the effect: no other factors or combinations of factors could produce this sort of effect exactly and identically. This is but a simple application of the principle of identity. E is an effect whose total cause (or, the totality of whose *sufficient* and *indispensable* antecedents) is $a + b + c$. But, if it is so, it cannot at the same time, being and remaining identical with itself, be the result of $a + b + c + d$, or of $a + b + d$, or of $a + b$, or of $m + b + y$, or of any other conceivable combination.[3]

222. SCIENCE AND THE DISCOVERY OF "CAUSES" AND "LAWS".—In popular thought, therefore, the notion of "physical cause" *usually* includes elements not indispensable to the production of the effect, though the notion of the "effect" does not include any element which is not necessitated by some element or other of the cause.[4] The reason for this peculiar difference

[1] The mediaeval Scholastics discussed "plurality of causes" in connexion chiefly with the *individual* effect, and the principle or ground of its individuation; proposing the problem in terms like these: "Would Alexander the Great have been the same individual had he been born of other parents than Philip and Olympia?" Their answer was usually in the negative. *Cf.* ZIGLIARA, *Ontologia* (46), vii.

[2] MELLONE, *op. cit.*, p. 277.

[3] *Cf.* JOSEPH, *op. cit.*, pp. 377-8.

[4] We say "usually," for there are evidences that the popular mind is quite familiar with some applications of the scientific conception itself: for instance, with the procedure at coroners' inquests, and with the convictions of criminals on circumstantial evidence. "The popular idea of the non-reciprocal character of the axiom of causation," writes Professor Welton, "is due to the fact that the 'cause' is much more frequently analysed than the 'effect'—using those words in the popular sense of temporal antecedent and consequent phenomena. Thus, when Mill says in support of his doctrine of the Plurality of Causes, 'Many causes may produce death' (*op. cit.*, Bk. III., ch. x., § 1), he is obviously speaking very loosely. Death is not the whole effect. Moreover, death can never be death in general, but only some one particular kind of death, and the death caused by a bullet through the heart is not the same kind of death as that due to drowning, and both again differ

between "cause" and "effect," in the popular sense of these terms, is not far to seek: it arises from the *practical* attitude of people in real life towards causality. When they want to produce some one given *kind* of effect (death, for example), it may matter little to them what *particular*, *individual* form or character this effect may assume, but it will evidently be of the greatest importance for them to have a large number of distinct *alternative*, *individual* "causes," or modes of producing the *generic effect*, to choose from. Hence, while people regard the "effect" in the abstract, contenting themselves with one generic name for it in all its varied individual manifestations, and care little to distinguish between these latter in the concrete, they behave in quite the opposite way towards the "cause": noting and distinguishing carefully, and often naming separately, its various concrete, individual modes or forms, and calling *each of these a "cause"*.

"The reason," writes Dr. Venn, "why we look out for a cause is not to gratify any feeling of curiosity, at least not primarily, but because we want to produce some particular effect. . . . What the savage mostly wants to do is to produce something or to avert something, not to account for a thing which has already happened. What interests him is to know how to kill somebody, not to know how somebody has been killed. Of course the past must interest him to some extent, because what has happened once may come to pass again, but this is a comparatively indirect or remote reference. What holds good of the savage does so also, though to a somewhat less extent, of the great majority of ordinary people: the explanation of the past will rationally be far subordinate in interest to the prediction of the future. . . . When we want to *explain* a fact an offer of several alternative solutions affords very little help. . . . The scientific student of early culture vexes his mind to ascertain in which of various possible ways fire was first produced, and employed by man; whether by lightning, by friction of boughs of trees, by sparks from flint chips, or so forth. But for those whose only care was to make a fire when they wanted it, such plurality of causes was all in their favour." [1]

And Dr. Mellone thus happily illustrates the same truth: "Sometimes what is practically most important is scientifically least important: it may be of great importance to know what circumstances will produce an event without knowing *how* they produce it. For instance, it may be of importance to clear

from death by poison, and so on. The effect as a totality differs in each case from that in every other case, and the very existence of the enquiries of coroners' inquests is a practical assertion of even popular belief in the reciprocity of the causal relation, as it assumes that by a careful analysis of the total 'effect' the cause is arrived at, and this assumption can be only justified on the ground that this totality could have had but one cause."—WELTON, *op. cit.*, pp. 27-8. *Cf.* JOSEPH, *op. cit.*, pp. 446-47.

[1] VENN, *Empirical Logic*, pp. 56, 63, 64.

the premises of rats ; traps, strychnine, phosphorus, and terriers are various 'causes' between which we must choose: but we do not as a rule hold post-mortems on dead rats."[1]

What Dr. Venn says of the savage and of the ordinary man is also largely true of the scientist: he, too, has a practical as well as a speculative aim in his researches. It is not his *sole* concern to *explain* an effect by bringing to light its necessitating *and indispensable* antecedents, *i.e.* its reciprocal or commensurate cause: he *also* wants to discover all the alternative combinations of existing agencies and conditions which *embody* the indispensable factors (in inseparable conjunction, perhaps, with many superfluous or indifferent elements)—combinations which constitute so many practical alternative modes of producing the effect in question.

"Properly speaking," writes Mr. Joseph, "to give the cause of anything is to give everything necessary, and nothing superfluous, to its existence. Nevertheless we should often defeat our ends if we gave precisely this ; if our object in seeking the cause of a thing is that we may be able to produce or prevent it, and if something is necessary to its existence which is a property of an object otherwise superfluous, it would be of no use specifying the property necessary unless we specified the otherwise superfluous object in which it was found." This the author illustrates by remarking: "It may be the texture of the pumice-stone that fits it to remove ink-stains from the skin ; but it would be of more use to tell a man with inky fingers to get a piece of pumice-stone, than to give him a description of the fineness of texture which would render a body capable of making his fingers clean"[2]. Similarly, with regard to the "elasticity" of the air (or other elastic medium) as a cause of the transmission of sound: "We want to know what possessed of the necessary elasticity is present when we hear at a distance ; nor could anyone without knowing that prevent the transmission of sound by removing the elastic medium ; for he would not know what to remove".[3]

In so far, then, as the scientist has this *practical* aim before him, he will rest content with discovering "causes" in the wider sense of this term—the sense in which an effect can have a "plurality of causes," *i.e.* of alternative modes in which it may be produced. Under the influence of this "practical" view of inductive science, Dr. Venn regards this wider conception of cause as "the most serviceable for purposes of inductive logic".[4] And in this he is undoubtedly right. But he goes farther, and asserts that the stricter concept of cause—that which makes the causal relation reciprocal—

[1] *op. cit.*, p. 275 ; *cf.* JOSEPH, *Logic*, p. 446. [2] *ibid.*, n.
[3] *ibid.* [4] *Empirical Logic*, p. 71.

"necessarily results in rendering it useless for *any* purposes of inference". "Make it perfectly complete and accurate," he continues, "and you make it at once hypothetical and the statement of what is to all intents and purposes a mere identity."[1] [Such an over-refinement of the law of causation] "renders that law suitable only for hypothetical conclusions, in other words, renders it useless for positive inductions about matters of fact".[2]

Now, it is of course true that if we make our concepts of "cause" and "effect" so comprehensive and closely connected as to involve each other reciprocally, we are not likely to get beyond the hypothetical "If *A* then *C* and *vice versa*," to the categorical "This *A* will always involve this *C* and *vice versa*". It is true, too, that knowledge of the *one immediate, indispensable* "cause" of *C* is of less *practical utility* than knowledge of the numerous alternative groups of antecedents in which this one "cause" is operative. But it is likewise true that if we want a *scientific*, even though conditional, *knowledge* of *C*, a knowledge of *how* it is produced, we must try to seize the process at the instant of the production of *C*, and to detect—if we can, or as far as we can—all that is *indispensable* for its production. In other words, when our aim is not directly practical—like that of the "savage" for instance: to compass a person's death in some way or other—but rather speculative—like that of the coroner, for instance: to discover how this person's death has been compassed—we must obviously seek, not for all the alternative ways in which the person could have been killed, but for all the factors indispensable to the way in which *this particular person* has been killed. So, too, when we want to *explain* a *kind* of result, we must seek, not for the various modes of producing it in different sets of circumstances, but for *that which is common to all the modes*, and which, being always "sufficient" and always "indispensable," will produce it in any and every conceivable set of circumstances. For instance, that "kind" of effect called "death"—that which is common to all individal instances of death and in virtue of which we call each of them a "death"—will be scientifically explained by us only when we have succeeded in discovering what precise factor (or group of factors) is present in every conceivable mode of producing "death," as being *sufficient and indispensable* for its production. A full scientific knowledge—were such attainable—of the relation between a "natural" or "necessary" cause and its effect, would thus show the relation to be *reciprocal*. One and

[1] *Empirical Logic*, p. 71 (italics ours). [2] *ibid.*, p. 52.

the same *genus* of effect (*e.g.* death) can have only one and the same *genus* of cause (*viz.* that *generic* element which is common to all *species* of causes of death, making them all alike destructive of life); one and the same *species* of effect (*e.g.* death from smallpox) can have only one and the same *species* of cause (*viz.* the microbe of smallpox); any one *individual* effect (*e.g.* the death of Julius Cæsar) can (in its individual totality) have had only one *individual* (total) cause, *viz.* that to which it was actually due.[1]

Now, in the mathematical sciences we establish numerous universal truths which are reciprocal.[2] But is it always possible to establish *reciprocal causal relations* in the inductive sciences? On the contrary, it is rarely possible. Some logicians set it up as an ideal at which the scientist must always aim. He is told to *commence* his scientific investigations by working with the *popular* concept of cause, which admits "plurality of causes," and to try to approximate gradually towards the *scientific* concept which excludes plurality. Dr. Mellone's expression of this theory is clear and accurate. "In the absence of scientific knowledge of the immediate cause, we have to bear in mind that different combinations of circumstances may bring about the same event. Practically we have to begin the investigation by examining those different combinations of circumstances in which the event is produced — considering them, at first, as so many different 'causes'. They are not the immediate cause; but *it is operative in them.*"[3] We are to commence, therefore, with the various distinct modes ("causes" in the popular sense) of producing a certain kind of effect, and to finish by abstracting what is *common* and *essential* in all of them (the "cause" in the stricter sense).

But this ideal is often unattainable, and, if attained, would be often comparatively useless and uninstructive; and this is so because "the phenomenon under investigation is often highly complex, and subject to all sorts of variation on the different occasions of its occurrence, through variations in the

[1] *Cf.* JOSEPH, *op cit.*, p. 65: "Whenever science tries to find the cause not of a particular event, such as the French Revolution (whose cause must be as unique as that event itself is), but of an event of a kind, such as consumption, or commercial crises, it looks in the last resort for a *commensurate* cause. What is that exact state or condition of the body, given which it must and without which it cannot be in a consumption? What are those conditions in a commercial community, given which there must and without which there cannot be a commercial crisis?" The same is true, of course, in regard to the cause of a "particular event"—except that this is regarded as belonging to the domain of history, not of science, which is "of the *universal*".

[2] *Cf.* JOSEPH, *op. cit.*, pp. 443, *supra*, 212.

[3] MELLONE, *op. cit.*, p. 277.

objects or events contributing to its production; *not the whole nature* of the objects or events under whose influence it occurs *is relevant* to its occurrence, but only *certain particular properties or modes of action;* and it is possible *to formulate severally the principles of action involved*, from which the joint result may be seen to follow, where it would not be possible to assign to the phenomenon any group of concrete objects or events as cause, about which we could say not only that, given them the phenomenon must be given, but also that, given the phenomenon, they must have been given too "[1]. . .

"For example, we may ask what is the cause of the monsoons—that is, of the regular and periodic winds that blow steadily in certain regions for one part of the year and for another in the opposite direction? If we said that they were due to periodic alterations in the distribution of atmospheric pressure, it would not be very instructive; for we really want to know what events happening in those regions, produce these differences. Yet the events which contribute to determine the deviation and direction of the monsoons are numerous and variable. . . ."[2] And these numerous and variable events are due to the variously combined influences and activities of *sun* and *sea* and *land* and *air* and *aqueous vapour*—among other things. Now, in such a case as this, to seek for the *reciprocating* or *indispensable* "cause" of the monsoon would be futile. "To give the cause of monsoons, without deficiency or superfluity, would mean that we must not mention the sun (because only the heat of its rays is material), nor the sea (because only its fluidity and its power of giving off vapour concern us, and a lake, if it was big enough, would do as well), nor any other of the concrete things which act in the way required, but only their requisite actions."[3] But no one would dream of giving the cause of the monsoons without mentioning those various agencies; and in giving them "we shall have to include in our statement of the cause elements at least theoretically superfluous". Shall we, then, rest content with a bare enumeration of these partly superfluous agencies; simply stating, in explanation of the monsoons, that these are due to the combined influences of sun and air and land and sea? No; something more than this is expected, even though an exact statement of the "commensurate cause" is not expected. There is a middle course, a third alternative, which is expected, and it is this: that we "look for the *principles* in accordance with which [these] objects [or agencies] *act under certain circumstances;* then we can show that the monsoon is only the complex result of the action of a number of objects under the particular circumstances of the case, and *in accordance with the principles of action* which our 'laws' express"[4]. In other words, "we alter the form of our problem. Looking upon the phenomenon as the complex result of many conditions, we attempt to determine not [merely] what assemblage of objects or events will produce the result, nor [again, attempt to determine] on what properties or events therein it depends [the reciprocating cause]; but *what is the principle of action in [the] different objects or events*, in virtue of which some one particular condition necessary to the production of the phenomenon is realized in them. For the reciprocating cause of a complex phenomenon we substitute as the object of our search the *principle in accord-*

[1] JOSEPH, *op. cit.*, pp. 445, 446 (italics ours). [2] *ibid.*, p. 444.
[3] *ibid.*, p. 445. [4] *ibid.* (italics ours).

ance with which a certain kind *of object or event acts.* Our problem is *better expressed as that of discovering laws of nature than causes.*"[1]

In explanation of the monsoons, for instance, we are expected "to point out the difference in the power of the sun at any place produced by the varying directness of its rays; how the sea gives off vapour; how vapour absorbs part of the heat of the sun's rays; how the heated water circulates with the colder; how the earth absorbs and retains the heat of the sun; how air is expanded by heat; how the principle of atmospheric pressure acts under conditions of different expansion; and so forth. Then we can see that if a certain combination of events occurs, a particular complex result must arise; if the sun travels from over the surface of the sea to over the interior of a continent, we shall find monsoons; for the difference between summer and winter temperature will in the interior be very great, but on the sea, owing to the way in which the moisture of the air absorbs part of the heat, and the currents in the water carry away part, it is not so great; hence as summer is ending, the air inland will be hotter and have expanded more than out at sea, as winter is ending, it will be colder and have contracted more; so that at one time the current of air sets inland in accordance with the laws of atmospheric pressure, and at another time it sets shoreward".[2]

Here we have an admirable example of the explanation of a complex effect by stating the *laws* according to which the various contributing agents act in such conjunctions as to bring about this effect. It is not the "laws" or their combinations that *produce* the effect; it is the various "causes" or "agents" that produce the effect, by acting each according to its own uniform "principle of action," that is, according to the "law" of its nature (217). These "laws" are both *descriptive* and *explanatory*: descriptive of the modes of action of the *causes*, and explanatory of the *effects* by showing how these latter are brought about by those causes (255).

JOSEPH, *Logic*, chaps. xix., xxii. WELTON, *op. cit.* ii., pp. 1-60. VENN, *Empirical Logic*, chaps. i., ii., iii. MERCIER, *Logique*, pp. 298-332. MELLONE, *op. cit.*, pp. 264 *sqq*. JOYCE, *Logic*, chaps. xv., xviii. MILL, *Logic*, iii., v.

[1] JOSEPH, *op. cit.*, p. 444 (italics ours). [2] *ibid.*, pp. 444-5.

CHAPTER IV.

PRESUPPOSITIONS OF INDUCTION: UNIFORMITY OF NATURE.

223. INTERPRETATIONS OF THE PRINCIPLE OF UNIFORMITY IN NATURE.—In the preceding chapter attention was called to two principles presupposed by induction: "sufficient reason" and "causality". There is another principle which it postulates still more directly and explicitly, the *Uniformity of Nature.* The aim of induction being to reach—as far as may be—general truths or laws about certain domains of our experience, it does and must assume in a special way that the agencies which it studies be uniform in their modes of operation throughout space and time. Only in so far forth as these agencies act regularly, uniformly, will our generalizations about them be reliable. About what is variable, unstable, capricious, we can make no certain or scientific general statement.[1] We can have science only of what is orderly and amenable to law. Therefore, underlying the inductive process by which we establish general laws of nature, there is the postulate known as the *uniformity of nature.* It has been stated in many alternative ways by logicians, philosophers, and scientists, the most usual formula, perhaps, being this one: "*The same physical causes, acting in similar circumstances, produce similar results*".[2] There has also been much discussion about its precise import and relation to induction, about the origin of our belief in it, and the grounds on which we yield it our assent.

Before examining these questions, a word about the sphere of application of the principle may not be out of place. Strictly

[1] *cf.* JOSEPH, *op. cit.*, p. 374.

[2] Compare the formula of Duns Scotus: "Whatever has resulted *regularly* and *constantly* from the action of a non-free cause cannot be due to chance, but must be connected with the *nature* of that cause, and will therefore always result from it" (*supra*, 208). Other alternative statements are: "Nature is uniform in its mode of action"; "the future will resemble the past"; "the unobserved will resemble the observed"; "the unknown will resemble the known" (*cf.* VENN, *op. cit.*, pp. 119 *sqq.*).

speaking, it applies only to the action of *non-free* or *necessitating* causes; these we call "physical" or "natural" causes in the present context, as distinct from the free, self-determining activity of the human will. The action of the former produces physical uniformity, that of the latter only uniformity in the wider sense —moral uniformity. But it would be a mistake to imagine that this looser and less reliable sort of uniformity, which characterizes the phenomena dependent on human activity, is an insufficient groundwork for *scientific* knowledge of these domains: the very existence of the various social and economic *sciences*, their co-existence with human free-will, disproves any such assumption.[1] About the generalizations of the latter sciences we can, of course, have only moral certitude, not physical; and it is to non-free causes, and to the law of *physical* uniformity, that we must mainly direct our attention here.

Is the law of uniformity, as understood to apply to the action of non-free causes, an axiomatic, *self*-evident, necessary, "analytic" principle—like the principle of causality, for example, that "whatever happens has a cause"? Or is it rather a derived, "synthetic," *mediately* evident truth, to which we assent only on grounds of experience? Some have held the former, some the latter view. As a matter of fact the principle can be, and has been, interpreted in two ways. Understood as a *hypothetical* judgment, it is a self-evident, axiomatic truth; regarded as *categorical*, it is a truth of experience.

The hypothetical judgment, "If, or whenever, or wherever, the same physical (non-free) cause acts in similar circumstances (and therefore unimpeded, not interfered with by other causes), it will always produce the same sort of effect,"—is an axiomatic, analytic, self-evident judgment. For, as Father Joyce expresses it, "the very concept of a natural agent, devoid of free-will, involves that, under the same circumstances, its action will be of the same kind".[2] It is a judgment whose truth the mind grasps directly and intuitively from an adequate understanding of the notions involved in it: "physical, non-free cause," "repeated action unimpeded," "similarity of effect".

But the principle, thus stated, makes no *categorical* assertion

[1] *Cf.* MAHER, *Psychology*, 4th edition, pp. 423-4. Mr. JOSEPH (*op. cit.*, p. 375) seems to identify man's free actions, with *capricious*, *motiveless* actions, and to regard them as, therefore, "incalculable". But this is not an accurate conception of the "libertarian" view of the will. *Cf.* MAHER, *op. cit.*, pp. 396 *sqq.*

[2] *Principles of Logic*, p. 237.

about any individual case. It "supposes the First Cause to preserve the ordinary operation of natural laws".[1] This supposition is explicitly contained in the reference to "similar circumstances". A case of interference by the First Cause would *alter* the circumstances. Such a case would not come under the principle. As stated, therefore, the principle is a metaphysically necessary one. It is, moreover, *self-evident* to anyone *who understands the import of the concepts involved in it.* These, however, are complex concepts, and to acquire them is a work of time and experience; for which reason we may admit that the principle, even understood hypothetically, is, to use the words of Mill,[2] "by no means one of the earliest which any of us . . . can have" reached. It is a *propositio per se nota in se* (86). That is, it is an analytic, *a priori* proposition, whose truth is grasped intuitively by the mind as soon as the concepts involved in it are fully analysed and juxtaposed in thought. But we may freely admit that it is not a *propositio per se nota quoad omnes*, that it is not—like "two and two are four"—immediately evident to everybody, because not everybody has clear and definite notions about the nature of a physical or non-free cause, its activity in similar conditions, and uniformity of effect.

We need to become familiar with the ordinary operations of nature in order to conceive the notion of *natural cause, i.e.* of a cause which is *not free to determine itself*—as the human will does—to produce this, that, or the other effect: a cause which has one definite, fixed line of action, one stable tendency which it endeavours as it were to realize and satisfy by its action. But, as soon as a person *has* formed, from his experience of the uniform recurrence of natural phenomena, the idea of a "*physical or natural cause or agent, acting repeatedly in similar sets of circumstances,*" he will see intuitively, by an analysis of that concept, and comparison with the concept of "*uniform production of the same effect,*" a metaphysically necessary connexion between them.

The principle of uniformity, understood in this *hypothetical* or *formal* sense, is, however, nothing more than a *purely formal* generalization of an abstract judgment, which prescinds from the actual existence or occurrence of any such entity as a "physical or non-free cause". It does not *imply* that *there are* such causes in existence, nor that they act repeatedly in similar circumstances,

[1] JOYCE, *op. cit.*, p. 238. [2] *Logic*, III., xxi., 2.

but merely states that "*If* such causes do exist and act thus, they will always produce the same classes of effects".

It may, perhaps, be objected that we could not have formed the notion of "non-free causes" at all, unless there *were* such causes in the world revealed to us by our senses. This, however, is scarcely so. The data of our sense knowledge must, of course, have presented such uniformity as *suggested* the idea of "non-free" causes to us. But we might conceivably have been mistaken in adopting that suggestion, and judging that the causes of those phenomena are really non-free: just as those philosophers who deny free-will maintain that we are really mistaken in concluding from the facts of our own internal experience that we have free-will. However this may be, the *hypothetical* statement of the principle of uniformity evades this question of fact in regard to non-free causes. The categorical statement of the same principle, however, implies and asserts the fact of their actual existence.

It is not, therefore, in the *formal* generalization of the abstract principle—in the assertion that "*If* (*whenever, wherever, as often as*) any physical cause acts in the same circumstances, it will produce similar effects"—that the difficulty lies, but in its *material* generalization, *i.e.*, (*a*) in asserting that *there are and have been and will be such causes* in existence, and (*b*) in proving that the various cases which we allege to be actual instances illustrative of the principle are indeed such.[1]

In order, for instance, to be able to apply the abstract principle of uniformity in (*a*) establishing by induction the general law that "an iron bar is lengthened by the application of heat,"[2] and in (*b*) applying this law to any particular case, we must be able not merely to assert the *formally* general (hypothetical) principle that "natural or non-free causes produce the same results if they act repeatedly in similar circumstances," but we must be able furthermore to assert categorically (*a*) that *heat acting on iron* is such a cause, and will therefore always lengthen an iron bar, and (*b*) that this particular case is really a case of an iron bar acted on by heat.

The general *categorical* assertion, that "*the causes which are at work in the physical universe are non-free, or fixed by nature in their mode of action, and that therefore they always have acted and always*

[1] *cf.* MERCIER, *Logique*, p. 330.

[2] We assume here, with Mill, whatever about the conventions of formal logic, that all such physical laws and general truths, reached by experience, *imply the existence or occurrence* of the things and events to which they refer (*cf.* 128).

will act uniformly," goes distinctly farther than the hypothetical principle that "*if a cause is fixed necessarily to one mode of action it will act uniformly in similar circumstances*". Yet those two distinct and separate statements are sometimes identified, or rather confounded, under the common designation of the "uniformity of nature". And those who rightly distinguish between them usually limit the latter title to the abstract, *hypothetical* principle, describing the *categorical* assertion as *belief*[1] "in the maintenance of the present order of things in the universe". Thus Dr. Mellone, in his *Introductory Text-Book of Logic*,[2] draws "an important distinction between two meanings of the uniformity of nature: (1) the uniformity of causation, (2) the maintenance of the present order of things in the universe. Experience [he continues] shows us that there are general 'laws'—*i.e.* kinds of orderly succession in the outward course of events: such as appear in the succession of day and night, summer and winter, seed-time and harvest, life and death. The regular succession of events in a thousand different ways accustoms us, from force of habit, to expect things to happen in a regular order; and we find that the expectation is fulfilled. This constitutes an overwhelming presumption in favour of the maintenance of the present arrangements in nature; but it does not show that derivations from this order are impossible. An expectation, bred by experience and custom, that events will occur in a certain way is not the same as a knowledge that they must so occur; and this knowledge is not in our possession. We have no grounds for affirming that the sun *must* rise to-morrow morning; there is only an overwhelming presumption in favour of the expectation that it will. But the principle of uniform causation tells us nothing as to the permanence of the present 'choir of heaven and furniture of earth'. It only says that the same cause will have the same effect; and to this there are no exceptions. The same cause may conceivably never act again; but this does not affect the truth of the principle that *if* it did it would have the same effect".

But, then, is the inductive process, by which we establish a law of physical nature ("*If S is M it is P*": "*If a bar of iron be heated it will be elongated*"), an application merely of the

[1] This "belief" extends to the past no less than to the future, to the distant as well as to the near; it is a conviction which has for its object the *existence* and *operation*, throughout time and space, of *natural* or *necessitating* causes.

[2] pp. 281-2.

hypothetical "principle," or does it also involve the categorical "belief"? The answer is that if the laws of physical nature are anything more than statements of mere *abstract possibilities;* if they are taken to imply the *actual existence* and *operation*, throughout space and time, of the agencies they refer to, then the inductive process by which we reach them *does undoubtedly* imply not only the hypothetical principle, but also the categorical belief. And that physical laws are interpreted in this latter sense, as informing us not about mere *abstract possibilities*, but about *concrete actualities, past, present, and future*, there can be no doubt. In reference to such a law, for example, as "*Heating* (M) an iron bar (S) *causes* its *elongation* (P)". Dr. Mellone[1] says that "the connexion between M and P is independent of time and place. We can reason backwards to unobserved cases in the past, and dip into the future and be sure that P will always be produced by M".

But *how sure* can we be about this latter? No surer than we can be that heat and iron (M and S) will continue to exist; for unless they continue to exist, the operation can never take place. And what certitude have we that they *will* continue to exist? The physical, hypothetical certitude which Dr. Mellone describes as an "overwhelming presumption". In fact, we cannot extend or apply a single physical law to a single *future* case —or to a single *past* or *distant* case for that matter, if it lies outside our actual experience—without assuming (a) that the causes it refers to are "necessary," "natural," or "non-free" causes, and (b) that they have acted, are acting, or will act, in the case contemplated, without any obstacle or impediment from the intervention of other causes.

Similarly, Father Joyce[2] seems to take the "principle of uniformity" as embodying not merely the abstract judgment that "a non-free cause acts uniformly in similar circumstances," but also the judgment that "such causes *do* exist and act in the universe," when he says that "in that principle we have the guarantee that our universal judgment will be verified in fact. Our judgment that *A as such* is the cause of *a*, would help us but little, unless we further knew that in the real order the same cause does actually always produce the same effect."

This being the sense in which Mill understood the principle, it

[1] *op. cit.*, p. 265. [2] *op. cit.*, p. 219.

is no wonder that he regarded it as synthetic, as reached by experience, not as analytic and self-evident like the mere hypothetical statement of uniformity. If, therefore, the principle of uniformity be understood to assert categorically our belief in the actual existence and operation, throughout space and time, of non-free causes, we have to determine (1) what are the ultimate rational grounds on which we assent to this principle, and (2) what are its relations to the processes of induction and deduction respectively.

224. ULTIMATE RATIONAL GROUNDS OF OUR BELIEF IN UNIFORMITY: THE SCHOLASTIC, EMPIRICIST, AND IDEALIST VIEWS.—Firstly, as to the rational grounds of our assent to the principle. We must bear in mind that it is a synthetic, or *a posteriori* generalization from experience, about which we have physical certitude.[1] Our concept of physical or non-free cause is not innate. We form it gradually from our acquaintance with *uniformity* in the processes of physical nature. From our experience of the uniform activities of the physical universe we abstract the notion of a *necessary* or *non-free* cause, fixed in its mode of action: just as from our internal experience of our own activity, and from observation of the activities of men in general, we abstract the notion of a *free* or *self-determining* cause, not fixed to one mode of acting in similar circumstances. Having, then, defined for ourselves a non-free or physical cause as "one which will always act the same way, by a necessity of its nature or constitution, in similar circumstances," we *deliberately judge that the causes of which we have experience in physical nature verify our definition:* we judge that they will always act the same way in similar circumstances, in the future as in the past, *provided something unwonted, extraordinary, unforeseen, does not occur.* Thus, while we quite recognize that at least apparent exceptions to uniformity have occurred in the past, that our knowledge of the forces of nature is limited, that some of its phenomena "seem altogether capricious,"[2] that unknown agencies, not calculated by us, may have interfered, and may again interfere, and surprise us by upsetting our expectations: nevertheless, we consider it prudent and reasonable to base upon

[1] *Cf.* MAHER, *Psychology* (4th edition), p. 420: "The latter generalization [that the 'laws of nature are constant'] is a *contingent* truth which we can easily conceive subject to exceptions".

[2] MILL, *op. cit.*, III., iii., § 2.

our actual experience of general uniformity, imperfect and possibly interrupted though this may be, a firm belief or expectation that the same regularity which has obtained within the limits of our actual experience will obtain also outside these limits.

Thus, actual experience of uniformity may be regarded as the *proximate, psychical* ground of our belief in uniformity beyond this experience. But we must go further if we are to assign an *ultimate rational justification* for this belief. If I am asked why I believe that nature is uniform beyond the actual range of my own personal sense experience, it will not suffice to answer: "Because I have found it uniform within this range". No doubt, this is the true psychological account of the genesis of my expectation that the uniformity will obtain beyond my experience; and, no doubt, I may quite prudently and reasonably act upon this belief. I may go on investigating nature as a scientist, observing, experimenting, conjecturing general truths or laws, generalizing from experience, and in this way passing beyond experience, even discovering and establishing laws of physical nature: I may do all this without once pausing to inquire what rational grounds I have at any time for going a single step beyond my actual experience of nature and inferring anything with any rational certitude about what is beyond this experience. But what right have I to infer that because a thing has existed, or an event happened, in a certain uniform way within my very limited experience, it therefore does, or will, or must exist, or happen, in the same way beyond? What right had Leibniz to think or say that "'Tis all like here . . . The present is pregnant with the future; the future may be deciphered in the past. . . . The distant is mirrored in the near"?[1] The "*leap*" *beyond experience* takes place in every single induction we make, because we believe in the "uniformity of nature". But what right have we to believe in it? What view of nature will afford us a rational justification of this belief?

Scholastic View.—Philosophers differ in assigning an ultimate rational ground for our belief in the uniformity of nature, because they differ in their views about the ultimate nature of the universe itself. The justification Scholastics offer—in common with all who admit creation, and the dependence of all nature on

[1] "C'est tout comme ici. . . . Le présent est gros de l'avenir; le futur se pourrait lire dans le passé . . . l'éloigné est exprimé par le prochain."—*apud* VENN, *Empirical Logic*, p. 81; *cf.* 124.

the Providence of an All-wise Deity[1]—is simple and intelligible. By reasoning from effect to cause, by means of *a posteriori* arguments whereby we apply self-evident principles, like the principle of causality, to the facts of sense experience, we establish with certitude the existence of an All-powerful, All-wise, Supreme Being, who has freely created the universe, freely conserves it in existence, and freely concurs with the activity of all created agencies; who has manifestly *ordered* and *arranged* and *designed* the universe, the "cosmos" as it is rightly called; who has evidently endowed the agencies of this visible universe with *fixed* tendencies, in virtue of which they act uniformly unless whenever or wherever He chooses to interfere (miraculously) with the established *physical* order for some higher (*moral*) end. Knowing all this, we know that natural causes will continue to exist and to act uniformly *in accordance with His will and as long as He wills*. Knowing, too, that He is All-wise, we know and believe that He will not interfere with the uniformity of physical nature *capriciously*, so as to render our reliance on it uncertain. Since He created its agencies "for man's use and benefit," this Divine purpose forms a firm basis for our trust in their stability. His occasional miraculous suspensions of its laws are for our greater good, and cannot in any way weaken our belief in its general uniformity (*cf.* 217).

Thus it is that our conception of physical nature as the work of an All-wise Creator and Ruler, forms the ultimate rational justification of that belief in the uniformity of nature, which is partially embodied in the formulation and application of every physical law. This, of course, does not mean that we must have deliberately convinced ourselves of God's existence, creation, and providence, *before* we can make a single inductive generalization from actual experience in any department of natural research: we may *assume* the uniformity of nature *provisionally*, and utilize our postulate *as scientists*, without justifying to ourselves the use we make of it. But if we want to justify this usage *philosophically*, we must, of course, put some rational interpretation on both nature and thought—*i.e.* on our experience as a whole.

[1] The Scottish school of philosophers are content to say that this belief is the natural expression of an innate, instinctive law. There is no denying the natural tendency to the belief; but to say that the latter must be "the effect of instinct, not of reason," is hardly to explain it. The tendency to the belief should not be called an "instinct"; for, although its exercise is spontaneous and unreflective, still, on *reflection*, we can assign a *rational* basis for it.

The existence of God can be proved independently of the assumption that nature is uniform in the sense in which this uniformity has just been explained.[1] Hence, the Scholastic justification of the postulate is free from all circular reasoning, in addition to being intelligible and adequate. But perhaps this fallacy is involved in applying belief in uniformity to individual inductive generalizations *before* we have explicitly assigned to this belief its ultimate rational basis? No, because every such generalization is merely provisional: the assumption of uniformity involved in it awaits whatever rational justification we may be able to supply for this assumption when we reflect upon it.

Empiricist View.—Mill was right, as we have seen above, in saying that belief in the general uniformity of nature (in the categorical sense) is "by no means one of the earliest"[2] of our beliefs: it is not a mental assent which must *precede* every scientific induction we make: it is partially embodied in each, and gradually extended over all nature.[3] But he failed utterly to assign any ground for rational, scientific certitude, whether about this widest law of uniformity of nature, or about any minor generalization reached by induction. He sought to show that the minor generalizations we make without explicit advertence or assent to the general uniformity of nature, can be only mere *enumerative* inductions, *i.e.* more or less hazardous extensions of *observed* uniformities to the region *beyond* our actual experience; that our belief in the general uniformity of nature is a gradual summing up of these hazardous conclusions; and that, nevertheless, this summing-up process gives us the *highest attainable scientific certitude* about this law of uniformity, this widest of all generalizations. The general uniformity of nature is, he teaches, a generalization from a number of less general uniformities, themselves reached by a "loose and uncertain mode of induction *per enumerationem simplicem*". The law of the uniformity of nature "is itself an instance of induction, and by no means one of the earliest which any of us, or which mankind in general, can have

[1] The same *observed and experienced uniformity* which prompts us to act on the assumption of its universality, also furnishes, of course, part of the data from which the existence of God is demonstrated. But in using the data for this purpose we are not assuming the principle of uniformity.

[2] *Logic*, III., xxi., § 2.

[3] *Cf. infra*, 225; VENN, *op. cit.*, pp. 134 *sqq.* For the opposite view—that belief in, or assumption of, *general* uniformity—is necessarily antecedent to any and every inductive generalization, see JOSEPH, *op. cit.*, pp. 386 *sqq.*

made. We arrive at this universal law by generalization from many laws of inferior generality. . . . As, however, all rigorous processes of induction presuppose the general uniformity, our knowledge of the particular uniformities from which it was first inferred was not, of course, derived from rigorous induction, but from the loose and uncertain mode of induction *per enumerationem simplicem*."[1]

And of this latter process he had already said: "It consists in ascribing the character of general truths to all propositions which are true in every instance that we happen to know of. . . . In science it carries us but a little way. We are forced to begin with it; we must often rely on it provisionally, in the absence of means of more searching investigation."[2] There is here, apparently, no rational basis assigned, on which this "loose" process can produce scientific certitude. Yet, it is by this process we ascend to the "particular uniformities," and, by a second application of it, from these to the "general uniformity," on which the validity of the whole inductive process is to be based. The principle so obtained must necessarily be, as Professor Welton expresses it, "untrustworthy in a twofold degree; for it is an inference, uncertain in its very essence, from other inferences of the same dubious character. . . . Mill's argument on this point is indeed nothing but a *petitio principii*. We are, he says, 'to consider no minor generalization as proved except in so far as the law of causation confirms it' (III., xxi., § 3), and yet that law is to be derived from those very same minor generalizations which it is called upon to 'confirm'."[3]

Mill is, of course, mistaken in thinking that we cannot make a *strict, scientific* induction without having *previously* justified our belief in the general uniformity of nature. We have pointed out above that this is not necessary; that we may accept the principle provisionally and base our scientific inductions upon it. Mill, however, thinks we can only make "enumerative" inductions; and upon these alone he endeavours to base our belief in that general uniformity, which will then turn around and confirm them. His attempt to avoid the charge of inconsistency in basing the validity of the "rigorous" process upon the "loose and uncertain" process, reveals once more a rather naïve *petitio principii*. The difficulty he had to face was this: Enumerative induction,

[1] *Logic*, III., xxi., § 2. [2] *ibid.*, iii., § 2.
[3] Welton, *Logic*, ii., pp. 42, 43. *Cf.* Joseph, *op. cit.*, pp. 388, 391.

i.e. generalizing from the mere *counting* of instances, is admittedly a hazardous process and cannot give certitude. How, then, can we be certain of the uniformity of nature, and through it, of our *scientific* inductions, if uniformity itself is grounded on this hazardous process of enumeration? Mill commences his answer[1] with this statement: "Now, the precariousness of the method of simple enumeration is in an inverse ratio to the largeness of the generalization". Assuming this, he points out that the subject-matter of the law of uniformity—which is the "largest" generalization of all—is "so widely diffused that there is no time, no place, and no combination of circumstances, but must afford an example of its truth or of its falsity," and that it was "never found otherwise than true".[2] From this he concludes that the law of uniformity "takes its place among the most firmly established as well as the largest truths accessible to science". This is a plausible piece of reasoning until we advert to the fact that its opening statement *assumes what is to be proved.* The reason why we regard a wide enumerative induction as safer than a narrow one, the reason why one which is found to range without exception over an extensive region of time and space yields higher certitude, is because we are made morally certain by it that the special observed uniformity in question is not a *casual* but a *causal* one, and *because we* ARE ALREADY CONVINCED, *or* ALREADY ASSUME, *that a* CAUSAL *uniformity will persist beyond and outside our experience*, in other words, THAT NATURE IS UNIFORM. Did we not already believe in the uniformity of nature, all enumerative induction, whether wide or narrow, in fact all inference beyond actual experience, would be equally hazardous. To assume that we can thus differentiate between wide and narrow inductions, in an attempt to prove that we can believe nature to be uniform, is simply to beg the question at issue.

Mill's attempt, therefore, to assign a rational basis for belief in the uniformity of nature breaks down. And hence he is unable to justify the individual scientific inductions by which we establish isolated laws of nature; for in every one of these inductions there is a partial application of the principle of uniformity; every one of them transcended the actual sense experience of the individual;

[1] *Logic*, III., xxi., § 3.

[2] What about his previous recognition [III., iii., § 2] of phenomena, which "seem altogether capricious," about the "course of nature" being "not only uniform" but "also infinitely various"? Again, what about miracles? Or about the impossibility of inferring what *must* be, or even what *will* be, merely from what *was* or *is*?

every one of them "did most certainly outreach the boundaries of observation as then and there obtained";[1] and in the Empiricist philosophy, which reduces all knowledge to sense experience, there is nothing to justify a single step beyond the present data of the individual's sense consciousness. This philosophy recognizes no channel of knowledge beyond the senses, and reduces all nature, all reality, to a mere flow of conscious sensations in the individual mind. The step, therefore, beyond what is *actually observed*—in fact, the step beyond the contents of the present transient moment of consciousness—is, for the phenomenist, at best a presumption, a "hazard," a "leap,"[2] a speculation, about the validity of which we may have a more or less strong *expectation, hope, opinion, probability;* but not *certitude* proper: at least, not a *scientific* or *reasoned* certitude, for which any sufficient rational grounds can be assigned (*cf.* 219).

Idealist View.—In the sensist philosophy there is room for knowledge of individual *fact* or *phenomenon* alone; for *law, necessity,* the *universal,* there is no logical place. In the Scholastic doctrine, that the universe is dependent on the free-will of an All-wise Creator and Ruler, there is an intelligible place for *physical* or *conditional* certitude about the nature, activities, and laws of physical agencies, conceived as subject to the will and wisdom of that Creator. The *idealist* philosophy errs in the opposite extreme from sensism by attributing to the processes of external nature an absolute, metaphysical necessity to which they can have no real claim. The advocates of this philosophy —to which we have already called attention (*cf.* 215)—prefer to speak of the *unity* of nature, rather than its uniformity. They tell us that "the world must be conceived as a systematic totality, with a thoroughgoing interrelation of parts . . . that nature is a unity . . . a system which remains identical with itself amidst the unceasing changes of relations between its parts, and which, by its own nature, necessitates and determines those changes".[3] And they assert this "unity" as a postulate or "presupposition," without which intelligible experience would be impossible.[4]

Now it is true, undoubtedly, that unless the world were a harmonious system of interrelated elements, *regular, uniform, consistent with itself* throughout all its changes, we could not arrive

[1] VENN, *Empirical Logic*, p. 131.
[2] BAIN, *Inductive Logic*, book iii., chap. i., § 1.
[3] WELTON, *Logic*, ii., pp. 4, 5. [4] *ibid.*

at a rational knowledge of it; for knowing implies defining, arranging, and classifying things; and the validity of these processes obviously *depends on the condition that their objects have abiding, permanent natures.* Whatever is knowable, therefore, is reducible to order within a system. But in this sense the unity implied in reality is of course *unity of order, unity by relation, not unity of being* or *essence*, as these philosophers would seem to imply. This pantheistic postulate will not stand the test of critical analysis. In the real world, as revealed to us through our senses, we detect a *unity of order*, but not a *unity of being;* we see in it manifold evidences which justify us in inferring that it is created, conserved, and ruled by some guiding intelligence distinct from it; but we do not by any means see in it only such *logically necessary* connexions and relations as would justify us in believing it to be a mere manifestation or evolution of the activity of some immanent intellect. We can prove that the "choir of heaven and furniture of earth" are dependent on Divine providence, on the wisdom and free-will of the Deity, and we can therefore be *physically, hypothetically* certain of the generalizations we reach by means of induction about the modes of existence and activity of agencies created in time and space; but *absolute* or *metaphysical* certitude about these modes of existence and activity, the very nature of these agencies, and the essential limitations of the human mind itself, preclude us from ever reaching.

Modern logicians may, perhaps, be tempted to deprecate the introduction, into a treatise on logic,[1] of such metaphysical theses as that God has created and conserves and governs the universe and concurs with its activities, and that man is endowed with free-will, for the purpose of explaining the nature and grounds of physical and moral certitude. But the fact is that these latter cannot be satisfactorily explained, either in logic or outside it, without adopting some attitude or other as to the ultimate nature, origin, and mode of existence, of this visible universe which furnishes the human mind with all its data for knowledge. Metaphysical assumptions of some kind are inevitable in logic, even although it is in metaphysics and not in logic that they should be justified. If John Stuart Mill introduces into his logic, as he does, the assumption of the empiricist or phenomenist philosophy, that all reality is ultimately analysable into spontaneously associated *sensations* of the conscious mind, and if Professors Bosanquet and Welton build their logical doctrine on the idealist assumption of Hegel and Green, which *identifies reality with thought* by declaring the former to be *constituted* by "thought-relations," Scholastics need not apologize for rejecting both the one and the other assumption as unsatisfactory and erroneous, for attributing a larger rôle to intellect

[1] *cf.* JOYCE, *Logic*, pp. 237-8; RICKABY, *First Principles*, pp. 89, 93, 102.

than the Empiricists, and a larger rôle to sense than the Idealists, for replacing the Agnosticism of the former, and the Pantheism or Monism of the latter, by the philosophy of Christian Theism, which teaches that the world was created by an All-wise Deity, and is conserved and governed by His power and providence.

Writers in sympathy with a spiritualist or idealist interpretation of experience have furnished very destructive criticisms of Empiricism. But their own substitutes are often far from satisfactory. We may instance the account of uniformity given by Mr. Joseph.[1] He deals with the principle as interpreted in the categorical sense, *i.e.* as believed by us to be *de facto* applicable to the universe revealed to us in sense experience. He shows clearly and conclusively that it cannot be established by induction in the manner propounded by Mill.[2] His own view is that uniformity is a *postulate*, an *assumption* which must be made antecedently to all induction: "all induction *assumes* the existence of universal connexions in nature".[3] He points out also, and rightly, that belief in the *uniformity* of the causal relation really involves belief in its *necessary* character, belief that it is a *law*.[4] But he goes on to draw a distinction between "conditional" and "unconditional" laws or principles. A "conditional" principle he defines as one whose truth "depends upon conditions which are *not stated in it*";[5] such a principle, therefore, may "admit of exception"[6] when any of those unmentioned conditions are not verified. An "unconditional" principle is, of course, one which is true absolutely and unconditionally, one "that can have no exception".[7] The uniformity of nature he apparently holds to be an *unconditional* principle or law, for he says it "involves the truth, without exception or qualification, of all unconditional laws".[8] Let us see, then, how he attempts to show that it is unconditional. For, if the principle of uniform causation is unconditional, it undoubtedly "becomes . . . important to determine, if possible, when we have discovered an unconditional law".[9] He gives us two tests, one admittedly satisfactory; the other admittedly less so. The *first* is simply cogent self-evidence: "if a principle is self-evident it must be unconditional".[10] Such truths, therefore, as "two and two are four," "*ex nihilo nihil fit*," "a thing must be itself," etc. are unconditional because self-evident. So, too, is the abstract, hypothetical statement of uniformity—"*if* natural causes have fixed, stable modes of acting . . . they will produce similar effects in similar circumstances"—"unconditional" because it is "self-evident". But is the *categorical* statement, that "nature actually is and must be uniform," a self-evident proposition? It certainly is not.[11]

[1] *op. cit.*, c. xix. [2] pp. 387-389. [3] p. 371. [4] pp. 376 *sqq*.
[5] p. 381. [6] p. 386. [7] p. 382. [8] p. 381.
[9] p. 382. [10] p. 386; *cf.* p. 384.

[11] We cannot claim the categorical principle to be self-evident unless we claim that the application of our *abstract*, universal concepts (and of the evidently necessary and universal *abstract* truths which the mind enunciates by comparing these concepts with one another) to the *concrete* data of our sense experience (for the interpretation of these latter), is an *evidently* valid process; in other words, unless we claim that the doctrine of *Realism* in regard to the significance of our intellectual concepts is, in some form or other, an *evidently* true doctrine: an indefensible claim, because some forms of realism are not true, and the true form is not evident. Yet,

Let us therefore apply to it the *second* test, which is this: "if without assuming [the principle] to be true, it is impossible to account for the facts of our experience, we should have to suppose it unconditional; though such impossibility may be hard to establish".[1] The law of uniform causation is supposed to fulfil this test, to be the only principle on which we can "account for the facts of our experience," and, therefore, to be unconditionally true. And this supposed impossibility of otherwise accounting for "the facts of our experience" is also alleged as the ultimate ground and justification of our belief in the law: "With what right then do we assume it? The answer to this has been given in discussing what we mean by it. To deny it is to resolve the universe into items that have no intelligible connexion".[2]

This whole position calls for a few considerations. *Firstly*, to prove in this indirect way that a principle must be true—because, namely, it is the only one that will "account for the facts of our experience"—is a perfectly legitimate procedure when the principle is not self-evident. It is a difficult method, of course, to apply; but, failing self-evidence, it is the only one; nor do we see why, having applied it carefully, "we should not be fully satisfied with it,"[3] as Mr. Joseph thinks we should not. Is he himself, then, not fully satisfied with the only way in which the law of uniform causation in its categorical or applied sense can be shown to be true?

Secondly, if the law *is* established in this way, is it not based on facts, and established by experience, as we have contended that it is?[4] It is *assumed*

although Mr. Joseph nowhere states that the law of (uniform) causation (in the categorical or applied sense) is self-evident, he does assert that our belief in it "rests . . . on the perception that a thing must be itself. If it is the nature of one thing to produce change in another, it will always produce that change in that other thing; just as, if it is the nature of a triangle to be half the area of the rectangle on the same base and between the same parallels, it will always be half that area" (*op. cit.*, p. 390 n.). But, manifestly, the parity between those two examples holds good only on an assumption which is, to put it mildly, not self-evident: the assumption that the same necessity which characterizes the relations between static, abstract thought-objects, or possible essences, in the conceptual order—or a like necessity—also characterizes the relations between the concrete sense phenomena that actually exist in the ever changing conditions of space and time (*cf.* 219). Apparently, Mr. Joseph has failed to distinguish between the self-evidence of the abstract law of uniformity within the conceptual order, and the entirely different grounds on which the application of this law to the concrete, actual domain of sense experience must be maintained as valid.

[1] p. 382. [2] p. 390.

[3] p. 382. The feeling that this method is not quite satisfactory seems to us to reveal that attitude of mind which would restrict the terms "knowledge" and "science" to *self-evident* truths, and conclusions derived from these by *cogent demonstrative* reasoning.

[4] We interpret Mr. Joseph's account of the principle, as given in his *Logic*, pp. 380, 391, to propound the view that this principle is "unconditional"; that we know it to be "unconditional," because, although it is not self-evident, the *facts* force us to admit it, because the denial of it would "resolve the universe into items that have no intelligible connexion". But this implies that the law is based on experience, and reached *a posteriori*. Yet, elsewhere he seems to hold that the principle is self-evident: *cf.* p. 390, n. (n. 11, p. 107); also p. 401, where he writes: "The law of the uniformity of nature itself, as we have seen, is not arrived at in that way [*i.e. a posteriori*], since if we once doubt it, it is impossible to show that

of course, not prior to, but in and with, all our experience;[1] but when we *seek rational grounds for our assumption* of it, where can these be found but *in* the facts, *in* our experience? When a law is established by this procedure, we must, of course, recognize that "had the facts been otherwise, we need not have admitted the law; and [that] we do not see, except on the hypothesis that the law is true, why the facts might not have been otherwise".[2] This is the reason why Mr. Joseph regards such procedure as unsatisfactory; but if we believe in the law of uniform causation because it is the only principle that will "account for the facts of our experience," surely we must be prepared to admit that "had the facts been otherwise we need not have admitted the law";[3] and in this there can be nothing unsatisfactory. But if this is the way we justify our belief in the law, then, obviously, that belief is not a prerequisite condition for experience, entirely prior to, and independent of, experience, but is rather psychologically simultaneous with, and philosophically grounded on, experience.

Thirdly, it must be carefully noted that the *abstract, hypothetical* law—which alone is self-evident, being, in fact, reducible to the principle of identity, as Mr. Joseph shows, and as we have already pointed out—*does not and cannot, of itself*, "account for the facts of our experience". It belongs to the conceptual or ideal order, the order of abstract objects of intellectual thought; whereas "the facts of our experience" belong to the phenomenal order, *i.e.* is to the order of realities actually existing in space and time, and subject to all the changeful conditions of such existence. But no purely abstract, conceptual principle can, of itself, account for the actual existence or permanence, in space and time, of the present "choir of heaven and furniture of earth".[4] It is the categorical principle alone—"Nature has been, is, and will be, and *must* be, uniform"—that can give us any intelligible account of *the actual world of our experience*, as distinct from a merely hypothetical world constructed by our own thought from intellectual concepts. And hence the supreme importance of determining *in what sense* nature *must* be uniform, of "discussing what we mean by"[5] this "must," and of assigning a rational ground for our belief in this necessity, in the sense in which we interpret it.

Fourthly, as already explained, we believe this "necessity," this "must," to be *conditional*, contingent, dependent on the *Fiat* of a Divine and All-wise

the facts are any more consistent with its falsity than with its truth". The abstract principle is, of course, self-evident, but the validity of its application to the actual world of sense experience is not.

[1] It is quite true that "if we once doubt" the truth of the principle as applied to the actual universe, "it is impossible to show that the facts are any more consistent with its falsity than with its truth" (*op. cit.*, p. 401), or, in fact, to reason at all about events in space and time beyond actual experience; but from this it does not follow that assent to the principle must be antecedent to, and independent of, all experience. The principle, even in its applied sense, is not reached by any process of *logical inference*. None the less, it is based on experience. From experience we *abstract* the concepts embodied in the principle. Experience *suggests* the abstract principle as validly applicable to the real world; we assume that it is so applicable; and further experience justifies the assumption.

[2] JOSEPH, *op. cit.*, p. 382. [3] *ibid.* [4] *Cf.* MELLONE, *op. cit.*, p. 282.

[5] JOSEPH, *op. cit.*, p. 390.

Creater and Ruler, whose existence and providence can be proved from "the facts of our experience": to us the principle means that "The course of physical nature must be uniform if, and provided that, and in so far as, the will of God makes it so". If, then, we wish to formulate an ultimate, *unconditional*, or absolutely necessary, law, for physical nature, or indeed for all contingent reality, we shall find it in the simple statement that "The whole course of contingent or created reality must be as God, the Necessary Being, wills it to be". If we accept J. S. Mill's definition of laws of nature in the strict sense, as "the fewest and simplest assumptions, which being granted, the whole existing order of nature would result"; [1] the law we have just enunciated would be the really ultimate "law of nature," though this was very far indeed from Mill's own thought. We have already referred to Mill's inability to transcend the "conditional," or to give any account of the nature of that ultimate, outstanding condition on which "the present constitution of things" [2] is dependent. Let us see whether Mr. Joseph is any more explicit in regard to the nature of this final and most important condition.

Understanding a law to be unconditional when its truth is not dependent on any outstanding condition other than those explicitly stated in the formulation of the law,[3] he goes on to inquire: "are there any unconditional laws known to us?" [4]. He first refers to the mechanical view of the physical universe, which purports to interpret and explain "all physical changes" as "determined altogether according to physical laws," and to be all "purely mechanical": [5] according to which view these mechanical laws, while conditioning the existence and course of all physical nature, would be themselves unconditional. He very rightly declines to accept this view on the ground that it is "impossible to *account* on physical principles for the facts of consciousness" [6] . . . "Thus to a physical theory of the world consciousness remains unaccountable; such a theory therefore cannot be complete or final".[7] He then suggests in a mild way that "we are perhaps sometimes too hasty in supposing that we see the necessary truth of physical principles".[8] Such a supposition is, of course, not only too hasty, but also erroneous, seeing that such principles, referring as they do to the order of concrete physical facts, cannot have the purely abstract necessity of mathematical truths: "it might be said that in the first law of motion it is self-evident indeed that a body will persist in its state of rest or uniform rectilinear motion until something interferes with it, *but not that interference can come only from another body;* that the mathematical reasoning in physical science is necessary, *but not the physical principles which supply the data* to which mathematical reasoning is applied; and that the doctrine that a body can only be interfered with by another *body* is one of these".[9] All this points to the conclusion that "the fundamental physical laws are only conditionally true," [10] that is, dependent on conditions

[1] *Logic*, III., iv., § 1. [2] *Logic*, III., v., § 6: *cf. supra*, 219.

[3] JOSEPH, *op. cit.*, p. 381. [4] p. 382. [5] *ibid.* [6] *ibid.* [7] p. 384. [8] *ibid.*

[9] p. 385 (italics are ours, except the last). The assertion that "a body can only be interfered with by another *body*" is not really a physical "*principle*," nor can physics even prove it to be *true:* what it is meant to convey is simply this, that "*physical* science prescinds from all but material agencies" (*Cf.* MAHER, *Psychology*, p. 518, n. 30).

[10] *ibid.*

which are not themselves physical, and whose nature, therefore, it is beyond the scope of *physical* science as such to explain : "supposing that there are, if we may so put it, spiritual conditions upon which the movements of bodies in the last resort depend . . . then physical science at any rate cannot deal with those conditions ".[1] Of course it cannot, since it does not purport to deal with *all reality* : but we expect from the physical scientist that he should not go on to *deny* the existence of such conditions merely because they fall beyond his scope and methods as a scientist :[2] he is doing a real service to his science by recognizing its limitations. But physical science and philosophy are both brought into disrepute by those who *gratuitously deny the existence of ultra-physical conditions and causes*, who contend that mechanical laws are unconditional, and that all existing reality can be explained by these laws, when, as a matter of fact, such laws offer no *ultimate* explanation *even of the material universe*. Mr. Joseph fails to note, however, that there is this still more fundamental reason for rejecting the mechanical view : that it purports to explain the *actual*, *concrete existence* and uniform *course* of nature, by the mere formulation of some one or some few mechanical laws. How could any abstract, intellectual formula about atoms, mass, motion, energy, etc.—even were such a formula self-evident—account for the actual existence and course of nature? An *abstract law* cannot *account* for *existing facts*, or for the uniformity or necessity[3] of actual processes. Actual facts demand an actual *cause*, and so does the mode—whether uniform, or necessary, or otherwise—in which they happen. If, then, all physical nature is dependent on, and refers us to, an ultra-physical or spiritual domain of reality, and if even the highest and widest *physical* laws are not absolutely ultimate, but conditioned by the reality or realities of this other domain, it is obviously the highest duty of the *philosopher* to determine the nature and influence of these conditions. But is it not also the duty of the *logician* to take note of, and call attention to, *all* the *leading alternative ways* in which these conditions have been, or may be, conceived by philosophers? or at least not to convey the impression that these alternatives are fewer than they really are? Now, according to Mr. Joseph, if we are dissatisfied with the "mechanical" alternative, "philosophy suggests

[1] JOSEPH, *op. cit.*, p. 385.

[2] *cf.* MAHER, *Psychology*, p. 420 : "The student should always remember that physical science simply *assumes* the *law of uniform causation ;* that its universality is merely a *postulate* to be justified only in *metaphysics ;* and that the metaphysician, who recognizes moral convictions to be not less real nor less weighty facts than those of physical science, is bound to qualify, limit, or interpret the law when applied to moral actions in accordance with his wider and more comprehensive view of experience". *Cf.* also pp. 517-24, especially p. 519, n. 32.

[3] It is no *ultimate* explanation of this necessity to say that it is "mechanical". If all nature is merely one vast machine or mechanism, who made it? The necessity we ascribe to the course of *actual* nature in time and space is not the necessity we ascribe to abstract judgments about possible essences : it is not purely intellectual : it is a manifestation of intelligence *and will and power*. The only immediate source it can have is our experience of the order, regularity, uniformity of all nature, compelling us to interpret the latter as a *cosmos*, as the work of an *Omnipotent Will* directed by *Supreme Wisdom*. The only necessity for which we can rationally account in actual nature is that by which it pursues the course marked out for it by the Divine *Fiat*. To say *as a last word* about the course of nature that it is "mechanical," is no better than to ascribe it to mere chance, or to pronounce it an insoluble enigma.

that in the last resort, instead of explaining consciousness in terms of physical law, we shall have to see in physical law a manifestation of intelligence. The whole material order is an object of apprehension; therein, however it stands related to minds that apprehend it, it and they together form the complete reality, or *res completa*; and they cannot be understood except together".[1]

As a final word on the problem, this is hardly satisfactory. At least, the statement might be a little more explicit. In justification, presumably, of his brevity, the author adds: "It is not our business to discuss here this central metaphysical problem". That is so; but from what he does say, and leave unsaid, about it, we are left in doubt whether or not he is really committing himself to the philosophy of idealistic or spiritualistic monism. "The whole material order," and "minds," "together form the complete reality, or *res completa.*" That sounds like monism. A few pages further on he writes: "If the whole series of events in time can be regarded as an expression of the activity of that which is in some way exempt from subjection to succession, then what appears in time as future may have to be taken into account in giving a reason for the present and the past, though of course the future cannot determine the present in the same way as what precedes it does".[2] But this statement—which apparently refers to the influence of *final cause*, or *purpose*, in the course of events—is equally compatible with theism or with spiritualistic monism. And we get no clue as to which alternative the author himself adopts; he merely adds: "The present chapter is perhaps already more than sufficiently metaphysical".

But there is a graver inconvenience in his treatment of the question: what he has managed to say, and to leave unsaid, may seriously mislead the student. When he chose to set over against the mechanical, materialist view of nature, the teleological, spiritualist view, he made mention of only one form of spiritualism, the pantheistic or monistic form. Why has he passed over in silence the other well-known alternative, the philosophy of theism? Theism is at least a possible alternative to monism. Therefore it claims a mention from the logician. But, according to Mr. Joseph, if we reject mechanical materialism, "philosophy suggests that *in the last resort*, instead of explaining consciousness in terms of physical law, we shall have to see in a physical law a manifestation of intelligence".[3] Philosophy does not suggest this as a "last resort". And, even if it did, the suggestion would be ambiguous: Of *what* intelligence are we to regard physical law as a manifestation? our own individual intelligences? or an immanent cosmic intelligence—an *anima mundi?* an intelligence *with will*, or one without will? an unconscious, or a self-conscious, intelligence? And what sort of manifestation?—a manifestation of one reality, itself to itself, by an inner process of self-evolution, so that the one reality is substance and process and law and cause and effect all at once? These are all various forms or phases of monism, which "philosophy," *i.e.* mature reflection on the facts of experience, may suggest. But, besides all of them, philosophy has at all times persistently suggested an alternative omitted by Mr. Joseph, the alternative which we believe to be the true one, *viz.*, that physical law is a manifestation, to men's minds, of the intelligence and will of a Necessary, Self-existent, Divine, All-perfect Being, really distinct from the finite, contingent, dependent, and conditioned universe of sense experience, the "world"

[1] Joseph, *op. cit.*, p. 384. [2] p. 390, n. [3] p. 384 (italics ours).

which He has freely created, conserves, and rules, according to the eternal dictate of that wisdom whose work must needs be a *cosmos*. The explicit mention of theism, as at least a possible alternative to mechanical materialism and monistic spiritualism, would have considerably enhanced the value of Mr. Joseph's able treatment of the uniformity of causation.

We pointed out already, in connexion with the *principle of sufficient reason* (215), as well as in the preceding paragraphs, that it is a mistake in method to suppose that we must justify the particular view of nature as a whole, or the particular interpretation of its uniformity, on which we base our inductions and inferences, *before* we proceed to make any of those inductions or inferences. It is one thing to set out in the investigation or discovery of truth by making certain assumptions, and to justify these assumptions in due course: it is another thing altogether to demand an ultimate justification of them before we set out at all, and as a condition for setting out. The former procedure is rational, the latter demand is irrational. While, for instance, it is undoubtedly true that unless reality were intelligible, knowledge would be impossible; it does not follow that this truth must be *explicitly assumed* and placed as the *necessary foundation* and starting-point of all search for truth; just as we saw that it is not necessary to assume a knowledge of the existence of an All-wise Ruler of nature before believing anything else about nature. We must start by *assuming* these principles of *sufficient reason*, *causality*, and *uniformity:* they are *presuppositions* of *induction:* it is by experience—in the broadest sense—that we afterwards *justify* them.

There is an analogous assumption discussed in epistemology regarding the *capacity of the mind to discover truth:* an *intelligible reality* and *faculties capable of understanding it* are necessary for an actual knowledge of reality, but *to prove beforehand that our faculties are capable* does not seem to be a necessary condition for arriving at such actual knowledge of reality. A good stomach and wholesome food are necessary for a good digestion; but a *knowledge* that we have either the one or the other is by no means necessary for the desired result. The sceptic has no right to *prejudge* the question of the possibility of knowledge, or to decide it in the negative sense; but neither does it seem justifiable to *prejudge* it and decide it *a priori* in the positive sense. It may not be decided *a priori*, but only by *experience*, by *testing* our faculties, by *letting them work and observing their mode of operation*. No doubt, it is the self-same faculty, which, by reflection, observes and estimates the value of its own operations. But this involves us in no *circulus vitiosus;* for the philosopher's critical reflection on the spontaneous workings of his own cognitive faculties does not purport to be a *logical proof* of their soundness, but a psychological process by which he proceeds to guarantee their soundness to himself, and to satisfy himself that they have not been deluding him. And if the reflecting mind sees no reason to doubt the validity of its own spontaneous assents, after a careful examination of these, it is justified in rejecting scepticism as unreasonable. This larger question, however, is not for logic, but for epistemology.

225. RELATION OF THE PRINCIPLE TO INDUCTION AND TO DEDUCTION.—Passing now to the second question raised above

(223, p. 99), we may inquire what is the precise rôle played by the principle of uniformity in every process by which we establish inductively a general physical law: what exactly is its relation to induction? The principle is a *standard* according to which we generalize, both formally and materially, every abstract relation of cause and effect which we discover in the physical universe; it is a rule of the widest generality, the indefinite scope of which we gradually realize by the application of it to wider and wider generalizations in various departments of nature. If we have determined, by the methods of inductive analysis, that a certain kind or species of physical agency, *A*, is the physical cause of *a*, we can forthwith generalize our discovery that "*A* as such is the physical cause of *a*," by stating that "Whenever and wherever *A* is operative, there will *a* be found"; and in doing this we are only making a special application of the wider principle of uniformity which tells us that "Whatever can be predicated of a physical cause or nature *in the abstract* (as *causally* connected therewith) can be predicated of all instances of that cause or nature". It is not that the *general* law of uniformity is reached first, and the *narrower* law (that "*A* will produce *a*") *deduced logically from it*. In neither case—and indeed in no case—is the discovery of a general law a *ratiocinative process*, a *logical inference* (197, 212). Inference may have been involved in the subsidiary processes by which we *verify* the abstract judgment "*A* as such is the physical cause of *a*"; but the immediate mental process by which the law is *reached* is a process of *judgment* (following on *abstract conception*), not a *logical inference* in the strict sense of a conscious derivation of one judgment from another, or others, which imply the former logically.[1] But if induction is not an *inference*, there can be no meaning in the statement we meet so commonly in logical treatises, that the principle of uniformity is the *major premiss—whether immediate or remote—of every induction.*[2] The principle does not help us to *reach the abstract truth* connecting cause and effect ("*A as such* is the physical cause of *a*"). It is in *generalizing* the latter (to "*All*

[1] *Cf.* Joyce, *op. cit.*, pp. 217, 227; though elsewhere he defines induction as the "legitimate *inference* of universal laws from individual cases" (p. 215): he uses the word here, presumably in the wide sense of *derivation*, not in the sense of a *logically* "inferential process" in which the principle of uniformity would be a major premiss (p. 218). *Cf. supra*, 212.

[2] *Cf. Palaestra Logica*, p. 130; Mellone, *op. cit.*, p. 384; Mill, *Logic*, III., iii., § 1.

A's will produce *a*") that the principle finds a partial application; just as in applying this generalized truth to particular cases by the syllogism, the Aristotelean *Dictum de omni* is partially applied. There is, therefore, a sense in which the law of uniformity bears a relation to the mental ascent from particular to universal, analogous to that which the axiom of the Aristotelean syllogism, the *Dictum de omni*, bears to the descent from universal to particular (170, 191).

Every deductive syllogism in the first figure is a special or narrower application of the *Dictum*. For instance, the syllogism "Man is mortal, Socrates is a man, therefore Socrates is mortal" may be thus expressed: "Mortality, which is predicated of the class man, can be similarly predicated of Socrates, who belongs to that class"; from which it appears, too, that the *Dictum* cannot be regarded as an *ultimate major premiss* of all syllogisms in the first figure, but rather as a *fundamental, standard syllogism* ("*All M is P; S is M; therefore S is P*") symbolizing that type of mental process, and by its self-evidence justifying the latter (192).[1]

So, too, induction is a distinct mental process of ascent from particular to universal; and every such ascent is a narrower and more special exercise of the fundamental, standard, typical induction, by which we reach the widest law of physical nature, *viz.* that *natural causes act uniformly*—that *whatever (a) has been discovered to be really due to a physical cause (A) in any observed instance or instances, will be always and everywhere produced by that cause.* And, just as the *Dictum de omni* is not a principle whose truth must be consciously grasped by the mind *beforehand*, as a condition for reasoning validly by the syllogism, but is rather a generalization of the syllogistic process, implicitly involved in every syllogism and explicitly grasped only by a deliberate, reflex analysis of this process itself, so the principle of the uniformity of nature is not a truth which must be grasped as a logical antecedent to justify the generalization made in each separate induction, but is rather itself a wider induction partially involved in every special induction, and explicitly grasped and formulated in its fulness only when the mind comes to analyse those special inductions afterwards.[2]

[1] *Cf.* VENN, *op. cit.*, p. 126: "When the *Dictum* was assigned as the ground of the individual inference, all that we were doing was to generalize this latter".

[2] Mr. JOSEPH (*op. cit.*, pp. 407, 408) rightly rejects the view that uniformity of

It was not by supposing "belief in uniformity" to be "by no means the earliest of our beliefs," but in supposing it to be *reached* by a certain kind of "inference," while at the same time supposing this kind of "inference" to depend for its validity on an antecedent profession of this belief, that Mill fell into the fallacy of *petitio principii*: just as we should fall into the fallacy were we to suppose that our knowledge of the *Dictum de omni* is an *antecedent condition* for the validity of the syllogism, and is itself reached by a syllogism. Belief in the uniformity of nature (in the categorical sense) is not a mental assent which must precede every induction we make: it is partially embodied in each, and is gradually extended by us to all nature.

In every scientific induction of a physical law, belief in the uniformity of nature is, therefore, operative. For we embrace the belief that the causes we are dealing with are *necessitating* causes (*i.e.* causes invariably followed by the same effects), when, in the first or *abstractive* stage of the process, we convince ourselves, from an observed case or cases, that "the nature *A* is necessarily connected with the effect *a*".[1] And in the second or *generalizing* stage, in which we pass from this *abstract* judgment to the *universal* judgment, "All *A*'s will always and everywhere produce *a*," we still more explicitly assent to what is a partial application of the general principle that "in the real order the same cause does always actually produce the same effect".[2]

But, if the principle of the uniformity of nature is thus shown to be a general expression or summing up of the mental process by which we pass *from* observed cases, *through* the *abstract*, *to* the *universal* judgment—*from* "Some (observed) *M*'s are *P*," *through* "*M* as such is *P*," *to* "All *M*'s are *P*": is not the self-same principle equally involved in the downward process by which we pass deductively or syllogistically from the universal "All *M*'s are *P*" to its special applications in the conclusions "These or those *S*'s, which are (other, new, hitherto unobserved) *M*'s, are

nature "is the ultimate major premiss of all inductions". He further admits that "it is not, indeed, necessary, in a particular investigation, to assume this uniformity to extend beyond the department of facts with which we are dealing"; but contends that it is, though only partially applied, nevertheless universally assumed, in every particular induction (p. 407). It is not so assumed *explicitly*; but when we come to reflect on the grounds of our inductions we see that the universal principle was implicit or latent in them: that otherwise we could not make our experience intelligible: that our success in making experience intelligible through it justifies our belief in it.

[1] JOYCE, *op. cit.*, p. 219. [2] *ibid.*

also *P*"? Undoubtedly, the principle of uniformity is involved *in the application of the syllogism to any actual sphere of reality.* The *Dictum de omni* informs us that "Whatever can be predicated of a class can be predicated about any member of the class". But in order to make the predication about any *new instance* of a class in any actual sphere, we must (*a*) *identify the instance as a member* of that class, and (*b*) assume that *all the members have a stable, uniform nature, which constantly demands the same predicates.*[1]

When dealing with the merely *formal* aspect of the syllogism, we regarded the terms of the latter as expressing *abstract concepts* of *possible class-essences*, apart from the question of their *verification* or *realization* in any actual sphere of reality. We *supposed* each abstract thought-object to be fixed, stable, unchanging. We had not, therefore, to raise the question whether *there is really* a corresponding uniformity, regularity, stability, in the actual spheres within which we suppose these concepts to apply.

It is when we pass from the purely *formal* and hypothetical processes of arranging and *dividing abstract concepts* logically according to intension and extension, and then reasoning "*consistently*" from them,—to the *material* and categorical processes of *classifying things*, of *verifying* our *definitions* of the latter, and reasoning "*truly*" or "*demonstratively*" about them,—that we feel called upon to justify our belief in that *real* uniformity in things, which is the objective ground and condition of our thinking, judging, and reasoning rightly about them.[2]

Dr. Venn, in his *Empirical Logic*,[3] asks the interesting question: How is it that an analysis of induction raises the question as to the origin of our belief in the uniformity of nature, while no corresponding difficulty is supposed to be felt in respect of deduction? He takes the example of a man bitten by a cobra.

[1] *Cf.* JOSEPH, *op. cit.*, p. 378; MELLONE, *op. cit.*, p. 252 (referring to Ueberweg, *Logic*, § 101): "the worth of the syllogism as a form of knowledge depends on the assumption that general laws of causation hold in nature, and may be known".

[2] "Geometrical proofs rest on the intuition of spatial relations, and algebraic on the intuition of quantitative relations. . . . In fact, our belief in the uniformity of space, and in the uniform formation of the numerical series, stands to mathematical reasoning as our belief in the uniformity of nature stands to inductive. Deny them, and in either case no general proposition remains possible any longer. Nay more, no demonstration remains possible even about a particular case."—JOSEPH, *Logic*, pp. 506, 507.

[3] p. 124.

We believe the man will die. We may assign our reason in either of two ways:—

"*Deductive:* All men who are bitten die. The man XY is bitten. Therefore XY will die.

"*Inductive:* The men A, B, C . . . were bitten and died. The man XY has also been bitten. Therefore XY will die."

Ask him who gives the deductive answer why he considers that the reason he assigns is a sufficient one: he will tell you that it is so because "what holds good of a class holds good of every member of that class". Now ask a similar question of him who gave the inductive answer: ask him why does he consider the fact that "A, B, C . . . and all men who have been bitten died" to be a sufficient reason for believing that XY will die: he will tell you finally that he considers it to be a sufficient reason "because nature is uniform". Now, why is the man who gives the deductive answer let alone at this point and not called on to explain why he believes that "what holds good of a class holds good of every member of that class," while the man who gives the inductive answer is not let alone, but has to justify his belief that "nature is uniform"? The only reason for difference of treatment would be because the deductive reasoner is not supposed to be concerned with the *application* of his class-concepts to the real world, but only with their *consistency* within the sphere of abstract thought, in which they have been conceived as fixed, static, unchanging: while the inductive reasoner is supposed to be concerned with the *real validity* of those concepts, with *their application to the real world*, and, therefore, with the *existence of uniformity in the real world itself*.

But the moment a person attempts to *apply* a syllogism within any domain of actual reality—in other words, to *demonstrate* or *prove* anything as *true*—he is committing himself to a belief in the "uniformity of nature" regarding certain classes of things within that domain. Hence, those logicians who are inclined to view their science as concerned exclusively with the *consistency* of thought refuse to go behind such ultimate *logical* generalizations as the *Dictum de omni* and the *Uniformity of nature* for the purpose of justifying these. Understanding by a *logical* ground or reason for assent to a judgment, always some *wider generalization* which includes the latter (198), they observe that there is no possible wider generalization than either of the two in question; and

they conclude that the justification of our assent to such principles falls within the province of psychology or metaphysics, rather than of logic.[1]

JOSEPH, *Logic*, chap. xix., pp. 407 *sqq.* VENN, *Empirical Logic*, chaps. iv., v., and xv. JOYCE, *Logic*, chap. xv. MILL, *Logic*, III., iii., iv., v., and xxi. MERCIER, *Logique*, pp. 326 *sqq.* MELLONE, *Introd. Text-book of Logic*, pp. 280 *sqq.* MAHER, *Psychology*, pp. 420, 517-24. WELTON, *op. cit.*, ii., pp. 1-30.

[1] *Cf.* VENN, *op. cit.*, p. 128.

CHAPTER V.

HYPOTHESIS: ITS NATURE, FUNCTIONS, AND SOURCES.

226. FUNCTIONS OF SCIENTIFIC HYPOTHESIS.—We have seen that the aim of science is to discover the causes and laws by which we may explain the facts of our experience. Our knowledge of these causes and laws is embodied in universal judgments, and these universal judgments it is the function of induction to establish. But we can neither discover nor verify a universal judgment unless we are first led somehow or other to suspect or suppose it to be true. Such suspicion or supposition we call an *hypothesis*.

Not every supposition, however, is an hypothesis in the strict or scientific sense of this term. For example, in order to help our imagination in the study of phenomena due to gravity, we suppose that if the total mass of a body were concentrated in a mathematical point, called the *centre of gravity* of the body, that point would manifest the same force and have the same weight as the whole body. We imagine the earth as a mathematical point. We conceive its total gravitation-force to be concentrated in that point—its "centre of gravity". We find it easier in this way to *measure* that force, to bring home to ourselves the law by which it acts on bodies on or near the earth's surface, than if we tried to conceive the several particles of the earth's mass acting each in its own place and independently of the others, on those bodies. But we know, all the time, that the latter is really the case, that our conception of "centre of gravity" has no *fact* for its object, that the conception is from beginning to end a mere fancy, a purely *subjective* conception having no other object than an imagined possibility.[1]

Again, in order to help ourselves to conceive great distances or magnitudes we often have recourse to mental images which we call suppositions. To realize the distance of the moon from

[1] *Cf.* MERCIER, *Logique*, p. 339.

the earth we may suppose or imagine a cannon-ball travelling at a velocity of five hundred yards per second and reaching the moon after eight days: that image helps us to bring home to ourselves a distance so great that a mere statement of the number of miles in it can hardly be pictured by us. But such a supposition is not what we understand by a scientific hypothesis: it belongs to the sphere of *imagination* exclusively, while a scientific hypothesis is a *judgment bearing on our knowledge of reality*. The image of the cannon-ball gives us a clearer apprehension of something we already knew; an hypothesis aims at teaching us something we did not know before.[1] Here is a simple example of a scientific hypothesis: The juice of the grape ferments: the origin and nature of fermentation were at one time unknown: Pasteur conjectured that it was due to germs that swarm on the grapes, leaves, and stems, of the vine-tree. That was a scientific hypothesis.

An hypothesis, therefore, *is an attempt at explanation: a provisional supposition made in order to explain scientifically some fact or phenomenon*.

The construction of hypotheses is not confined to the inductive sciences. The process described in connexion with deductive reasoning, by Aristotle and his mediaeval commentators, as "*inventio medii*," "discovery of a middle term," *i.e.* of true and proper premisses to prove a conclusion, is really identical with what we nowadays call the conception or construction of an hypothesis. But it is in the positive or inductive sciences that hypothesis plays an all-important rôle. And in these sciences we understand by it the conception or supposition of some cause or law capable of explaining certain observed facts. Without hypothesis we can make no progress in scientific investigation. We cannot *find* the causes of phenomena without first *suspecting* their existence and whereabouts. Our experiments will lead nowhere unless made with the object of *verifying some supposition*. To direct our investigations along certain lines towards the discovery of laws: such, in a word, is the function of hypothesis.

All hypotheses should have their origin in the observed facts which we are attempting to explain (233, 234). But the actual conception of hypotheses is amenable to no logical rules. It is just here that the sagacity, genius, and originality, of the scientist and

[1] *Cf.* MERCIER, *Logique*, p. 334.

inventor will have free scope (197). Wrong hypotheses will be usually conceived before right ones. Kepler is said to have conceived and disproved *nineteen* successively, before arriving at the laws of planetary motion.[1] It must not, however, be imagined that hypotheses are useless unless they turn out to be true; they often admirably fulfil their function of directing investigation, and do immense service to science, even though they be afterwards disproved. Thus, in astronomy we have the famous example of the Ptolemaic or geocentric hypothesis, which gave place to the Copernican or heliocentric hypothesis in the sixteenth century.

The conception of an hypothesis which is likely to prove useful, and helpful to the progress of science, is usually possible only to the well-trained mind that is stocked with information about the matter under investigation, and is accordingly quick to detect and utilize analogies. To such a mind, even the most commonplace facts may suggest invaluable lines of speculation and experiment—as the falling apple did for Newton, and the dancing lid of the steaming kettle did for Watt. Whewell was therefore right in emphasizing, as against Mill, the great importance of hypothesis, or as he called it, "colligation of facts by means of an exact and appropriate conception," in the whole inductive process.[2] But there are various kinds of hypotheses; and some are "more far-reaching in their effects than others; for some are much more general, and apply to a much larger number and variety of facts. . . . Scientific hypotheses consist for the most part not in the mere coupling in the mind, as cause and effect, of two insulated phenomena (if the epithet may be allowed): but in the weaving of a large number of phenomena into a coherent system by means of principles that fit the facts".[3] This brings us to the consideration of some of the principal types of scientific hypothesis.

227. SCIENTIFIC VALUE OF VARIOUS KINDS OF HYPOTHESIS.—We have described an hypothesis as a provisional explanation of certain observed facts. Now, we *know* a fact *scientifically* when we know *all its causes*, and the mode of its connexion with, or dependence on, these causes. If we take any phenomenon, or group of phenomena, involving within it a multiplicity of elements, changes, motions, activities—for example, the motions of the

[1] *Cf.* WELTON, *op. cit.*, ii., pp. 66, 86. JOSEPH, *op. cit.*, pp. 435-6.
[2] *Cf.* WELTON, *op. cit.*, pp. 48 *sqq.* JOSEPH, *op. cit.*, p. 434.
[3] JOSEPH, *ibid.*, pp. 432, 433.

planets, or the phenomena of the refraction and reflexion of light—we may conceive and verify hypotheses as to the exact quantitative relations between those various elements and motions, without for the time inquiring either into their origin or their *raison d'être*, their *efficient* or their *final* causes. We may, by accurate observation and experiment, seek to arrive at an *exact quantitative expression* of the various events and agencies which make up the whole phenomenon. We may aim, in other words, at weighing and measuring the facts, at *describing* them with mathematical precision, at establishing formulæ which will be "descriptive statements of the exact character of the phenomena to be explained, when their relations to other phenomena are not in question";[1] at reaching expressions which will describe concisely and accurately the quantitative side of those phenomena. Now, the scientist's supposition or conjecture as to the exact quantitative relation or ratio between some or all of the various elements or motions in a given series of phenomena, has been commonly called an *Hypothesis of Law*. And when such supposition is verified, and formulated in clear and concise language, it is what is commonly recognized in the physical sciences as a *Physical Law*. A "law" of nature, in this sense of the term,[2] tells us "*how*" a phenomenon takes place, *i.e.* in what exact measure and proportion the various constituent agencies and energies must be present and operative; it is simply an exact mathematical description of the measure in which a certain phenomenon regularly occurs. Such, for example, are the "laws" of refraction and reflexion of light; or the "law" which states that the strength of an electric current varies directly as the electromotive force and inversely as the resistance of the circuit; or the "law" of gravitation, that any two bodies in the universe tend to move towards each other with an acceleration that varies directly as the product of their masses and inversely as the square of their distance apart: "The business of physical science," writes Mach, "is . . . the abstract quantitative expression of facts. The rules which we form . . . [for this purpose] . . . are the laws of nature."[3]

But though this mathematical measurement of phenomena gives us a clearer *description* of them, still it does not give us a

[1] WELTON, *Logic*, vol. ii., p. 91.
[2] For another and deeper sense of this expression, see above, 217.
[3] *apud* WELTON, *loc. cit.*

full insight into them, it does not *explain* them ; for explanation reveals not merely *how*, or *in what manner*, an event happens, when it does happen ; but also *why* it happens (its *final* cause), and *what makes it happen* (its origin or *efficient* cause). And while, as physical scientists, we may seek to establish "physical laws," in the sense of quantitative descriptions of the relations between the various elements of a given regularly recurring phenomenon, we cannot, as rational beings, rest content with such partial explanation, but are impelled by our nature to ask ourselves further about the "whence" and the "wherefore" of the whole phenomenon, and so to connect it with *all its causes*. We cannot help asking those further questions, because the "sufficient reason" of any phenomenon is not to be found in the isolated phenomenon itself, but in the sum-total of all its causes. The Positivist school of philosophers would, indeed, have science study "only the laws of phenomena, and never the mode of production"[1] of these phenomena by their causes. But science—or philosophy—*will* insist on studying the latter. Man *will* and *must* seek the *causes*, both efficient and final, as well as the mere description and measure, of the phenomena surrounding him. And the hypotheses he makes about the *causes* of any given phenomenon—as distinct from those he makes with a view to arriving at a more exact quantitative presentation and description of the constituent parts of the phenomenon—have been called, in contradistinction to the latter, *Hypotheses of Cause*.

For example, Newton's gravitation hypothesis was an *Hypothesis of Law* in so far as it simply aimed at giving *an exact quantitative expression or description* of the various relations of mass, distance, and rate of motion, that make up the whole complex group of phenomena presented to us in our experience of falling bodies and of the motions of the moon and the planets: and in so far as the conception of "gravitation" includes all these phenomena in one common quantitative description, it has been verified, and is now an established "theory" or "law". That is to say, we now recognize as a verified and accurate *description* of the *manner* and *measure* in which these motions of matter occur throughout the universe, the statement that "the acceleration with which any two distant bodies in the universe, M_1 and M_2, tend to move towards each other through space,

[1] WELTON, *Logic*, vol. ii., p. 91.

varies directly as the product of M_1 and M_2, their masses, and inversely as the square of their distance (D^2) asunder— $\frac{M_1 M_2}{D^2}$

But, beyond all this lies the question as to what is the determining *cause* of those motions. And as to this, we are indeed free to assume that it is a certain *natural property*, an *active power* or *force* (228), in the bodies themselves, in virtue of which they determine or bring about these motions in this precise manner and measure. But beyond the mere fact, which the principle of causality forces us to admit, that there *is* in the bodies which constitute the visible universe *some* adequate *cause* of those motions, *some force* that produces them, we *know* as yet practically nothing. What is the *nature* of that force? How does it determine and bring about those motions whose magnitude we can accurately estimate by an already verified law? What sort is that influence? How is it exerted through space, independently, as it would appear, of intervening bodies? We describe it as "attraction," but what idea does this term convey to our minds? Thus, as to the *nature* of the cause in question, as to *what kind* the force of gravity *is*, and *how* it *acts*, we are still largely in the dark. Here, then, is a field for further hypotheses, *hypotheses of cause*, whose purport will be to *explain* the *known fact* of gravitation. Various hypotheses have been framed at different times to connect this fact with the hypothetical all-pervading ether, and with the fundamental constitution of matter.[1] So far, however, these are conjectures hazarded to help our imagination in picturing the phenomenon to ourselves, rather than possible explanations of it. Hypothesis, as we have defined it (226), is essentially *explanatory*: a supposition that does not offer an *explanation* of phenomena, but merely aids us in conceiving and describing them, would appear to fall outside our definition. Every *hypothesis of cause*, *i.e.* every supposition of some definite antecedent (or group of antecedents) as being the real, actual cause of the phenomenon in question, is necessarily explanatory: it offers—provisionally—an *explanation* of the phenomenon. *Hypotheses of law*, on the other hand, in so far as they merely describe with mathematical exactness the manner in which phenomena occur, are rather *descriptive* than *explanatory*; but nevertheless, inasmuch as a correct quantitative estimate of

[1] *Cf.* NYS, *Cosmologie*, p. 125. LESAGE, *The Unseen Universe*, § 140. PICTET, *Étude critique du matérialisme et du spiritualisme*, p. 239.

those changes or activities suggests or reveals to us, at least partially, the nature of their causes and of the laws according to which these causes interact—for "*operari sequitur esse*,"—and inasmuch as hypotheses of law thus inevitably suggest hypotheses of cause, the former as well as the latter have some claim to be called *hypotheses* in the stricter sense, *i.e. explanatory hypotheses.*

Some hypotheses, whether they appear descriptive or explanatory, may be recognized from the beginning as having little or no probability. We may know so little about some unfamiliar, unexplored, complex, many-sided phenomena—such as those of electricity, for example—as to be scarcely able to make any supposition at all as to their real nature, laws, and causes. But *some* provisional supposition as to the nature of the agent we call electricity, is absolutely necessary if we want to collect, arrange, describe, and discuss, in intelligible language, its various manifestations, and their connexion with their supposed common cause. And this supposition, moreover, if it is to be of any use in helping us towards an *explanation* of the phenomena, must be *based on some analogy with some known agent*. If we make any supposition at all, from which we can infer anything, about the unknown thing we call "electricity," we must suppose the latter to be something *resembling in some way or other* some *known* natural agent. Accordingly, the supposition was made by Franklin, if only as a starting-point for investigation, and to see how it would work—in other words, as a *working hypothesis*,—that electricity was a *fluid* of some sort. This hypothesis, though scarcely probable, and merely "better than none," the best perhaps in the circumstances, purported from the beginning to be mainly descriptive, *but was none the less, of its nature, explanatory also*, inasmuch as it supposed the *real cause* of the phenomena in question to be some sort of fluid. That hypothesis was never verified, and gradually lost ground, at least in its original form; though the "electron" hypothesis, which is closely analogous to it, is now in turn taking the place of the two-fluid hypothesis.

In other cases, however, such working hypotheses may for a long period grow steadily in probability according as they are found capable of explaining a larger area of phenomena, as did the Ptolemaic or geocentric hypotheses (in astronomy), with their cycloids and epicycloids to account for the apparent motions of the planets.[1] So admirably did this group of hypotheses "ex-

[1] *Cf.* Joseph, *op. cit.*, p. 435.

plain" the known phenomena, that it was pretty generally (and erroneously) regarded, for centuries, as a fully verified or established system. But, as St. Thomas pointed out with his characteristically prudent reserve, some other hypothesis might perhaps account after all equally well—or even better—for the apparent motions of the heavenly bodies.[1] And so after-events proved, culminating in the substitution of the Copernican for the Ptolemaic astronomy.

Or again, two conflicting hypotheses may appear to account equally well for certain phenomena. These latter will be *differently described* on either hypothesis. And both descriptions will *appear* to be equally accurate. But evidently *at least one* of the descriptions must be *de facto* inaccurate: the assumptions involved in the very language used—based as this is on the supposition that the phenomena are of such and such a nature, due to such and such a cause, while they are quite otherwise in reality—such assumptions necessarily falsify the whole description.

We may conclude, then, that there is no *fundamental* difference between *working*, *descriptive*, and *explanatory* hypotheses, hypotheses *of law*, and hypotheses *of cause*, provided only and always that they are suppositions which *have for their objects the* REAL CAUSES *of the observed phenomena, and the* REAL LAWS *according to which the changes wrought by those causes in the observed phenomena actually take place.*

228. NATURE AND VERIFICATION OF CAUSAL OR EXPLANATORY HYPOTHESES.—The first essential, then, of a scientific hypothesis, is that it be a supposition of something *real*, equally real with the phenomenon it is to explain. This at once disposes of the class of suppositions referred to above (226), to which we have recourse merely in order the better to realize some phenomenon. But, furthermore, even when the object of our supposition is not a mere fancied possibility, when we mean the cause or law we are supposing, to be real, to be a fact, even then the question arises: Have we *always* or *necessarily* a *scientific hypothesis*, as distinct from what some writers, for want of a better name, call *systematic* or *synthetic conceptions*?[2] What sort, in other words,

[1] "Licet enim talibus suppositionibus factis apparentia salvarentur, non tamen oportet dicere has suppositiones esse veras, quia forte secundum aliquem alium modum, nondum ab hominibus comprehensum, apparentia circa stellas salvantur."—*In Lib. ii. De Cœlo et Mundo*, l. xvii.

[2] MERCIER, *op. cit.*, p. 338.

must we suppose our cause to be, in order that our supposition of its existence be a *scientific hypothesis?*

The controversy as to what kind or concept of cause it is legitimate for us to employ in our hypotheses about the phenomena of nature, is one of very long standing. No supposition of ours, as to what is the cause of a phenomenon, will be of any use unless it be *verifiable*. But what kind of cause must we suppose to be operative, if our supposition is to be verifiable? Newton insisted that the object of our hypothesis must be a *vera causa*, a real cause—meaning, thereby, to exclude arbitrary, fanciful, *a priori* suppositions and prejudices, not suggested by facts of experience. This, of course, is obviously right and proper. Must the cause, however, be supposed to be itself a *phenomenon* of some sort, *i.e.* something itself perceptible by the senses, so that the only valid verification of such hypothesis would be actual discovery, by sense perception, of the supposed cause, and actual observation of its visible causal connexion with the effect? Such a requirement is rightly repudiated by scientists, though it is only such a sort of cause that answers to Mill's definition.[1] Followers of Mill's phenomenist philosophy contend that it is a mere waste of time, and a hindrance to real scientific progress, to refer the various phenomena of mind, or of external nature, to corresponding "faculties" or "powers" or "forces" in either domain. And no doubt, such reference of individual effects, or classes of effects, to corresponding efficient principles, whether these be called "faculties" or "forces," would be calculated to retard further investigation, if such reference were taken *as an ultimate rational explanation* of those effects; if, for instance, men were so foolish as to think they had said the last word as to why opium induces sleep by declaring opium to have a *vis dormativa*—to use the old familiar example.

It is true that in the Renaissance period some of the decadent camp-followers of the great mediaeval Scholastics left themselves open to this reproach by taking refuge in such verbal explanations of natural phenomena.[2] But up to quite

[1] *Logic*, III., v., § 2. *Cf. supra*, p. 74, *n.* 3; *infra*, p. 131.

[2] *Cf.* De Wulf, *Scholasticism Old and New* (2nd edition), pp. 147 *sqq.*; *History of Medieval Philosophy*, p. 503. It was not, however, the fault of those mediaeval philosophers that the "forces" or "causes" in question were then, and are still for the most part, "occult," *i.e.* such that we have no positive imagination of the mode of their action. Modern scientists who are loudest in their ridicule of those "occult forces" are themselves obliged to have recourse to "motions" and "masses" and "ions" and "electrons" and "ids" and "biophors" and a whole host of such things,

recent times it was the fashion with modern philosophers and scientists, in their boasted ignorance of mediaeval thought, to impute this and all manner of absurdities to Scholasticism generally, and with the inevitable result that the ridicule they heaped upon their predecessors is now seen in the light of history to recoil upon their own heads. The thirteenth-century Scholastics, no less than their later critics, realized the importance of observation and experiment, the necessity of noting analogies between phenomena, of endeavouring by analysis of these analogies to reduce gradually, as far as possible, the number of distinct "forces," or "powers" postulated for the explanation of phenomena. They were never content to refer each separate phenomenon in nature to a distinct and corresponding cause supposed to be capable of producing that effect alone—to be *sui generis*, so to speak. They pushed investigation as far as the conditions of their time permitted. And those who, in modern times, have inherited the best traditions of Scholasticism, have always welcomed every careful attempt of the positive and experimental sciences to unify our experience of external nature by tracing large and varied and apparently unconnected fields of phenomena to the operation of some one or some few common "agencies". They have nothing but approval for the methods whereby scientists have formulated and tested hypotheses for the exploration of hitherto unsuspected natural "forces," or for the explanation of phenomena by referring these to already known "causes," with which such phenomena were previously thought to have no connexion. They themselves adopt these methods in physical science. They are not content to say that the varied phenomena of external nature must have causes, must be due to the operation—and co-operation—of nature's forces and agencies. They endeavour to discover in what groups of phenomenal antecedents the agencies productive of a given effect are operative. They try to bring to light "the sum-total of the [phenomenal, perceptible] conditions, positive and negative taken together, the whole of the contingencies of every description, which being realized the consequent [effect or phenomenon] invariably follows"—which is Mill's own conception of the discovery of a "cause".[1] And it is only when the inductive methods fail for want of analogies on which to base hypotheses, *i.e.* in investigating the remoter causes of wider fields of phenomena, and the Ultimate Cause of the whole phenomenal universe, that they use the simple *a posteriori* argument to prove that such remoter causes—and such Ultimate Cause—must exist, and to discover about the nature of these just as much as the effects will warrant us in attributing to the latter.

But Scholastics have held to the doctrine that while the senses stop at phenomena, intellect or reason can discover, in these phenomena, "substances," "causes," "faculties," "forces," which constitute and permeate the world of sense experience, and which reveal themselves to intellect by acting in and through the phenomena of sense. And they have held to this doctrine in obedience to such self-evident dictates of reason as that "every event must

just quite as occult, in their own hypotheses. "How could *masses* and *motions* that must remain *occult* be any more acceptable . . . than the *occult powers* of the ancient Scholasticism?"—Duhem, *L'Évolution de la mécanique*, p. 190. *Cf. Dublin Review*, April, 1906, pp. 332, 337, where Professor Windle suggests a comparison of some of Weismann's hypotheses with the famous *virtus dormativa*.

[1] *Logic*, iii., v., § 3.

have a cause," "every change must be a change of some state," and "every state must be a state of some subject or substance". Of course, such principles will not of themselves unlock the secrets of science by telling us whether this and that event, or change, or state, have any underlying agencies, or causes, or substances *in common*, or how many of the latter there are in the world of sense experience altogether. Yet positivists and phenomenists appear to think that something like this should be expected from those principles. For, not finding in the latter the key to any new *positive* information about nature, they proclaim that "substance," "power," "force," "efficiency," "purpose," etc. —in a word, all such objects of thought as lie beyond the ken of the senses—are "occult" and "unknowable," and should therefore be discarded.[1] But as a matter of fact these objects are not "occult" to the *intellect*—of Positivists any more than of Scholastics. The former, despite their disclaimer of agnosticism, know just as much, or as little, about such objects of thought, as the latter: they discourse about "substances" and "causes" and "forces" and "faculties" no less than the latter: and we are all alike guided, in our ascent to such thought-objects from the data of sense, by the scholastic principle that from the *operations* of things we judge of their *natures: operari sequitur esse; qualis est operatio talis est natura.*

But positivists pretend to be able to "explain" the universe without calling in the aid of any "hyperphysical" entity[2]—we shall see presently with what effect,—and blame Scholastics for not discarding the "antiquated" metaphysics of "substance" and "accident," of "faculty," "power," and "force," in the philosophy of external nature. They have tried—unsuccessfully, of course—to deliver human reason from the supposed bondage of theology and metaphysics, by eliminating from their system of thought all such "Scholastic" notions. We may be pardoned if we hesitate to exchange the "antiquated" system for the teaching of those later philosophers—who resolve all reality into "states" or "phases" or "processes," while denying that there is any substance or agent of which these are the states, phases, or processes; or into a transient "flow" of sensations in the individual's consciousness, while denying that there is any permanent mind other than the said flow of sensations, or any abiding, substantial *ego*, or individual, to experience and interpret these sensations, and thus to remember past experience and to expect and anticipate future experience. The fact is that this phenomenist philosophy has made itself unintelligible by "divesting the human mind of its most fundamental conceptions"[3] —or, rather, by pretending to accomplish such a hopeless task: for it really *smuggles into its explanations*, at every turn, under the mask of a new terminology of course, *the very conceptions it pretends to dispense with.*

In opposition to the traditional philosophy of those so-called "occult" causes, Mill boldly proclaimed that he would deal only with causes which were themselves "phenomena," *i.e.* entities which would be in themselves perceptible by the senses: "I pre-

[1] *Cf.* I. E. RECORD, April, 1910: "Some Current Phases of Physical Theories," p. 403; January, 1910: "The New Knowledge and its Limitations," p. 27.

[2] *Cf.* I. E. RECORD, April, 1910, *ibid.*

[3] WARD, *Naturalism and Agnosticism*, i., p. 65. *Cf.* I. E. RECORD, *ibid.*, p. 400.

mise," he wrote, "that when . . . I speak of the cause of any phenomenon, I do not mean a cause which is not itself a phenomenon; I make no research into the ultimate or ontological cause of anything".[1] The trammels he thus sought to impose upon human thought were soon deemed too irksome, not only by philosophers, but even by the scientists who professed a general sympathy with the positivist philosophy. It is, indeed, conceivable that *scientists* might agree to confine their efforts exclusively to the discovery of coexistences and sequences between *phenomena*, and to eschew all thought and all mention of *non-phenomenal* or *imperceptible* entities, even as mere aids to investigation.[2] But of course they have refused—and rightly—thus to debar themselves from using their *imagination* at all events, in addition to their *senses.* They have given a very wide interpretation indeed to the term "phenomenon," if the "causes" which they contemplate nowadays in their hypotheses are to be regarded as phenomena. Not only are some of the objects of current scientific hypotheses —*i.e.* some hypothetical *causes* of the phenomena of nature—not perceptible themselves by the senses, but they are not even in any true sense positively picturable by the imagination. We are very far removed indeed from the "phenomenal antecedents" of Mill when we are introduced into the domain of "ethers," "vortices," "corpuscles," "ions," and "electrons," by the physicist, or into the domain of "ids" and "biophors" and "biotic energies" by the physiologist. Indeed it is not so clear that scientists have not returned to the "occult" entities of the "antiquated" metaphysics,[3] and merely rebaptized, in a more mechanical terminology, the "*materia prima*" and "powers" and "efficiencies" and "vital forces" of Aristotle and the Scholastics! As a matter of fact, it is now beginning to be recognized by scientists that all attempts to explain nature, whether organic or inorganic, by collocations and motions of material masses in space and time, *i.e.* by purely

[1] *Logic*, III., v., § 2.

[2] *Cf.* POINCARÉ, *Science and Hypothesis*, p. 223: "The day will perhaps come when physicists will no longer concern themselves with questions which are inaccessible to positive methods, and will leave them to the metaphysicians. That day has not come yet; man does not so easily resign himself to remaining for ever ignorant of the causes of things."

[3] See article, "Weismann and the Germ-Plasm Theory," in the *Dublin Review*, April, 1906, where Professor Windle suggests the comparison of Weismann's hypotheses with the "*vis dormativa*" and other such "*virtutes occultae*" of the older philosophy. *Cf.* also, *What is Life?* by the same author (Sands & Co., 1908); and I. E. RECORD, April, pp. 398 *sqq.*

perceptible or *picturable* factors, and without the aid of purely *conceptual* or *intelligible* factors, such as force, power, efficiency, purpose and design, have proved futile; that concepts of hyperphysical entities and influences, however "occult" to sense or imagination, are indispensable for a rational explanation of nature's processes; in a word, *that the cause or principle of action which may be the object of a legitimate scientific hypothesis need not be itself a phenomenon, directly perceptible by the senses.*

It must, however, be such an agency, or group of agencies, that, though not directly perceptible itself, it is perceptible *in its effects:* it must be supposed to dwell *in phenomena*, to become operative in certain *combinations of phenomena*, and to produce therein directly perceptible effects. This indirect perceptibility of the supposed causes, *in their effects*, is necessary and sufficient for the object of a scientific hypothesis. In this way *alone* are "atoms," "electrons," "ions," "sub-atomic motions," "biophors," and all the infinitesimally minute "causes" of modern scientific hypotheses, perceptible or "phenomenal": in their *effects*, in the phenomena which they are supposed to actuate or constitute; and in this they differ *in no way* from the "*materia prima*," "*forma substantialis*," "qualities," "forces," "faculties," "natures," "properties," etc., of Scholasticism: for these too are perceptible indirectly, in their effects.

Properly speaking, all such explanatory factors of our experience are "intelligible" or "noumenal," rather than "sensible" or "phenomenal". The need that impels us to look for an explanation of sense experience obliges us to conjecture or suppose the real existence and operation of such—really *supra*-sensible—agencies. The whole process of conceiving the latter, and reasoning from such conceptions, is a process of the faculty which transcends the faculties of sense—the intellect. It makes comparatively little difference whether these conceptions, these hypothetical "causes," are more or less immersed in, and supported by, concrete imagination-pictures.[1] The visible,

[1] It might, perhaps, be argued that hypotheses having for their objects abstract "powers," "forces," "natures," etc., in phenomena, cannot be so accurately verifiable, nor, therefore, so fruitful to science, as hypotheses which contemplate only such directly calculable factors as "atoms," "electrons," "undulations," etc. This is scarcely true, for mathematical values may be assigned to the former as easily as to the latter. It cannot be said that the British scientists have in any striking way excelled the French in their contributions to science; yet the former have been always far more addicted than the latter to *concrete, picturable, mechanical* conceptions. *Cf.* DUHEM, *Évolution de la mécanique* (Paris, 1903, ch. xv.); Professor Windle's article in the *Dublin Review*, already mentioned; art. on "The Contrast of English and French Concepts of Physical Theories," by the Rev. P. DE VREGILLE in the *Month*, April, 1907, pp. 350 *sqq.*; H. POINCARÉ'S *Science and Hypothesis* (Eng.

measurable phenomena, in which they are supposed to be operative, are equally amenable to observation and experiment, whether the hypothetical "causes" be conceived as "properties," "forces," "affinities," "qualities;" or as "atoms," "electrons" "vortices," "undulations," etc. Quantitative values may be assigned to such factors, by whatever names we call the latter. Since they are supposed to be factors operative *in material phenomena*, there must be a quantitative aspect in their *modus operandi;* only we must not forget that this is not their sole aspect, and that we have not "explained" the facts fully by "calculating" the *measurable aspects* of these factors. Yet this is likely to be forgotten by scientists who are influenced by the empirical philosophy. Their tendency naturally is to assume that "all perceptible facts are measurable" [1] in terms of material masses and mechanical motions, and that science can attain to nothing that is not thus measurable. But, for instance, exhibitions of "talent, prudence or self-denial" [2] are perceptible facts. Yet, surely, their "magnitude" cannot be measured by any mechanical standard. The "attainment of precise mathematical law" is a proper ideal *for those departments* of research *whose laws are capable of assuming* "the form of precise *quantitative* statement"; [3] but to assume that *all reality* is thus quantitatively measurable, and that exact measurement exhausts all we can know about it, is utterly unjustifiable in point of method, as well as being erroneous in fact. An unfortunate outcome of this tendency has been already instanced (201, 224, *cf.* p. 141) in the hopeless attempts of some scientists and philosophers to explain all the phenomena of the universe on the hypothesis that they are all ultimately reducible to *mechanical motions of atoms of matter*, a supposition which has absolutely nothing to recommend it but its *excessive* simplicity.[4] Of course, the *physical scientist* as such may confine himself to the conception and verification of hypotheses that are *empirically* verifiable, hypotheses about the proximate, phenomenal antecedents of this or that series or group or order of phenomena; he may abstain from *philosophizing*, from seeking the *ultimate* causes of all physical phenomena: in which case he will have no occasion to "invoke the agency of beings whose existence cannot be empirically verified," [5] *i.e.* beings like angels, spiritual souls, human free-will, God—for whose *modus operandi* known *physical* agencies and laws furnish no analogy. He may *abstract from* the influence of such agencies until he reaches the point at which mere physical antecedents begin to appear insufficient or unsatisfactory for the explanation of his facts. Up to this point, being concerned with proximate causes, he has no need to inquire into ultimate causes: hence, *as a*

tr.), ch. xii., pp. 213 *sqq.*, and *Introduction* by Professor LARMOR, pp. xiv.-xvi. An instructive illustration of the British frame of mind is to be found in a passage from one of Lord Kelvin's lectures at the Johns Hopkins University (quoted by DUHEM, *op. cit.*, p. 194, from the author's *Lectures on Molecular Dynamics*, p. 132; also by WARD, *op. cit.*, i., p. 119, from *Nature*, vol. xxxi. (1885), p. 603: "I never satisfy myself till I can make a mechanical model of a thing. If I can make a mechanical model I can understand it. As long as I cannot make a mechanical model all the way through, I cannot understand. . . ." As to which Dr. Ward pertinently asks: "Why must mechanism '*all the way through*' be the one and only means of intelligibility?" (*op. cit.*, p. 120).

[1] WELTON, *op. cit.*, ii., p. 160. [2] *ibid.*, p. 161. [3] *ibid.*

[4] *Cf.* WELTON, *op. cit.*, ii., p. 209. [5] JOSEPH, *op. cit.*, p. 429.

physical scientist, he may say with Bacon, "*Deum semper excipimus*"; [1] for, as Newman has somewhere said, science is *a-theistic*, or *non-theistic*, in the sense that God does not come within its immediate scope. But all physical investigations lead up sooner or later to a point at which physical, empirically verifiable antecedents begin to appear unable to account for all the facts. If, at this point, the scientist chooses to contend that physical antecedents—as, for instance, atoms and motion—*are* still sufficient to account for *everything*, he is indeed at liberty to propound this mechanical conception as an ultimate philosophy of the universe—as Laplace appears to have done when he told Napoleon that he had no need of the hypothesis of God in his *Mécanique Céleste*; [2]—but he cannot contend that physically verifiable hypotheses are alone "legitimate," nor can he disallow hypotheses which postulate *ultra-physical* causes, by the *a priori* assumption that such hypotheses are not "scientific". If no physical causes we can postulate are sufficient to explain the physical universe as a whole, it is not only perfectly legitimate, it is even logically necessary, and therefore "scientific," for us to postulate causes which are ultra-physical. Such hypotheses cannot be described as "unscientific," or "scientifically inadmissible," [3] for there are causes other than physical or phenomenal, and laws other than mechanical or mathematical, of which we can, nevertheless, have *scientific* knowledge. No doubt, the terms "science" and "scientific" are often narrowly used nowadays as synonymous with the exact sciences of mathematics, abstract mechanics, and physics conceived and treated mechanically; [4] and sometimes with the mischievous insinuation that in these departments alone is to be found *certain* knowledge; but when we speak of "the aim of *science as such*, and of the logical conditions under which that aim can be realized," [5] it would be misleading to identify science with physics, instead of understanding it in the philosophical sense of *all certain knowledge* of things through their causes. Although, therefore, when there is question of discovering the *proximate* causes of "a particular natural event," our hypothesis "should be," as Mr. Joseph holds, "of such a nature that observable facts, *if we could find them*, might prove . . . it" [6] by disproving all its rivals; yet we cannot place this restriction on the conception of certain wider and more fundamental explanatory theories to which science leads, and to which we shall presently refer; [7] nor does Mr. Joseph appear to insist on such a restriction in these cases: [8] on the contrary, in regard to such "postulates," or "fundamental assumptions," he consents to "enlarging . . . the liberty of the mind" in a way we cannot profess to understand, for he says "the fundamental assumptions of a science may be metaphysically untenable, and we enlarge it [the "liberty of the mind"] to extend to all which these assumptions cover, however it may be ultimately impossible to think the facts in terms of them". [9] If a scientist

[1] *De Principiis atque Originibus*, ELLIS AND SPEDDING, iii., p. 80;—*apud* JOSEPH, *op. cit.*, p. 429.

[2] *Cf.* JOSEPH, *ibid.*; WARD, *Naturalism and Agnosticism*, i., pp. 3, 4, 45, 46, 64; POINCARÉ, *op. cit.*, Introduction by Professor LARMOR, p. xiv.

[3] JOSEPH, *ibid.*

[4] *Cf.* WARD, *op. cit.*, i., Lectures v., vi., and *passim.*

[5] JOSEPH, *ibid.* (italics ours).

[6] *ibid.*

[7] *infra*, p. 137.

[8] *Cf. op. cit.*, pp. 468, 476-7; *infra.*

[9] *ibid.*, p. 430, n. 2.

goes on constructing and testing hypotheses based on fundamental assumptions which he knows to be metaphysically untenable, he must surely know that, whatever practical utility they may possess as possible aids to experiment, they cannot be *true* theories of *reality*, or ever form a constituent portion of science proper, of *truth*, whether physical or metaphysical. The assumptions Mr. Joseph has in mind are probably, among others, "the independent existence of matter, the action of one independent thing on another, the production of a conscious state by a process in a physical organism": [1] these he regards as "unable to resist metaphysical criticism," [2] and as affording, therefore, only a provisional validity to the scientific hypotheses and explanations based upon them. No doubt, if such metaphysical theses are held as *unproven* assumptions, the scientific theories based upon them will be only provisional. For instance, the scientific theories based upon the hypothesis of an ether-medium in space can be true only on the assumption that there is no *actio in distans* in the actual physical universe. But it is one thing to accept any such fundamental assumption provisionally, and proceed to build upon it, and another thing altogether to regard such an assumption as "metaphysically untenable". We take this latter expression as equivalent to *rationally indefensible*; and, obviously, to build on such an assumption would be worse than illogical, for it would be irrational. Mr. Joseph's difficulty, from the point of view of metaphysics, against such assumptions of science as those referred to, seems to be that they "are all unintelligible" [3]. But this brings us again to the question referred to in connexion with the principles of *Sufficient Reason* and *Uniformity of Nature*: What is the criterion of intelligibility? Is that alone intelligible which is *imaginable* after *mechanical* analogies, and describable in terms of the purely *quantitative* concepts of mathematics and dynamics? and is it only such laws and principles that are to be recognized as *scientific*? [4] Such a restricted conception of the domain of the "intelligible" and the "scientific" we regard as a good example of those metaphysical assumptions which are fairly open to serious criticism.

229. THE RÔLE OF ANALOGY IN VERIFICATION: ULTIMATE SYSTEMATIC CONCEPTIONS.—The "cause" which forms the object of a scientific hypothesis need not, then, be itself a phenomenon. Further, it need not be an agency which is *already known* as such to be operative elsewhere in nature: otherwise no *new* natural agency could be discovered by way of hypothesis. But it must be conceived to bear *some analogy or resemblance* to some such known agency. The reason of this requirement is not difficult to find. It is only in so far as we conceive our supposed cause to be analogous in its *modus operandi* to some known cause or other, that we can infer anything as to the conduct of the former in this or that particular set of circumstances. And it is only by observing such operation—experimentally, if necessary and pos-

[1] *op. cit.*, p. 469. [2] *ibid.* [3] *ibid.*
[4] *Cf.* WARD, *op. cit.*, i., pp. 119, 120.

sible—in varied circumstances, and by seeing whether this tallies with what we should expect, that we can hope to *verify* our hypothesis. The only means we have of going beyond the general assertion we can make in virtue of the principle of causality—that the phenomenon (C) has *a* determining cause—the only means of *discovering* the latter, of *detecting its whereabouts*, and *bringing it to light*, is by supposing it to be some definite cause of a certain kind (say X), and then trying to verify this supposition by the employment of some processes that may lead with certainty to X as the *only possible* cause of the phenomenon in question, the only possible element, amongst all the surroundings of the phenomenon, which can be the determining cause of the latter. When we have discovered that the supposed cause, X, is the *only possible* cause of the phenomenon, then, and then only, can we infer, from the reality of C, that the supposed cause, X, is its real cause; for X is an antecedent of which C is the consequent, and from the reality of C we can infer the reality of X, only when the latter is not merely the sufficient or *necessitating*, but also the *only possible* cause of the former.

But it is obvious that we cannot bring any such independent experimental processes to bear upon X, to determine if it be real, unless we suppose it to be of a nature *at least partially known*, *i.e.* to have some analogy with known causes, to be of such a kind that we can deduce from it *something else besides the bare phenomenon for whose explanation it was postulated.* If we cannot draw any *other* inference from it except *that;* if it is so unique, so unknown to us otherwise, that all we can say about it is that " it is a something which is the determining cause of this phenomenon "; if we cannot study it in varying sets of conditions, and conceive what its effects would be, and how its influence would be manifested therein, and see whether these inferences tally with the phenomena that are observed to occur in these conditions: if we cannot do all this, manifestly we cannot hope to be able *rigorously* to *verify our hypothesis;* for it is only by doing all this that we can sift the surroundings of the phenomenon and prove that the supposed cause, X, is the *real* one, by proving that it is not only a sufficient (necessitating) cause, but the *only possible* cause, that could *determine* or bring about the phenomenon. Unless, for example, we supposed the so-called luminiferous *ether* to resemble matter so far at least as to be subject to the laws of motion, unless we supposed it to have some

analogy with the elastic medium, air, which propagates sound, we could infer nothing at all about it in explanation of the propagation of radiant heat and light. If it "were wholly different from anything else known to us, we should in vain try to reason about it".[1]

But now, granting all this; granting that if the hypothesis is to be *verifiable in this sense*, *i.e.* empirically, by being brought to the test of facts, the supposed cause must have some analogy with known causes; the question at once arises: Is it always possible to make an hypothesis of this kind? Or must we not be sometimes satisfied with supposing, as the real cause of the phenomena under observation, some cause for the conception of which we can have no independent evidence, *no analogy* to aid us; for the real presence of which we have no *independent* evidence, *i.e.* other than the actual phenomena under investigation; and about whose nature, therefore, we cannot hope to learn anything further than what we can attribute to it *as cause of these phenomena?* The answer is, that certainly we must sometimes be satisfied with this latter sort of supposition. In searching for the immediate causes of the smaller sections of reality examined in the various special sciences, analogies are more abundant. But according as we seek the remoter and wider causes of more extended regions of reality, our sources of analogy must of necessity become fewer and fewer, and we are forced to fall back upon the supposition of causes about whose nature we can get practically no other information than what the study of the effect itself—the larger field of phenomena in question—will yield us. This is the case with all those wider and more fundamental speculations, or "systematic conceptions," about the ultimate nature and properties of the phenomenal universe, about the constitution of matter, the cause of gravitation, the arrangement and motions of the heavenly bodies. Our hypothesis may account *sufficiently* for all the facts that suggested it; but who will say that it is the *only* one that can account for them? The most we can say is, that of all the alternative hypotheses it is the one that accounts *best* for the facts; and this may give us moral certitude that it is the right one, even though, strictly speaking, we cannot pass from the affirmation of consequent to the affirmation of antecedent unless we know that the latter is the *only possible* antecedent of the consequent in question.

[1] JEVONS, *Principles of Science*, p. 512.

For centuries, the Ptolemaic system of astronomy was accepted as being the only one that could account for the facts. It was only a man of the rare penetration of Aquinas who could point out that perhaps on some other hypothesis these could be equally well explained: "*forte secundum aliquem alium modum nondum ab hominibus comprehensum apparentia circa stellas salvantur*";[1] and the substitution of the Copernican system, three hundred years afterwards, justified his suspicions. Probably no one at the present day would venture to doubt the truth of the latter system. But there are in current science several "systematic conceptions" of such a character that they can scarcely ever be verified in the stricter sense of being shown to be the *only possible* explanations of the facts. A few of these will help to illustrate how such conceptions differ from strictly verifiable scientific hypotheses.

Lord Kelvin's theory that the ultimate atom of matter is a vortex-ring in a perfect liquid, is rejected by Clifford as unscientific because "a perfect liquid is not a known thing but a pure fiction . . . a mere mathematical fiction";[2] and, since we can deduce nothing from the absolutely unknown, the hypothesis is unverifiable.

Laplace's hypothesis—that the solar system was at first a rotating nebula from which the planets got detached, and from them in turn their satellites or moons, all of which condensed and cooled down gradually by radiation, and so solidified—is not verifiable either by *observation* or by *experiment*. It does not bear upon the *immediate*, but upon the *remote*, far-distant origin of our earth and solar system, at a time when the conditions of the natural forces at work may have been very different from any with which we are at present familiar. What hope, therefore, can we have of ever *proving* that these planets could have developed in *no other possible way?* It remains, therefore, a mere hypothesis, and may have its function in science; but it is not a *scientific* hypothesis in the strict sense, if by the latter we are to understand a supposition whose truth can be rigorously established, to the exclusion of all alternatives, by that experimental method of which Pasteur has so well said that it "leads to *absolute* and *unanswerable demonstration* . . . and deceives none but those who make a bad use of it".[3]

The same remarks apply with equal force to the conception of an all-pervading ether as a medium for the propagation of light, of radiant heat, of electric and magnetic influence, etc. The *existence* of *some* medium is the only alternative to *actio in distans*, the absolute impossibility of which is not easily demonstrable; and the *nature* of the supposed ether, the *properties* with which it is endowed, are not by any means agreed upon by scientists.[4] In those circumstances, no prudent scientist would venture to say that the ether as he conceives it, and the modes of transmission of those various influences as

[1] *In Lib. II. de Cœlo et Mundo*, lect. xvii. Cf. *Summa Theol.* 1^a, P. Q. 32, a, 1, ad 2. DE WULF, *Scholasticism Old and New*, p. 32.

[2] CLIFFORD, *Lectures and Essays*, p. 169.—*apud* WELTON, *op. cit.*, ii. pp. 74, 98.

[3] MERCIER, *op. cit.*, pp. 340-1.

[4] *Cf. The "New Knowledge" and its Limitations*, in the *Irish Ecclesiastical Record*, January, 1910, pp. 23 *sqq.*—the fourth of a series of articles in which are examined in some detail the implications of another of those "systematic conceptions"—the electrical theory of matter. *Cf.* art. on *The Philosophy of Energy*, *ibid.*, February, 1910, and *Some Current Phases of Physical Theories*, *ibid.*, April, 1910.

he describes them, afford the *only possible or conceivable* explanation of those phenomena.

Again, are vegetable and animal species *fixed*, or *transformable?* Let a naturalist suppose them to be transformable. He must proceed to experiment on some cause supposed to be capable of effecting the transformation from some one specific type to a different type, and so submit his supposition to the control of facts. What might such a cause be? A possible cause is suggested by the phenomena of *artificial selection*, and *artificial cultivation or rearing*, which are found to be productive of new varieties and races. Darwin observed those phenomena carefully and minutely, and then *made the supposition* that there are at work in nature *agencies analogous* to those employed by the artificial breeder, and capable of producing not merely new varieties or races, but new species. Now, *if* there are really in nature some such agencies, they can become the object of scientific hypotheses, and their mode of action may be described—after the analogy of the intelligent artificial selection, or intelligent sorting, of the breeder—as "*natural* selection". But it remains an open question whether the actual existence of such transforming agencies in nature—if there are such—can ever be verified: and until their existence and mode of operation are at least shown to be capable of verification, the hypothesis will remain a mere systematic conception, an "*idée directrice*,"[1] a methodological view of the world of living things, rather than a "scientific" hypothesis.

Similarly, such an hypothesis as Weismann's germ-plasm theory to account for the fact of heredity, can scarcely be regarded as a *scientifically verifiable* hypothesis, involving, as it does, elements admittedly beyond the range of all possible experience. Professor Windle, writing about it in the *Dublin Review*,[2] remarks that "the theory is a tolerably complex one to be built upon a system of 'vital units' which no one has ever seen *or ever can demonstrate*".

Yet another instance of the same unsatisfactory class of conceptions is that of Sir William Crookes, regarding the renovation of energy in the universe: "that the heat radiations propagated outwards . . . are transformed at the confines of the universe into the primary—the essential—motion of chemical atoms, which . . . gravitate inwards, and thus restore to the universe the energy which would be lost to it through radiant heat."[3]

On the other hand, a good example of a thoroughly scientific hypothesis, afterwards experimentally verified beyond any possibility of doubt by the "method of difference" (241), was Pasteur's supposition that the fermentation of the grape was due to *germs* that settled on it, and not to mere chemical action. He first extracted the juice from the interior of the grape without allowing it to come into contact with the exterior covering, or with the air, sealed it hermetically in tubes, and found that it did not ferment. Again, in the month of June—before the appearance of the coating of germ-cells, which begins in July—he carefully surrounded certain grape-clusters with wadding, and so protected them from the germs. The grapes of those bunches were pressed and the juice obtained refused to ferment.

Equally convincing, perhaps, were the experiments of the same eminent

[1] MERCIER, *op. cit.*, p. 340, from which context the above example is taken.

[2] April, 1906, p. 334 (italics ours).

[3] *apud* GERARD, *The Old Riddle and the Newest Answer*, p. 26.

scientist, and of Professor Tyndall, in establishing the hypothesis of biogenesis, and disproving that of abiogenesis or spontaneous generation. Yet, there are scientists who still refuse to admit that the experiments of those two men finally established the former, or disproved the latter hypothesis: a good illustration of the possible differences of opinion as to the amount of verification that is to be deemed adequate in any given case. "Every effort," writes Professor Windle,[1] "to prove the existence of spontaneous generation, has so far failed. It is true, all this amounts to is that no experiment has ever yet succeeded in showing that spontaneous generation takes place, and there are those who urge that some experiment may yet turn out to be successful." The experiments of Professor Burke of Cambridge, upon gelatine acted on by radium, do not seem to lend any probability to the hypothesis of spontaneous generation. Weismann unscientifically regards the latter as the *only possible hypothesis*, though holding at the same time that the process will for ever escape observation.

Finally, a brief comparison of the *Atomic Theory* (in chemistry) with the *Mechanical Conception of the Universe* (in philosophy) will furnish an instructive contrast between a strict scientific hypothesis and an unverifiable systematic conception. The "atomic theory" in chemistry is an hypothesis which supposes chemically simple bodies to be aggregates of particles indivisible by any known chemical methods, and accordingly called *atoms*. This hypothesis is exceedingly probable, if it is not indeed fully verified by the various arguments on which it is based. One of these, for example, is drawn from the experimentally established *Law of Multiple Proportions*. For instance: nitrogen combines in constant ratios by weight—28 parts—with *varying* weights of oxygen, *but only on condition that these latter be always some multiple* of a *minimum combining weight* of oxygen—16 parts. Thus N_2, 28 parts by weight of nitrogen, combines respectively with 16, 32, 48, 64, 80 parts by weight of oxygen, to form five different oxides; N_2O, N_2O_2, N_2O_3, N_2O_4, N_2O_5. Now, does not this remarkable fact or law suggest, as a possible—if not its only possible—explanation, that the mass of oxygen represented by 16 is a fixed, constant, chemically indivisible portion of matter,—an *atom?*[2] Suppose 28 grammes of nitrogen and (say) 20 grammes of oxygen together submitted to the chemical agencies capable of effecting a combination: why, if the mass of oxygen were capable of *indefinite* division *by those agencies*, should 4 grammes of oxygen invariably remain over? Why should not that mass of 4 grammes divide itself around on the 28 of nitrogen, seeing that these have an equally strong affinity for all parts of the mass of oxygen? But if we conceive the 28 grammes of nitrogen as containing a certain number of *atoms—twice* as many as 16 grammes of oxygen contain—each *pair* of which nitrogen-atoms unites, under the play of the chemical forces at work, with *one* atom of oxygen, there will be evidently 4 grammes of atoms of oxygen left, each of which would *have to divide* into four parts in order to give an additional share to each pair of nitrogen-atoms. But the atom being, *ex hypothesi*, indivisible by the chemical agencies at work, the 4 grammes of oxygen must remain unincorporated into the compound.

Many scientists regard this hypothesis—that the chemically simple bodies are made up of chemically indivisible particles, or *atoms*—as an established

[1] *Dublin Review*, April, 1906, p. 340. [2] NYS, *Cosmologie*, p. 411.

theory. Recent researches into electrical phenomena, and into the properties of radium, have, however, led scientists to suppose that there may be at work, in nature, agencies other and more powerful than chemical forces, capable of disintegrating, and indeed actually and constantly disintegrating, the chemical atoms and molecules of all matter.

Quite different from this chemical *atomic theory*, though at first engrafted on the latter, and advocated as an extension of the latter, is what we have called the *Atomic or Mechanical Conception of the Universe.* While the former hypothesis gives a view of matter which is most probably if not certainly true as far as it goes, the latter is not only beyond the range of rigorous verification, but may easily be shown to conflict with multitudes of facts, and to be accordingly untenable. It would reduce all bodies, simple and compound, to aggregations of *homogeneous corpuscles* infinitesimally smaller than the hydrogen atom : it would endow those ultimate atoms with *local motion* in space, and suppose them subject to the laws of mechanical motion alone : it would then try to explain and account for all the phenomena of nature, all the forces of nature, all the properties of bodies, animate and inanimate, even all the phenomena of human life and mind, by the evolutions of those motions in obedience to the principles of mechanics ! There is something peculiarly attractive—or seductive—about a conception that is so vast, so simple, so clearly imaginable ; but, unfortunately for the mechanical conception, those attributes are no test of truth.[1] There is, indeed, in the human mind an innate craving to simplify the complex, to reduce the manifold to unity ; but we must not allow this craving to blind us to facts, or induce us to ignore the unexplained. "There are more things in heaven and earth . . . than are dreamt of in" the mechanical "philosophy" ! To weigh and measure phenomena exactly, is not to know *all* about them. Nor, in our endeavour to explain some one aspect of them, must we forget that there are others still unexplained.

230. VERIFICATION BY CUMULATIVE EVIDENCE.—It is clear, then, that we must not expect the same sort of rigorous verification in every department of scientific and philosophic investigation. In the special sciences, which seek the proximate causes of narrower fields of phenomena, we may hope to approach, if not to realize, the ideal of establishing reciprocating causal relations (221), by eliminating what is irrelevant—through the application of the "experimental methods" to be explained in the next chapter. But we must often be content to leave this elimination incomplete, and so to bring to light only a non-reciprocating cause.[2] Furthermore, there are multitudes of hypotheses in science, in regard to which we can scarcely ever hope to be able to assert that they are the *only possible* hypotheses that will

[1] *Cf.* WELTON, *op. cit.*, ii., p. 209.

[2] *Cf.* WELTON and MONAHAN, *Intermediate Logic*, chap. xxx., for distinction between the *Direct Development of Hypothesis* by the "experimental methods," and the *Indirect Establishment of Hypotheses* by inferences pointing to their superiority as compared with other conceivable alternatives.

explain the facts; all we can say of some of them is that they explain the facts more satisfactorily than any alternative hypotheses so far suggested; and if we find an hypothesis which was conceived in explanation of one group of phenomena to be capable of *extension* to many other cognate groups, and to explain these satisfactorily also, such "consilience of inductions" may make us morally certain that our hypothesis is the right one.

Our verification of such hypotheses will consist in our pointing to their superior power of explaining facts. It is well to emphasize this point; because, firstly, it is with the validity of these wider and more general hypotheses that *philosophy*, as distinct from the special sciences, is mainly concerned; and because, in the second place, the special sciences are full of them. As Mr. Joseph rightly observes: "many at least of the most general and fundamental of our scientific principles are accepted only because they explain the facts of our experience better than any we can conceive in their stead; they are therefore, or were at the outset, hypotheses, used in explanation of facts, and proved by their relative success in explaining them. We do not see why they are true, but only why we must believe them to be true. They are established inductively by the facts which they explain, and the failure of any rival hypothesis; the facts are explained from them".[1] Now are we to regard such hypotheses as *proved* or *verified*, because they explain the facts "better than any we can conceive in their stead"? Mr. Joseph adds: "it is important to realize that an hypothesis is not really proved by merely explaining the facts. But many hypotheses are provisionally accepted, which are not proved, on the ground that they explain the facts, and without the performance of what would often be the impracticable task of showing that *no* other hypothesis could equally well do so."[2] What kind and amount of credence, then, are we to give to such hypotheses, the evidence for which is cumulative, though not cogent? This is an extremely delicate and difficult matter to determine; and all the more so because the hypotheses in question are usually of considerable significance: they are the theories that shape men's convictions about the ultimate causes and nature of the universe. Each such theory must be judged on its merits; and the responsibility of accepting or rejecting it, or holding it provisionally, calls for the exercise of care, caution, and prudence.

231.—"POSTULATES" AND THEIR JUSTIFICATION: "TRUTH" OF VERIFIED HYPOTHESES.—There can be no reasonable doubt that this cumulative evidence may become sufficient to warrant an assent of moral certitude to such a theory. But men differ so much in their mental outlook—on account of the different intellectual atmospheres, traditions, and beliefs, in which they have lived and moved—that evidence which may satisfy one will be deemed insufficient by another. Hence the conflicting philoso-

[1] *op. cit.*, pp. 476-7. *Cf.* MELLONE, *op. cit.*, p. 332: "It is this demonstration that the consequences of a law do actually agree with facts, that forms for science the verification of that law".

[2] *ibid.*, p. 477 n.

phies and divergent world-views that have at all times prevailed among men. If we are to decide between these, to discern the truth that is in them, and to eliminate the error, logic can merely tell us that in this process we must be as unprejudiced, critical, careful, and judicious as possible.[1]

A thoughtful analysis of the various fundamental judgments which make up our general outlook on the inner nature and ultimate significance of the universe—whether these judgments be called assumptions, postulates, axioms, principles, or beliefs—may or may not have the effect of modifying some or all of these latter; but this effect it will undoubtedly have: it will show us that in choosing between various alternative theories about the remoter causes of things, in shaping our philosophical views about the universe, we are all alike influenced more or less by certain partly instinctive and implicit intellectual tendencies, which are often not clearly realized in consciousness, and which, when realized, are felt to be legitimate though they may not be capable of logical justification by reference to any definite principles lying beyond themselves. These tendencies or leanings have their root in our "belief that the universe is rational,"[2] and in our conception or "notion of what a rational universe should be".[3] This conception, and this belief, Mr. Joseph considers to be "not derived from experience"[4] inasmuch as they control our interpretation of experience. But it is not true that they are in our possession prior to, and independently of, experience. Prior to experience we have only our cognitive faculties—senses and intellect. These alone we bring to the interpretation of experience. It is sense experience, as interpreted by intellect, that gives us our "notion of what a rational universe should be". If that notion were prior to experience it should be the same in all men; but it is not: the agnostic, the monist, and the theist, have different conceptions; and the conception which works best, which proves most satisfactory, which fits in most harmoniously with human experience all round, is alone the true conception. Theism is the one which we believe to fulfil these conditions.

[1] *Cf.* Welton and Monahan, *op. cit.*, p. 398.

[2] Joseph, *op. cit.*, p. 469. [3] *ibid.*

[4] *ibid.* The tendency to endow the mind with constitutive thought-principles antecedent to all experience is very common among post-Kantian writers. It pervades the otherwise excellent work of Professor Borden P. Bowne on the *The Theory of Thought and Knowledge* (Harper Brothers, 1899).

Apart, however, from the question of the origin of such conceptions, there are undoubtedly in our minds the tendencies to which we have referred. They have been crystallized in the course of time by philosophers into maxims such as that known as "Occam's[1] razor": *Entia non sunt multiplicanda praeter necessitatem*—which may be interpreted as affirming "a presumption in favour of theories which require the smallest number of ultimate principles,"[2] for example, "in favour of the derivation of the chemical element from some common source, or of the reduction of the laws of gravitation, electricity, light and heat to a common basis".[3] It simply voices the innate yearning of the human intellect to unify, as far as possible, the manifold of experience. The same sort of prepossession is also expressed in the maxims: "*Simplex indicium veri*" and "*Natura non abundat superfluis sed delectatur paucissimis*". In other words, we are prompted to regard the *simplicity* of a conception or hypothesis as an *index of its truth*. We give our preference to the simplest of a number of equally probable alternative explanations, not merely from the motive of practical convenience, but with a feeling that because the actual universe is rational the simplest theory of things ought to be the true one.[4]

We can hardly say that the guiding principles embodied in such maxims are "preconceived ideas" pure and simple. Rather, they are gradually moulded in our minds by our progressive understanding of the universe. But further reflection will teach us that, if followed blindly and unquestioningly, *they may mislead us*. It would be unwise to demand simplicity in hypotheses merely on the ground that *Nature always acts in the simplest way*. "Even so," writes M. Rabier,[5] "to determine *a priori* what are the *simplest ways possible*, we should know what is the *minimum of complication necessary*. And since we have no data to determine the latter, it is quite useless to attempt an *a priori* solution of the former. . . . The idea of the simplicity of nature's methods, without its indispensable corrective, *viz*. a realization

[1] Occam was one of the later mediaeval Scholastics. He lived in the first half of the fourteenth century. *Cf*. De Wulf, *History of Medieval Philosophy*, pp. 420-5.

[2] Joseph, *op. cit*., p. 470.

[3] *ibid.*

[4] Hence, for instance, the ratio of the inverse square in the law of gravitation is regarded as the true ratio, though some more complex ratio might yield results deviating so slightly from those of the former as to escape detection in our actual measurements and observations. *Cf*. Joseph, *ibid.*

[5] *Logique*, p. 239.

of the inevitable requirements and difficulties of the facts, is the parent of shallow minds."

But, when we have made full allowance for the complexity of phenomena, and have to choose between theories all of which appear to offer equally satisfactory explanations of the latter, we should certainly choose the simplest theory. And the theist, at all events, will find a sufficient rational ground for doing so, not in any *a priori* postulate that the universe *must be* "rational" or "intelligible," but in his own reasoned, *a posteriori* conviction that the universe *actually is* the work of an All-wise God, governed by His law, and reflecting His Intelligence.

Some philosophers, evidently influenced by the impossibility of securing *cogent logical proof* of such ultimate hypotheses as we have been considering, believe that *no* scientific hypothesis can be *proved*. For instance, we find it contended that "a causal hypothesis is never proved in the strict sense of the word. It is neither true nor false; it is simply good or bad, useful or embarrassing, as the case may be".[1] In confirmation of this view, the authority of such well-known scientists as Quételet and Ostwald is invoked; the history of innumerable hypotheses that have had their day and are long since exploded, is also appealed to; and, finally, attention is directed to the formal law of the hypothetical syllogism: "Posito antecedente ponitur consequens; *at non e converso*. . . . We witness the reality only of the consequent, *i.e.* of the phenomenon: we cannot thence conclude to the reality of the supposed antecedent. . . . Between the observed phenomena and the scientific hypothesis there is a chasm that no reasoning can bridge. From the fact to the theory there is a dialectic somersault that no logic can justify."[2]

No doubt, logic will not justify the inference from consequent to antecedent unless we are certain that the inferred antecedent is the only one possible. Can we ever be certain of this? Yes, whenever we can exclude *all possible* alternatives. But how can we ever be certain that the excluded alternatives are exhaustive of all the possibilities (213)? No rules of logic will help us here, except indeed the general directions it lays down for *observation* and *experiment;* but, by the proper conduct of these processes, we *can* often arrive at physical certitude that our causal hypothesis is the right one because it is the only possible one.

In regard, however, to those wider and more general hypotheses and conceptions which cannot be verified in this rigorous experimental manner, our assent must be more or less provisional, although it may often prudently reach that high degree of probability which is sometimes described as moral certitude.

232. THEISM AS A VERIFIABLE HYPOTHESIS.—Of course, as long as we merely infer from actual phenomena the existence of an adequate cause, and make no supposition or postulate whatever as to the *nature* of this cause, beyond what the phenomena permit us to predicate about it, we are

[1] Père DE MUNNYNCK, O.P., in the *Revue Néo-Scolastique*, vol. vi., pp. 235 *sqq.*

[2] *ibid.*

obviously not employing hypothesis at all, but simply the *a posteriori* argument from effect to cause. Such arguments can undoubtedly reach *real causes*; and they are practically the only sort of inferences we can make when we push back our investigations to those wider and ultimate regions of reality where analogies for hypotheses fail us. And when, finally, we contemplate the phenomenal universe as a whole, when we are brought face to face with what Mill has called "the ultimate laws of nature (whatever they may be)," [1] and "the co-existences between the ultimate properties of things—those properties which are the causes of all phenomena, but are not themselves caused by any phenomenon, and a cause for which could only be sought by ascending to the origin of all things," [2] it is not by way of hypothesis and verification, but by *a posteriori* reasoning from effect to cause, we proceed to prove that the whole phenomenal universe, being contingent, not self-explaining, must have an originating First Cause, and that this Cause must be distinct from all phenomena, self-existent, and, as regards perfection, adequate to the production of all phenomenal reality *ex nihilo*—Creator, Conserver and Ruler of the universe.[3] It is *mainly*, at all events, by *a posteriori* reasoning of this kind that defenders of the philosophy of theism have traditionally established its fundamental thesis: the existence of an All-wise, Omnipotent Deity, really distinct from the phenomenal universe, which He has created, conserves in being, and rules by His providence. Now, this *a posteriori* reasoning combines in its premisses certain principles (like that of *causality*) which are claimed by those who employ them to be necessary truths, validly applicable to every conceivable sphere of reality; and certain truths of experience which are likewise claimed to be accurate interpretations of experience (224). But the accuracy of those interpretations, and the validity of those principles, are questioned by philosophers of other schools. Hegelian idealists deny the accuracy of the realist interpretation of the data of sense experience; Kantists and phenomenists furthermore question the validity and necessity of the realist's principles. Hence it is that the problem of establishing the truth of the philosophy of theism may be regarded as a problem of proving this latter conception of the universe to be the true conception by showing that it offers for all the facts of human experience an explanation vastly superior to those of empirical phenomenism, Hegelian idealism, or any other alternative that can be suggested; that the explanation offered by theism is, in fact, the only satisfactory philosophy of human experience as a whole. This method has, indeed, been already suggested by the comparison instituted in the preceding chapter (224) between the three conceptions just mentioned. It is the method we employ in establishing the realist interpretation of sense experience [that there exists an external, material universe, really distinct from the percipient mind] against such types of idealists as Berkeley, Hume, Mill, Bain, Spencer,

[1] *Logic*, III., v., § 6, n. 1. [2] *ibid.*, xii., § 2.

[3] But at this point phenomenists would have us abdicate the use of our reason: asking us to believe that we cannot and must not ascend "to the origin of all things" because such source of all phenomenal reality cannot be itself a "phenomenon". Of course it cannot; but is this any reason why we should doubt its reality? The things which "are not themselves caused by any *phenomenon*" must be caused by *something*. And, since we can know about their cause whatever we are able to infer from themselves, the contention of Agnosticism, that this cause is unknowable, must be rejected as erroneous.

Huxley, as also Kant and his followers.[1] "Briefly stated [writes Father Rickaby], the whole proof of the present thesis will consist in showing that the experienced facts of sensation are confessedly alike with our adversaries and ourselves, *and that only our way of accounting for them is adequate.*"[2] It will likewise be found that this same method is really, though perhaps only implicitly, involved in the traditional lines of reasoning by which the philosophy of theism has always been supported. And it is very desirable that in placing this philosophy before the modern world, in comparing its claims to acceptance with those of other current systems, its supporters should make more explicit use of this method; that is, that they should proceed explicitly by way of *hypothesis* and *verification;* comparing their hypothesis with the facts of human experience, and establishing it by showing its "relative success in explaining them,"[3] as compared with the relative failure of all competing alternatives. This would involve no real change of method on the part of Scholastic philosophers, who are the main upholders of theism; but only that they should develop more fully the analytical side of the Scholastic method (202) by meeting, discussing, and removing the more recently formulated difficulties against the general principles which they utilize in the deductive, synthetic stage of their systematic reasonings.[4]

[1] *Cf.* Rickaby, *First Principles of Knowledge*, pp. 270-290.

[2] *ibid.*, p. 268 (italics ours).

[3] Joseph, *op. cit.*, p. 477.

[4] This would remove even all apparent grounds for the really groundless reproach of non-scholastic thinkers that "our arguments are too *a priori* . . . abstract . . . technical"; that our "principles are far from evident, and appear to be gratuitously assumed," and so forth. *See* Irish Ecclesiastical Record, April, 1911, article on *The Pragmatic Value of Theism*, by Leslie J. Walker, S.J. (pp. 338, 339). The article is an earnest plea for the wider use of the method referred to in the text, for the defence of the philosophy of theism: because, on the one hand, it would be understood and appreciated by modern thinkers whose "modes of thought are almost all of one type," namely, that they "start with a hypothesis which they proceed to verify by showing that its consequences harmonize with the data of human experience" (p. 340); "nor," on the other hand, "would any of the old arguments have to be given up, for all are essential to the completeness of the methods. At most, traditional arguments would have to be stated in a somewhat different form. Axioms and principles would not be asserted merely on the ground of their self-evidence; but we should first of all formulate all principles and all doctrines provisionally as 'hypotheses,' not of course in the sense that we should for a moment doubt their truth, any more than St. Thomas doubts the truth of God's existence when he asks: *An Deus sit?* but merely as provisional positions shortly to be proved. We should then proceed to verify our hypotheses . . ." (pp. 352, 353). He applies the method himself in subsequent articles in the Irish Ecclesiastical Record (May, pp. 465-80). We are in no way detracting from the value of this method by observing that it, too, must accept some "principles" or "axioms" or "intuitions" on self-evidence alone, as starting points for rational interpretation of experience, and reasoning therefrom; for even though "the philosopher of this twentieth century, having grown familiar with inductive or scientific methods of proof, is no longer content with *a priori* reasoning from self-evident principles" (p. 340), it is none the less true that "intuition is . . . involved in [his own 'inductive'] process, and many statements are made [by himself] which cannot be proved, but which are none the less axiomatic or evident" (*ibid.*). When is it lawful, and when unlawful, to assume a judgment as a self-evident axiom (203)? This is a very grave question, which divides philosophers, and which logic is unable to answer. *Cf. infra*, 275, A, *c.*

233. SUMMARY OF LOGICAL REQUIREMENTS FOR A LEGITIMATE HYPOTHESIS.—We may now briefly recapitulate the conditions required for a legitimate scientific hypothesis.

1. *It must be based on preliminary observation of some fact or groups of facts, be invented in order to explain them, and therefore have for its object a real cause, a "vera causa".* This rule excludes all subjective suppositions employed as aids to the imagination (226). It also excludes all purely fanciful guesswork about causes. Observed uniformities in facts must *suggest* hypotheses; these must not be constructed entirely from imagination; they must have a basis in accurate and unbiased observation of the facts; they must not be merely preconceived notions which we allow ourselves to read into the facts. "The scientist," writes Claude Bernard, "should have an hypothesis to verify; but he ought to make sure that the facts on which it is based be accurately and impartially observed. Hence he should be an observer no less than an experimenter. As observer, he will simply and solely register the phenomenon under observation. He will be, so to speak, a *photographer* of phenomena: his observation will be a faithful representation of nature, free from all prejudiced and preconceived ideas. As observer, he will be passive, silent, receptive; he will listen to nature, and write under her dictation. Then, once he has carefully observed the phenomenon, he will conceive an hypothesis and proceed to test it experimentally."[1] We must, therefore, observe the facts without preconceived ideas; that is, we must observe before supposing, not *vice versa*. Simple as this recommendation is in its formulation, it is by no means easy to carry out in practice. Our initial observation and determination of the phenomenon to be investigated must be impartial, not biased by any preconceived views. Then, when we have conceived our hypothesis, and proceed to test it by renewed observation and experiment, we must resist all inclination to interpret the facts in favour of it. We must be ever ready to modify or reject it. Just as the wish can be father to the thought, so can attachment to an hypothesis easily misguide and distort our reading of the facts. It is difficult to guard against this undue influence while we are conducting our observations and experiments for the express purpose of testing our hypotheses. It is to secure impartiality in this testing process that Claude Bernard says to the

[1] CLAUDE BERNARD, *Introduction à l'étude de la médecine expérimentale*, pp. 39-40.

scientist: "On entering the laboratory leave your imagination with your overcoat in the vestibule, but take it with you again on your departure".[1] That is to say: reflect on your experiments and observations, and conceive and test hypotheses about them; but never allow your hypotheses to influence your actual reading of the facts when you are observing or experimenting.

2. *It ought to be self-consistent, and free from conflict either with established truths or undoubted facts.* Truth cannot oppose truth. Hence the demand for consistency, freedom from internal contradiction, intelligibility. This, as we have seen, is not to be confounded with *imaginability* (228).

Again, the hypothesis must not contradict other established truths or laws. Caution is needed, however, to make sure that the contradiction is real, that it cannot be eliminated by any possible restatement of such laws: for, sometimes an hypothesis sheds a new light on an established law and leads to a more accurate or more extended formulation of the latter. Sometimes, too, what is commonly believed to be an "established law" is in reality a false and misleading theory, *e.g.* the Ptolemaic Astronomy.

In comparing our hypothesis with facts, the chief danger to be avoided is a prejudiced interpretation or reading of the facts, in favour of the hypothesis. If we find that facts are not in accordance with our hypothesis, we must not say "so much the worse for the facts," but rather "so much the worse for the hypothesis". On the other hand, however, we need not reject our hypothesis until we are sure that it is really incompatible with the facts. And here we must take care that what we have regarded as facts are not already mixed with half-unconscious theories or interpretations which, in the light of our present hypothesis, we may now be able to eliminate from the mixture, and so leave the residue of real fact compatible with our present hypothesis.[2] Much, if not all, of what we commonly call "fact," is intimately interwoven with what are really *interpretations* or *theories*; and some of these may have been false from the start. Mere fact cannot be, interpreted fact must be, either true or false. Hence we apply the name "fact" to what we believe to be a truth, a true interpretation of fact (248). And in some of these we may be mistaken, as people were when they regarded it as a "fact" that the sun goes round the earth. When, therefore, we

[1] CLAUDE BERNARD, *Introduction à l'étude de la médecine expérimentale*, p. 44.
[2] *Cf.* JOSEPH, *op. cit.*, pp. 432-3.

find "facts" discordant with our hypothesis, we must make sure we have interpreted these facts rightly. Then we must see if they can be made to fit in with our hypothesis by such correction and modification of the latter as will incorporate in it factors to account for those facts. Only when we fail to effect such a modification of our hypothesis must the latter be rejected altogether.

3. *It must be based on some analogy with known causes: it must be capable of yielding exact deductive inferences: it must be verifiable by the submission of those inferences to the control of observation or experiment.* These are three alternative statements of one and the same requirement. We have seen already that it may not be always possible to conceive an hypothesis which will fulfil this condition: the degree in which an hypothesis does lend itself to such verification is the measure of its "exact" scientific character.

The combination of conditions 1 and 2 show how the functions of *reason* and *sense* alternate and aid each other in science. Initial *observations* suggest an hypothesis. This in turn must be verified: and to verify it the scientist must *reason* from it, and submit his conclusions anew to the control of *observation* or *experiment.* "Thus," writes Claude Bernard, "the mind of the scientist is placed between two observations, one which is for him the starting-point of a reasoning process, the other its conclusion."[1]

4. *It is "verified" or "established" when it is shown to yield not merely a sufficient explanation, but the only possible explanation, of the facts it purports to account for* (*cf.* 212). Our success in showing this will vary with the nature of the facts and the scope of the hypothesis. Sometimes the facts are subject to the control of experiment, and the hypothesis is comparatively restricted in its scope, so that we are able to eliminate and disprove all conceivable alternatives, and thus attain to the ideal of a rigorous verification. Again, the hypothesis may be shown to be capable of such extension, by consilience of inductions, that although we may not hope to prove rigorously that it is the only possible explanation of the facts, yet we are able to show that it does explain a vast field of fact, and does so more satisfactorily than any suggested alternative: in which case we may give it a provisional assent amounting to moral certitude.[2]

[1] *op. cit., ibid.*—*apud* MERCIER, *Logique*, pp. 344-5.
[2] It is often very difficult to distinguish, and there is often no practical distinc-

If, finally, in our inquiry into the *ultimate* cause and explanation of experience as a whole, the relation between the supposed cause and this experience be such that we can argue *a posteriori* from the latter to the existence and nature of the former—merely in virtue of the principle of causality—then our reasoning may reach certitude, provided we are able to show that the facts consist with no other interpretation. This we consider to be the case in regard to the philosophy of theism as an explanation of the whole field of human experience.

234. SOURCES OF SCIENTIFIC HYPOTHESES: ANALOGY.—In a general way, it may be asserted that all hypotheses have their origin in observation of facts and reflection on what we already know about facts. We may, however, distinguish a few of the more important immediate sources of hypotheses.

(1) Even reflection on the common class names and ordinary generalizations embodied in the language we use, may raise problems about the phenomena of experience, and suggest hypotheses in explanation of them. All inquiry into the causes of things presupposes, as its initial stage, the *classification* of similar things on the basis of common attributes (63, 68), and the *nomenclature* or system of class names or general terms concurrently embodied in common language (69).[1] So that in the very language we use, in the classifications embodied in it, and in the rough and ready generalizations of ordinary life, we have to hand an abundance of materials which suggest to the thoughtful mind new connexions and relations, as hypotheses for verification. Observation will often show the necessity of discarding or modifying customary classifications, and of re-grouping things according to newly detected points of similarity or dissimilarity. But these processes have their origin in the study and comparison of existing classes, and in analysis of accepted generalizations. Thus, the relation involved in an ordinary universal judgment—"All S is P," or "If S is M it is P"—may, perhaps, be a *reciprocal* relation: and

tion, between an extremely high degree of probability and what is commonly called certitude. And this is particularly true in the social and historical sciences. "Speaking strictly and in accordance with correct logical usage," writes M. Ernest Naville (*La Logique de l'Hypothèse*, p. 222), "the highest probability cannot become certitude. And yet it is an indisputable fact that there are crowds of hypotheses upon which we have no hesitation in acting as if they were absolutely certain. Practice is here in advance of theory, and does not follow quite the same law." The kind and amount of evidence required for verification, and for a certain assent, are not identical in all the sciences. They vary with the subject-matter.

[1] *Cf.* JOSEPH, *op. cit.*, pp. 413, 440.

the supposition that it is so is an hypothesis for verification. Or we may put the matter in this way: Ordinary observation discloses relations of uniform concomitance or sequence between phenomena. In these uniformities science endeavours to detect *reciprocal causal relations* (221). We know a phenomenon or fact *scientifically* only when we know its connexion with, and its dependence on, all that constitutes its *one sufficient and indispensable cause;* and when we place this cause and this fact in the relation of antecedent and consequent, or of subject and predicate, we know that the relation is not only universal but *reciprocal;* for example, not only that "all living organisms are mortal" but that "all mortal things are living organisms". But in order to establish such a reciprocal relation we must have made explicit the *one essential ground* of the consequent in question; we must, in other words, be certain that it can follow from this antecedent and from no other. "If man is a living organism he is mortal; and if man is mortal he is a living organism": because mortality is a *proprium* of organic life, a property in the strict sense of the word, "quod convenit *omni*, *soli*, *semper*, et *ubique*". "If a triangle is right-angled the middle point of the hypotenuse is equidistant from the three vertices, and *vice versa*": because right-angled triangles *alone* are inscribable in semicircles. From all this we see that the very observations which give rise to the enunciation of universal relations—whether categorically, *All S's are P*, or hypothetically, *If S is M it is P*—suggest the hypothesis that these judgments may, perhaps, be in reality *simply* convertible, although formally they can be converted only *per accidens:* into *Some P's are S* (which is equivalent to *All P's may be S*), and *If S is P it may be M.*

(2) We have already seen (217) that hypotheses may be suggested by *Enumerative Induction.* Even a single observed instance of a phenomenon may set one speculating or guessing as to its cause. But the suggestion comes more easily when we have observed a number of instances of the same coexistence or sequence of two phenomena, particularly if this persists through varying circumstances.[1] The sole scientific value of enumerative induction lies in its suggestion of an hypothesis as to the content or nature of the instances examined. For example, the observation of the facts that $1 + 3 = 2^2$, $1 + 3 + 5 = 3^2$, $1 + 3 + 5 + 7 = 4^2$, and so on, suggests the hypothesis of some *necessary*

[1] As in the "method of agreement" (241).

equality—springing from the very nature of numbers—between the sum the first n odd numbers and n^2.

(3) More important still than enumerative induction, as a source of hypotheses,—indeed by far the most fruitful source,—is *Analogy*. Certain resemblances of an unexplained phenomenon to some other already known and explained phenomenon, will suggest the direction in which we ought to look for an explanation of the former. Thus, Malus, accidentally observing, through a double refracting prism, the light of the setting sun reflected from the windows of the Luxembourg Palace, saw that the light disappeared at two opposite positions of the prism, *like light polarized by passing through another prism*; and he argued *by analogy* that *this*—and, *a pari*, all *reflected* light—was likewise probably polarized: an hypothesis which was speedily verified.

The term "analogy" is commonly understood nowadays to mean a resemblance of any sort; and by the "argument from analogy" is understood an inference based on such resemblances. Mill's description of it is simple and clear: "Two things resemble each other in one or more respects; a certain proposition is true of one, therefore it is true of the other".[1] It is an argument from partial resemblance between two phenomena (or groups or series of phenomena) to some further point of resemblance between them. A few simple examples will illustrate the nature of this mode of inference.

(*a*) Cholera has been proved to be due to the action of a certain known bacillus. Here is some other disease, which is seen to present many symptoms similar to those of cholera. Therefore, this disease also *probably* has its origin in the action of some bacillus.

(*b*) The planet Mars revolves around the sun, has light and heat from the sun, rotates on its axis, and appears to have mountains and rivers—*like the earth*. Therefore, it *may* also be the scene of vegetable and animal life.

(*c*) A is a man of certain character, disposition, opinions, etc. (say xR); and he has acted in a particular way in certain circumstances. B is also a man of the same character, etc. as A (say xR^1 : x being the known common points, and R, R^1, the partially differing and partially unknown residue in the case of each). Therefore, B will *probably* act in the same way when placed in similar circumstances.

[1] *Logic*, III., xx., § 2.

(*d*) Rocks exposed to glacial action are seen to become scored or striated.

Many rocks in this particular district are thus scored or striated.

Therefore, this district has *probably* been the scene of glacial action.

(*e*) In districts now exposed to glacial action we find perched boulders.

In this district we also find perched boulders.

Therefore, etc. as in (*d*).

(*f*) In districts now exposed to glacial action we find long lines of accumulated stones and debris, which are called "moraines".

In this district we find such "moraines".

Therefore, etc. as in (*d*).

From those examples we see that the argument from analogy naturally assumes the form of a syllogism in the second figure with two affirmative premisses; that, therefore, it does not *prove* the law suggested in the conclusion, but only makes the conclusion more or less *probable;* that this probability may, perhaps, be a practically worthless and groundless suspicion;—or that it may amount to moral certitude: especially when, as in (*d*), (*e*), and (*f*), we have a number of independent analogical inferences all pointing to the same conclusion.

The *formal* reason why we cannot derive a universal law with *certitude* as conclusion from the premisses of such an argument, is that it would involve the fallacy of *undistributed middle.* Until we can convert our major premiss *simply* [from "*All P is M*" to "*All M is P*"], and so construct a syllogism in the first figure, we cannot be sure of our conclusion. We are not sure in (*a*) that the symptoms in which the particular disease agrees with cholera can be due *only* to the action of a bacillus; or in (*b*) that the points in which Mars resembles the earth are *sufficient and indispensable* for organic life; or in (*c*) that the ground for *A*'s action is *x*, in which he agrees with *B*, rather than *R*, in which he differs from *B*;[1] or in (*d*), (*e*), (*f*), that the scorings, perched boulders, and "moraines" in question, might not possibly be accounted for otherwise than by glacial action. And the only way we can become sure of these things (and so convert our major premiss and prove our conclusion) is by a closer investigation of

[1] *Cf.* WELTON, *op. cit.*, ii., pp. 71-3.

the phenomena. In other words, the *material* reason why we cannot regard the suggested law as certain, or verified, is because we have not sufficiently analysed the facts. In each case, the points of resemblance between the facts under observation and other known facts, suggest that the causal law which accounts for those points in the known facts may also account for them in the facts now being examined. We are trying to extend a known law to a new case. But this extension is an hypothesis which awaits verification. The connexion of the two cases by a common law is not verified by mere analogy as such. Analogy, as such, "sticks in the particular instances," and gives only a more or less probable conclusion.

There are good and bad arguments from analogy. The probability of the conclusion may range indefinitely from zero and practical certitude. On the one hand, the conclusion may be far less probable than its contradictory, when, as in (*b*), the points of resemblance seem to have little causal relation to the conclusion inferred from them, and not to include such essential conditions for this conclusion as the presence of oxygen or air, and absence of extremes of temperature.[1] Or, again, the conclusion may be no more probable than its contradictory, when conflicting analogies produce absolute doubt; as, for example, in the case of some lower form of living thing which may present certain resemblances to animal life, and certain other equally marked resemblances to vegetable life. Or, finally, it may become evident that the resemblances are causally connected with the inferred property, in which case the argument passes beyond the stage of analogy, its premisses take the form of the first figure of syllogism, and its conclusion becomes a certainly established law. If some symptom *common* to cholera and the other disease in example (*a*), above, could be shown to be due to *no other cause* than the action of a bacillus, the conclusion would become *certain* that the disease in question was due to such action. In order, therefore, that an inference from resemblances may lie within the limits of analogy, it must be probable, but *only* probable, not certain, that the common characteristics are causally connected with the conclusion based upon them.

235. WORTH OF ANALOGY: ITS FUNCTION IN VERIFICATION.—How, then, are we to estimate the value or force of an argument from analogy? On what will the probability of its conclusion

[1] *Cf.* MELLONE, *op. cit.*, p. 263.

depend? It will evidently depend on the degree of likelihood there is that the common characteristics are in reality causally connected with the conclusion that is based upon them. And on what does this likelihood depend? According to Mill, it varies in proportion to the *amount* of resemblance between the two phenomena, *i.e.* to the *number* of independent points of *similarity* as compared with the number of independent points of *difference*, and with the *total* amount of known and unknown elements in the two phenomena.[1] But, not to speak of the impossibility of "counting" the number of qualities assumed to be "independent" of one another, this purely numerical test is a very misleading one because it ignores the *nature* of the characteristics, while it is precisely in their nature as active properties, in their *purpose* or *law*,[2] that their importance lies as a basis or ground for inferring some further common bond of law connecting the phenomena.

Two phenomena may resemble each other in quite a multitude of respects which may furnish no real ground for inferring resemblance in any additional property. Two boys may be of the same age, height, strength, colour of eyes and hair, have similar home surroundings, and attend the same school. Yet from these resemblances we cannot infer with any degree of probability that because one of them is very talented so must the other be likewise. It is not by the number but by the *importance* of the points of resemblance that the strength of an analogy is to be estimated. Now, their importance depends upon their nature as compared with the nature of the additional property inferred in the conclusion. "Importance" is a relative term. The points of resemblance are "important" towards what is sought to be

[1] *Logic*, III., xx., § 3. *Cf.* WELTON, *op. cit.*, ii., p. 79. MELLONE, *op. cit.*, p. 262. FOWLER, *Inductive Logic*, pp. 213-14.

[2] In determining the significance or importance of characteristics in relation to one another we are aided very materially by the consideration that the laws of their nature are an expression of the purpose or *design* they are intended to serve (217). "This is easily seen," writes Professor Welton, "when the cases with which an inference is concerned are the purposive works of man. For example, by analogy we conclude that certain flints found in the earth are remains of weapons, because they bear marks of artificial shaping of such a kind as to adapt them to be cutting or piercing instruments, and corresponding, moreover, to those of flint weapons made and used by savages at the present day" (*op. cit.*, ii., p. 78). But there is purpose in nature, organic and even inorganic (217), as well as in the works of man. In botany and zoology many important laws have been brought to light through analogies based on the connexion between the structure and development of organs on the one hand, and the functions which it is assumed that they are intended to discharge on the other. *Cf.* MELLONE, *op. cit.*, p. 324.

inferred from them, just in so far as they are likely to be causally connected with this latter. *The points of difference, too, must be noted;* and these will be "important" as militating against the analogy, and so weakening it, in so far as they appear to be of such a kind as would be incompatible with the supposed causal connexion. This supposition of a causal connexion has next to be tested—by observation and, if possible, experiment—according to methods set forth in the next chapter. Should these convince us that the resemblances were merely accidental in regard to the inferred characteristics, that the latter cannot really be affirmed of the phenomenon under investigation, then the analogy of the latter to the other phenomenon was bad and misleading from the start. If, on the other hand, further analysis reveals some sort of causal connexion which enables us either to verify our hypothesis, or to modify it in some way, to alter its scope and restate it, and to verify it in its altered condition, then the analogy will have been so far good and useful and instructive.

We have referred to analogy as the application of known laws to new sets of facts. Such attempts at extension usually lead to restatements which give such laws a wider scope; or to the suggestion and verification of new hypotheses. These are the most important functions of analogy in induction. We may illustrate them by the following example, for which we are indebted to Dr. Mellone's *Text-book*:—

"A conspicuous instance . . . is seen in the early researches of Pasteur and his friends into bacteriology, as described in the *Life of Louis Pasteur* by his son-in-law. The old belief was that many contagious diseases were due to a *virus* or poison introduced into the blood. Further research was undertaken on the assumption that the cause of the diseases was something in the blood, but not necessarily a *virus*. This was a suggestion by analogy with the former belief, and it was experimentally proved by inoculating healthy animals with a drop of the infected blood. Afterwards the presence of minute animalculæ, visible only by the microscope, was detected in the blood of diseased animals; but at first it was supposed that these minute organisms could not produce such great effects. But subsequently Pasteur proved that such a great effect as fermentation was caused by the growth of an invisible vegetable organism; hence analogy suggested that the animalculæ whose presence was detected in the infected blood, might after all be the true cause of the diseases in question. This hypothesis, being experimentally verified, was proved to be true by applications of the joint method [*cf.* 242]. The old theory, that these diseases were caused by a *virus* introduced into the blood, could only give a forced explanation of many known facts; and it had to give way to a new theory—harmonising all the facts. But the new theory was originally suggested by analogy with the old; and the speculations with

regard to the action of the *virus* which were based upon facts did not lose their value; they simply had to be *revised* by the aid of the new light shed upon the question."[1]

"*A pari*," "*a fortiori*," and "*a contrario*" arguments, are all arguments from analogy: "The planet Mars bears a close resemblance to our earth, therefore, *a pari*, it is probably inhabited". "Work in the mines is hard on the health of male adults; therefore, *a fortiori*, it is injurious to women and children." "The abuse of alcohol is a cause of national decay; therefore, *a contrario*, the suppression of that abuse will make for national prosperity."

Inference by analogy is a very common form of reasoning, and very liable to abuse; the real significance of resemblances may be misinterpreted: and the metaphorical use of language often increases this danger. Some examples of inconclusive analogies will be examined in the section on fallacies.[2]

The characteristic which we seek to prove of an observed phenomenon, by analogy, may be known to belong to a single other phenomenon, or to a whole class. More usually, perhaps, it is something we suspect or know to be true of a whole class, something embodied in a generalization or law, which law we now seek to extend to the newly observed phenomenon. Or, we may have in our minds only the two individual phenomena; but even here the inference—which Aristotle calls παράδειγμα, *Example*—is not made from particular to particular without the aid of an implicit generalization. "The inference is the same," remarks Aristotle,[3] "whether it be based on resemblance to one case or to many" (193).

236. THE "ARGUMENT FROM EXAMPLE" IN ARISTOTLE.—By the *Argument from Example* Aristotle meant an inference from one individual phenomenon to another similar phenomenon "by bringing the one under the same universal to which the other is known to belong".[4] Manifestly, therefore, the Aristotelean παράδειγμα is what we now commonly call inference from analogy. Comparing it with the inductive syllogism (207), he describes it as "proving the major term of the middle by a *term resembling the minor*".[5] This description he explains and justifies

[1] *op. cit.*, p. 325.

[2] *Cf.* also JOSEPH, *op. cit.*, pp. 494 *sqq.*

[3] *Anal. Prior.*, ii., 24 [21], (3).

[4] *Anal. Prior.*, ii., 24 [26], (4): Φανερὸν οὖν ὅτι τὸ παράδειγμά ἐστιν . . . ὡς μέρος πρὸς μέρος, ὅταν ἄμφω μὲν ᾖ ὑπὸ ταὐτό, γνώριμον δὲ θάτερον. *Cf.* also *Rhet.* i., 2, (15)-(17); ii., 20.

[5] *ibid.*

by the following illustration: "The war between the Thebans and Phocians was a war between neighbours, and was an evil; therefore war between the Thebans and Athenians, being a war between neighbours, will also be an evil". Here the major term ("evil") is "proved" of the middle term ("war between neighbours")—that is to say, the *implicit universal principle* is "proved" —by means of a term ("war between Thebans and Phocians") *which resembles* the minor term ("war between Thebans and Athenians"). So that the whole process here consists in (*a*) an enumerative induction based on the enumeration of a single instance; and (*b*) the consequent application of the empirical generalization thus reached, to a new case that is brought under it, by a syllogism in the first figure: the conclusion of the latter being only probable because its major premiss, the generalization, is only probable. We may express the whole (as we may express any argument from analogy) in a syllogism in the second figure thus: "This disastrous war (between the Thebans and the Phocians) was a war between neighbours (P is M); War between Thebes and Athens will be a war between neighbours (S is M); Therefore it will (probably) be disastrous (S is P)". If we could cite additional instances of disastrous wars being wars between neighbours, so much the better; for it would strengthen the supposition that "all wars between neighbours are disastrous". If, finally, we could verify this supposition and lay it down as an established truth, we could substitute for the *probable* syllogism in the second figure: "P is M; S is M; therefore S is P," a *conclusive* syllogism in the first figure "M is P; S is M; therefore S is P".

Hence we can understand what has been said of the relation between analogy and enumerative induction: "In the latter, because a number of instances of a class x exhibit the attribute y, we infer that all x are y; in the former, because two particulars a and b agree in certain respects x, we infer that y which is exhibited by a, will be exhibited by b also. In the latter, from the limited extension of an attribute over a class, we infer its extension over the whole class; in the former, from a partial agreement between two individuals in intension, we infer to a further agreement in intension. But the one passes gradually into the other, for the former may be called the application to a particular case of a general principle inferred in the latter from a larger number of instances than in the former. This is very plain in an illustration which Aristotle gives of the 'example' (his name for the argument from analogy). A man might have inferred that Dionysius of Syracuse designed to make himself tyrant, when he asked the people for a bodyguard; for Pisistratus of Athens asked for a bodyguard, and made himself tyrant

when he got it; and likewise Theagenes at Megara. Both these fall under the same general principle that a man who aims at a tyranny asks for a bodyguard."[1]

237. ANALOGY AS UNDERSTOOD BY ARISTOTLE.—We have just seen that what is nowadays called the argument from analogy Aristotle called *παράδειγμα*. We have now to consider what *he* understood by an argument from analogy. The term *ἀναλογία* originally meant *identity of relations.* Four terms were said to be analogous when the relation of the first to the second was the same as that of the third to the fourth. Now if the relations are identical, and if what is inferred from them depends on this identity alone, the inference is cogent or necessary. And this is pre-eminently the case in mathematics, where the terms, relations, and inferences are purely quantitative. Here, then, the term "analogy" meant *equality of quantitative relations or ratios,* "*ἰσότης λόγων*,"[2] and has been translated by the terms "*proportio*," "proportion".[3] If 2 : 4 : : 3 : 6, we can infer that because 2 is the half of 4, 3 is the half of 6; and our reasoning is cogent, like any other mathematical reasoning. But the name "*ἀναλογία*" was also applied to cases in which the terms, relations and inferences were not quite, or not all, quantitative. Thus, "x vibrations of the air: 2 x vibrations : : a note : its octave," where the second relation is *not of the same order* as the first; or again, "x vibrations of luminiferous ether: y vibrations : : the sensation of red: the sensation of green," where again the second relation is qualitative, not quantitative, but yet is so connected with the first that it varies with, and can be known from, the latter; or again, to quote Aristotle's example, intellect bears the same relation to the soul as sight does to the body—*νοῦς : ψυχή : : ὄψις : σῶμα*,[4] where there is no idea at all of number or quantity, but only of nature or quality.

Now if what we infer from an identity of quantitative relations does not depend exclusively on those relations, our inference is not necessarily valid: I cannot validly infer that it will be twice as expensive to send goods by rail from A to B as from C to D

[1] "*Rhet.* a ii., 1357*b*, 25-36. To make the inference to Dionysius necessary (of course it is Dionysius I. who is meant), the principle would have to be, that a man who asks for a bodyguard aims at a tyranny; and that is really what the suspicious citizen of Syracuse would have had in his mind."—JOSEPH, *op. cit.*, pp. 500, 501, n.

[2] ARISTOTLE, *Eth. Nich.*, bk. v., 3 (8).

[3] FOWLER, *Inductive Logic*, p. 210, n. 4; *Deductive Logic*, p. 71, n. 2. *Cf.* also WELTON, *op. cit.*, ii., p. 75; JOSEPH, *op. cit.*, pp. 492 *sqq.*

[4] *Eth. Nich.*, i., 6 (12).

because the former distance by rail is twice the latter. And when we pass out of the domain of purely quantitative relations it becomes difficult to know for certain (*a*) that the relations are really identical, and (*b*) that what we infer from the identity depends on it alone. Our inference is based merely on *similarity* of relations, and yields only a probable conclusion. This is the sort of inference illustrated by Aristotle's example of the similar relations of sight to body and intellect to soul. It is a very common form of inference. We may argue, for instance, that because the relation of colony to mother-country is like that of child to parent, the reciprocal rights and duties of the former pair should be the same as those of the latter pair. Or again, the colonies may be compared to the fruit that drops off from the parent-tree when ripe.[1] Obviously, inferences of this kind may be of any degree of value—or of worthlessness. It is such resemblances between things that give rise to the metaphorical use of language (*e.g.* "mother-country"). Metaphors are simply analogies of this kind compressed into a single word or phrase; and metaphors are often misleading.

Between this "Aristotelean" analogy, and analogy in the modern sense, there is no essential difference. In the former the inference is based on similarity of relations, in the latter it is based on similarity of qualities, properties, or characteristics of any sort. The former may be symbolically stated: "*a* is related to *b* as *c* is to *d*; from the relation of *a* to *b* such and such a consequence follows, therefore it follows also from the relation of *c* to *d*"; the latter: "*a* resembles *b* in certain respects *x*; *a* exhibits the character *y*, therefore *b* will exhibit the character *y* also".[2] Similarity of relations must involve some similarity of inherent attributes or qualities in the things related, and so it was quite natural and inevitable that the term "analogy" should be extended, as it has been, from the former to the latter.[3]

WALTON, *Logic*, ii., bk. v., chaps. iii. and iv. JOSEPH, *Introd. to Logic*, chaps. xxi., xxii. and xxiii.; pp. 492 *sqq*. MELLONE, *op. cit.*, pp. 251-64, 317-40. MILL, *Logic*, III., v., xx. MERCIER, *Logique*, pp. 334-49.

[1] *Cf.* JOSEPH, *op. cit.*, p. 494. [2] *ibid.*, p. 496. [3] *ibid.*, p. 498.

CHAPTER VI.

METHOD OF DISCOVERING CAUSAL LAWS BY ANALYSIS OF FACTS: OBSERVATION AND EXPERIMENT.

238. OBSERVATION AND SELECTION: INITIAL PRECAUTIONS.—It is the aim of induction to discover the causes of phenomena, to verify as far as possible the laws according to which we suppose these causes to act, and so to explain the phenomena by means of these laws and causes. We have examined, so far, the general scope of the inductive method, the ways in which a knowledge of laws and causes is suggested, and the logical character of the procedure by which hypotheses are verified. We have next to investigate, in some detail, the functions of *observation* and *experiment*, with the more directly practical purpose of bringing to light certain useful rules for the proper conduct and application of these processes.

The events or occurrences of nature are complex and interrelated with one another. None of them stands apart in a state of isolation. It is because they are so intimately interwoven that we find it difficult to single out and mark off a "phenomenon" for investigation, to analyse fully its surroundings, and to lay bare those amongst them with which it is causally connected. Yet we must do all this before we can say whether or not our supposed cause—the object of our hypothesis—is the real cause of the phenomenon.

We *observe* an event occurring in certain circumstances or surroundings. In our initial observation we have evidently to *select* the phenomenon, and limit it or mark it off from its antecedent, concomitant, and subsequent circumstances. We next want to bring to light its cause, to find out the natural law according to which it occurs. We therefore make some supposition or hypothesis as to the factor or group of factors with which it is causally connected. This, too, involves *selection* on our part. And finally, we want to verify our hypothesis, *i.e.* to find out, if possible, whether our supposed cause is the sufficient and indispens-

able cause of the phenomenon. Obviously, the only way to reach certitude on this latter point is by instituting a careful and detailed analysis of the whole phenomenon and its surroundings, and so satisfying ourselves—by further and more searching observation, under special and modified conditions if necessary—that, amongst all the surroundings of the phenomenon, none have a causal influence upon it except those supposed by us to have such influence.

All observation of facts is *selective.* That is to say, what we observe is largely determined by the nature and direction of the *interest* we take in what comes under our notice. Our minds are never purely passive, but are constantly *interpreting* the data of sense experience, and *reasoning* more or less unconsciously from those data. Hence arises an initial danger, that our observations may be vitiated by *bias.* What we observe in any phenomenon depends largely on our previous knowledge: the skilled engineer "sees" more in the locomotive than the uneducated peasant, though the latter may have *sight*, and all other *senses*, as sharp as the former. Superior knowledge it is that renders observations more fruitful. The well-stocked mind will perceive analogies where the ordinary mind will not. The example of Malus, above referred to, is a case in point. Yet this very psychological fact exposes the observer to the danger of falling a victim to preconceived notions. We often *think we see* what we only *imagine.*

Again, the selection and isolation of the phenomenon to be examined, and of the elements to be tested as its likely causes, are more or less arbitrary processes, in the sense that they must be determined and prosecuted by the individual himself: for their proper and fruitful prosecution an extensive knowledge of the matter in hand is the only guarantee. The investigator who is not well acquainted with his subject will not be likely to detect all the conditions that are operative, or to eliminate and disregard all that are inoperative, in conducting his observations and experiments. Against this, as against the danger of bias, logic can furnish no safeguard. Both will be referred to at greater length in connexion with *Fallacies.* There, also, we shall deal with yet another mistake which may easily be made: the error of inferring *non-existence*, whether of *instances* or of *conditions*, from mere *non-observation* of the latter. It is only when these are of such a kind that they could not have escaped observation had they existed, that such inference is legitimate.

239. EXPERIMENT: ITS RELATIONS TO OBSERVATION.—*Experiment* is simply *observation under special conditions brought about by the observer himself, and modifying the object observed.* The qualification introduced in the latter phrase is essential; for if the special conditions mentioned modify only the observer's point of view, or his power or medium of observation, leaving the object unchanged, we do not speak of such an observation as an experiment. Thus, "vivisection is experiment because the determinate conditions it produces enter as factors into the action of the organism observed"; while "common dissection is not experiment, though it introduces conditions in the way of separation and demarcation as definite as anything can be,"[1] because these conditions merely prepare, without changing or forming part of, the object under observation. So, also, we speak of *observations* with the microscope, telescope, etc. The transition from pure observation to definite experiment is, however, gradual; they differ in degree, not in kind. This is best illustrated in what have been curiously called *nature's experiments*, or *natural experiments.* In these the phenomena observed are altogether beyond our control and, therefore, cannot be influenced by any activity of ours; but we take advantage of specially favourable circumstances, or select a specially favourable medium or point of view, for our observations: as when, for example, astronomers select special times and places for their observations; or meteorologists climb high mountains, or ascend in balloons, to make climatic observations; or scientists lay long lines of wire, or erect magnetic stations, or construct seismographs, for the purpose of observing and recording electric, magnetic, or seismic disturbances.

Experiment may, therefore, be rightly regarded as a special kind or mode of observation: the latter being the genus, the former a species. There is no real opposition between them, although in observation the natural and passive aspects, in experiment the artificial and active aspects, predominate. The superior value of the latter, as compared with the former, arises from the fact that the common object of both—the acquiring of a full and exact knowledge of all the operative conditions influencing the phenomenon under investigation—can be attained only by calling in the aid of experiment, and not by simple observation alone. This will appear presently from an examination of the proper

[1] BOSANQUET, *Logic*, vol. ii., p. 145.

function of experiment. At the same time it is important to bear in mind that all experiment is subservient to, and terminates in, observation. Moreover, in many departments of scientific research—as, for example, in ethics, zoology, geology, astronomy—experiment is practically impossible, and accurate observation must be relied on as our only and ultimate channel of experience.

240. THE FUNCTION OF EXPERIMENT; DIFFICULTIES OF ANALYSIS.—When we remember that an hypothesis is verified by endeavouring to show that it is *sufficient* and *indispensable* to account for the facts (229), that the supposed cause is proved to be the real cause by showing that *no other* cause is adequate, we shall realize the necessity of analysing all the surroundings of the phenomenon under investigation, of isolating the latter, of laying aside all that is inoperative and unessential to its presence, in order that we may be able gradually to bring to light its connexion with all the factors and conditions that really determine its occurrence. This work of analysis—mental analysis in the first place, and real analysis, separation, isolation of factors, as far as this is feasible, in the second place—is really the most difficult stage in scientific investigation. There may be causes or influences at work, of whose existence we are totally unaware: they may be part of the total proximate cause of the phenomenon. And non-observation is no *proof* of non-existence. On the other hand, certain circumstances which, in our experience, have always accompanied the phenomenon, need not be in reality causally connected with the latter: uniform coexistence or sequence does not *prove* causal connexion. Hence our hypotheses may at first include either too little or too much. The only way to obviate these difficulties is by endeavouring to *discover fully and accurately* ALL *the circumstances accompanying the occurrence of the phenomenon*, so as to be able to eliminate the accidental and retain the essential ones. It is for this purpose, for securing a full knowledge of *all* the surroundings of a phenomenon, that *repeated* observations—whether simple or experimental—are useful. If we could be certain, *from the observation of a single instance*, that all the conditions we observed accompanying it, and none others, were requisite for its production, we could straightway formulate, and regard as established or verified, the *natural law* connecting that phenomenon with its causes. But this is very rarely, if ever, possible. Hence the need for observing a number of instances.

Again, when we are sure that we have taken note of all the circumstances that usually accompany a phenomenon, and set ourselves to discriminate between those that are merely *casual* or accidental, and those that are *causal* or essential, we soon advert to the necessity of securing instances *that differ from one another in various respects.* Instances that vary among themselves are superior in value to instances that are practically similar to one another, because the former enable us to eliminate as unessential to the phenomenon the facts or circumstances which we noticed to be absent in this, that, or the other, of the varied instances in which the phenomenon occurred: the important principle underlying this mode of elimination being that *whatever circumstances can be removed or eliminated without interfering with the happening of a phenomenon are not causally connected with that phenomenon.* It is here that the superiority of experiment over simple observation first becomes manifest. No doubt, in very numerous departments of research, nature presents us not only with similar instances, but also with varied instances, of the occurrence of a phenomenon. But it is when we can *control* the agencies of nature, and *modify* them by experiment, that we can secure the most fruitful variety of instances, the most fruitful combinations of circumstances in which a phenomenon may or may not occur. Experiment thus enables us actively to interrogate and cross-question natural events, to analyse them more effectively, and to disentangle more successfully the connexions which are casual from those that are causal. The raw material furnished by nature, in our sense experience, for scientific analysis and interpretation, is for the most part chaotic and complex. The simple observer has to take this material as he finds it; the experimenter can control and modify it, and determine for himself the special conditions under which his observation of it will take place.

The manner and order in which the experimenter is to handle his materials and to operate upon them, must be left largely to his own scientific knowledge, insight, and genius. His experiments will be guided by the hypotheses he has formed; but, like the actual formation of the latter, so, too, the actual procedure in the former cannot be subjected to the guidance of any set of mechanical rules. Logic can only analyse his procedure, and point to some general principles of which that procedure is usually an embodiment and application. Such is the principle underlying the variation of instances in which a phenomenon occurs—

that *whatever can be eliminated without interfering with a phenomenon is not causally connected with the latter.*

There is another principle of equal importance, at which we arrive by the following simple consideration. No variety of *positive* instances, *i.e.* instances in which the phenomenon *occurs,* —no matter how great the number and variety—ought to satisfy the investigator that he has successfully segregated all the essential conditions of the phenomenon from its accidental accompaniments, if he can make an experiment whereby he will be able to *remove* the *supposed causal conditions* from a positive instance, in order to see whether by so doing he will thereby remove or eliminate the phenomenon itself: thus changing the whole into a *negative* instance, *i.e.* one in which the phenomenon *does not occur.* For, if he can thus successfully change a positive into a negative instance (or *vice versa*) by removing (or introducing) the supposed cause, he will by this mode of elimination have secured greater certitude about the accuracy of his hypothesis than any variety of positive instances could give him. The principle underlying this mode of elimination is that *whatever cannot be removed or eliminated without interfering with a phenomenon is causally connected with the latter.*

In those two principles we use the terms "cause" and "causal connexion" in the strictest sense, *i.e.* as *reciprocating.* They are simple principles in theory, but often not easy to apply satisfactorily in practice. This we shall see presently, from an examination of the various experimental "methods," or "rules," or "canons," which are merely so many ways of attempting to apply the principles just formulated.

In the course of our observations, whether simple or experimental, an instance or instances may occur in which the effect is absent though the supposed cause is present, or *vice versa.* Such instances are called *exceptions,* or *exceptional instances.*[1] If on careful examination these turn out to be not merely *apparent* but *real* exceptions, they will, of course, oblige us either to modify our hypothesis or to abandon it for a different one. For this reason they are of the greatest importance in inductive research. One single real exception to an hypothesis, *i.e.* to a supposed *law,* is sufficient to disprove the latter as it actually stands.[2] But, at the

[1] The term "*negative* instance" is sometimes applied, in a restricted sense, to an instance in which the phenomenon does not occur though the supposed cause is present.

[2] The legal aphorism, *Exceptio probat regulam,* is sometimes applied in a confus-

same time, the "exceptional" fact often suggests the direction in which we ought so to modify our hypothesis that in its modified form it will be compatible with the fact, which will then be no longer an "exception".

No logical rules can remove the difficulties inherent in the process of analysis by which we seek to discover and prove the causal laws according to which phenomena happen. When we have fixed upon a phenomenon (say *p*) for investigation, we must mark off around it—by going backward in time and outward in space (216)—a region or sphere (say *S*) of antecedent and concomitant facts, among which we assume that we can find the total proximate cause of the phenomenon. By this very limitation of our field of investigation we have excluded the rest of the universe as presumably irrelevant to *p*. This initial limitation of the sphere of intended analysis is essential in every inductive process: for in no case can we hope to analyse the whole universe as a system, of which our phenomenon, *p*, forms an integral portion.[1] And so it is quite possible to fall into the error of excluding from all consideration, from the very start, some condition which may really be causally connected with *p*. No logical canon will avail to guard against this danger. It can be avoided successfully only by the investigator who has a sufficiently deep and extensive knowledge of his whole department of inquiry to safeguard him against thus excluding *ab initio* any really operative or essential factor. Observation of a number of instances of the occurrence of the phenomenon will, of course, be helpful here in enabling him so to circumscribe the field of investigation, *S*, that he will be sure it contains within it all that is *sufficient* for the production of the phenomenon, *p*,—*i.e.* the sum-total of the influences which, when present, will entail the occurrence of *p*. This limitation is specially "liable to error when"—as Professor Welton observes[2]—"the phenomena are complex, and such error can only be detected by extremely careful and varied experiments to determine whether any condition is operative which had not been suspected and had therefore been [unconsciously] relegated to the unanalysed" universe, as unessential to the phenomenon.

The next step, naturally, is the analysis of the whole sphere of investigation,

ing way to the process of proving inductive laws. In jurisprudence, the full statement of the maxim is: *Exceptio probat regulam pro casibus non exceptis*. It simply means that the existence of a case or group of cases known to have been specifically exempted or *excepted*, by the legislator, from the operation of a certain rule or law, is a sufficient proof that the said rule or law exists and is binding in all similar cases *not* specifically excepted. In induction, a merely *apparent* exception can scarcely be said to "prove" the (hypothetical) rule or law, except in the negative sense of not disproving it. And the only sense in which a *real* exception can be said to "prove" the (hypothetical) rule or law is that, by securing the modification of a wrong hypothesis in the right direction, it contributes to the establishment or proof of the right hypothesis.

[1] Nor is this necessary in the *special* sciences, which seek the *proximate* causes of things. But it is part of the function of *philosophy* to examine hypotheses which are based on a consideration of the world of phenomena as a whole (229, 232).

[2] *op. cit.*, ii. p. 120. He instances (from Jevons, *Principles of Science*, pp. 428-9) the discovery, by Davy, of common salt in the air—its previously unsuspected presence in the air having "caused great trouble" in connexion with electrolysis.

S, into separate elements or factors, $a, b, c \ldots m, n, o, p, q, r \ldots$, and the supposition that some one factor or group of factors (say m) is the cause of p. Here is where the difficulty increases; and where some writers have left themselves open to the charges of conveying false impressions as to the simplicity of the process by which the real cause is to be discovered, and of setting up wrong ideals of this process. The difficulty lies in the fact that the breaking up of any field of phenomena, coexisting in space or successive in time, into separate entities, capable of being expressed and dealt with symbolically, as a, b, c, etc., must be to a large extent a *mental* analysis, which cannot claim to give us an adequate view of the complex reality as it actually is. It is only abstract and incomplete *aspects* of the reality that can be so represented. No doubt, it is only by such analysis that we can hope to detect causal connexions between phenomena. But nature does not present its materials to us thus analysed or broken up into separate factors. Its agencies act and interact; they often counteract and neutralize one another. Its influences cross and recross and combine with one another in hidden and intricate ways. It is on our making suggestive and fruitful analyses and syntheses of the materials which constitute our sense-experience, that the progress of science depends. When this work is done on the proper lines, the materials for induction are *prepared*, and the rules laid down by Mill are easy and obvious; but this work of preparation is *the most difficult stage* in the whole inductive process. And the rules in question rather suppose it to be done than help us to do it. Mill made the mistake of practically overlooking this part of the process; his treatment of induction conveys the impression, the erroneous impression, that nature furnishes us with prepared materials: with simple, isolated causes and effects, which have only to be observed, enumerated, and expressed by separate symbols (*cf.* 245).

The same is true of Bacon and Jevons (209, 210). Their treatment of the subject ignores the difficulty of making a successful analysis of the field of investigation into separate elements, about each of which we may next go on to inquire whether or not it is the cause of the phenomenon, p. Into how many alternatives are we to break up the field of investigation when we attempt to state the problem in the form of a disjunctive proposition: "The cause of p is either a, or b, or c, . . . or l, or m, or n, or . . ."? In other words, how many hypotheses are we to make and test? This is certainly not determined for us by the way in which the data are given to us; for they are not given catalogued into elements. Nor is it possible to determine the number of hypotheses, as Jevons suggests, by any merely formal counting of instances and "calculation of mathematically possible combinations"[1] of elements in those instances; for even if the elements were in each instance really distinct and independent of one another, as they certainly are not, the number of instances would be an "unattainable infinite series,"[2] and, hence, no *certain* conclusion could be obtained by such a method.

How, then, are we to proceed? Are we, perhaps, to go on selecting empirically and by haphazard, one after another, all the factors, $a, b, c, d, \ldots$ which we can detect in the field of investigation, S, and test each separately,

[1] WELTON, *op. cit.*, p. 59; *cf. ibid.*, pp. 53-55; VENN, *Empirical Logic*, pp. 357-60.

[2] WELTON, *ibid.*, p. 59.

until we have sifted out the cause of p? "Did we know all the conditions present," writes Professor Welton, "and needed but to decide which were operative and which were not, it would appear to be theoretically possible to empirically determine this question by trying every possible combination of both the presence and the absence of these conditions. Practically the enormous number of experiments involved would render this impossible; and the fact that conditions are not independent elements of reality would add another difficulty. For the removal of one condition not infrequently affects the action of those left behind, and similarly the addition of a new condition may cause an alteration in the result which could not be produced by this condition by itself but only through its union with others.".[1] The latter difficulty would multiply our experiments hopelessly if we proceeded in this empirical manner, for we should have to test not only every separate element, but every possible combination of the elements.

It is imperative, therefore, that our analysis be guided by an examination of the *nature* of p and of S, an examination which will suggest some part of S, say m, as *sufficient* for the occurrence of p: the residue of S, say R, remaining for the present unanalysed, as being presumably irrelevant to p. We can next convince ourselves that m is *sufficient* for p by showing, if possible, that wherever m occurs p occurs, even in the absence of R. To prove this we have to secure, if we can, the elimination of R; but it is quite possible that in attempting this we may find that m is not sufficient for p, that some part of R, say l, is also needed (leaving a residue R^1 presumably irrelevant). When we have thus determined what part of S, say lm, is sufficient for the occurrence of p, we have established the hypothetical "If S is lm it is p". We have next to see whether the *reciprocal* of this is also true, *i.e.* whether lm is *indispensable* for the occurrence of p. That is, we have to verify, if possible, the proposition "If S is p it is lm," or its equivalent (contrapositive), "If S is not lm it is not p". It is much more difficult to prove that nothing in S—or, for that matter, outside S—except lm, can produce p, than to prove merely that lm is *sufficient* for p. It is attempted by endeavouring to secure *negative* instances, *i.e.* instances of S from which lm is removed, in order to see if its removal will entail the disappearance of p. In other words, we endeavour to find in S—or outside it, even—an instance of the occurrence of p without the occurrence of lm. It is our failure to find p anywhere without lm that proves lm to be indispensable to p. But here, too, it is possible that we may find p occurring *in the absence of lm*, and accompanied only by R^1 (or, indeed, by R^1 *plus* something outside S altogether). This will prove that what is indispensable to p is not really lm, but something which is to be found not only in lm but in R^1 (or in R^1 *plus* something outside S). It proves, in other words, that we had not sufficiently analysed lm, that lm *contained* the really indispensable cause of p (say x) *and something irrelevant as well;* and that this x is to be found in R^1 (or in R^1 *plus* something outside S) as well as in lm. Thus, it is only by a very careful, and possibly a very prolonged, observation, whether simple or experimental, of negative instances, that we can finally bring to light x as the reciprocating cause of p (leaving as an irrelevant residue R^2). And it will be seen how, in this process, our initial hypothesis (that m is the cause of p) may have had to undergo many modifications and remouldings.

[1] WELTON, *op. cit.* ii., p. 117.

This outline of the experimental discovery and proof of a cause, contemplates only a comparatively simple case. As a rule, the agencies operative in nature are so complex, so interwoven, and so dependent on one another, that the actual process of analysis cannot be adequately represented by any such simple symbolism as we have been employing. The outline given will, however, help us to realize that in every successful discovery and proof of a causal relation "there is a comparison of the phenomenon [p] both in the presence [$S = x + R^2 + p$] and in the absence [$S = R^2$] of that particular condition [x] we are investigating".[1] The observation of *positive* instances will help us to include in the field of investigation, S, all the really relevant operative influences in regard to the phenomenon under investigation, p; while the observation of *negative* instances will help us to determine what part of S is *indispensable* to the occurrence of the phenomenon, p. But in both sorts of instances there are difficulties to be surmounted. Practically speaking, we can never completely eliminate the "residue" from our positive instances; we can never get a positive instance of p without a residue R, R^1, or R^2, accompanying the supposed m, lm, or x. And hence we have to make sure (1) that this residue "includes nothing which is not known to be present, and whose influence, if it existed, could be determined and allowed for";[2] and (2) that this residue, if it cannot be totally eliminated, is at all events really irrelevant to p. We try to make sure of these points—and so to convince ourselves that there is nothing really operative unknown to us in the positive instances, besides the supposed cause—by directing our attention to the residue, and analysing it in a series of *negative* observations or experiments, *i.e.* instances in which the phenomenon, p, does not occur. We remove, if possible, the supposed cause, leaving the residue, in order to see if the phenomenon will disappear. Or, finding a case in which both the latter and the supposed cause are absent, and the residue alone present, we introduce, if possible, the supposed cause, to see whether the phenomenon also will appear. This process of comparing positive with negative instances is likely to bring to light operative influences of which we were previously unaware, if there were really any such present in the unanalysed residue in the positive instances. It is by the negative instances that we prove our supposed cause to be *indispensable*, and everything else irrelevant, to the effect.

But this proof will be cogent only in so far as we are sure that in passing from the positive to the negative instance, or *vice versa*, we have eliminated, or introduced, *nothing else but* the supposed cause. And it is not easy to be sure of this, on account of the complexity and interdependence of the causal agencies that are operative in nature. In experimenting, our working principle should be, as far as possible, *never to introduce or eliminate more than one element at a time*. If we vary more than one element at a time, we shall not know to which of the elements, so varied, any resulting change is to be attributed.

It will now be sufficiently clear that the conduct of analysis, by observation and experiment, cannot be reduced to any set of mechanical rules or formulæ. The history of the inductive sciences, of the ways in which great scientific truths have been *de facto* discovered and established, makes this fact still more evident. A number of interesting and instructive examples are given by Professor

[1] WELTON, *op. cit.*, p. 118. [2] *ibid.*, p. 121.

Welton[1] in a section which he concludes with these words: "Even such an imperfect outline as the above makes abundantly manifest that induction is by no means an easy process, or one that can be reduced to mechanical rules; that the procedure starts from and is guided throughout by hypotheses; that number of experiments is appealed to only as a guarantee that only known conditions are operative; that the procedure of perceptual analysis is to establish a positive connexion, to purge this of exceptions and to limit and corroborate it by negative instances; and that one inductive enquiry gives rise to others."[2]

Many valuable and instructive examples will also be found in Mr. Joseph's *Introduction to Logic*, especially in chapter xx.,[3] where the author sets forth a number to illustrate "the truth of the contention that inductive conclusions are established disjunctively by the disproof of alternatives".[4] We have already set forth this theory of the inductive process (212), and it now remains to examine briefly the "experimental methods," or rules according to which this process of eliminating alternatives may be conducted.

241. THE "RULES" OR "CANONS" OF INDUCTIVE ANALYSIS; "METHODS" OF "AGREEMENT" AND "DIFFERENCE".—By a study of the various plans of procedure actually adopted by scientists in inductive research, logicians have been able to formulate certain practical *rules* or *canons* of which an explicit knowledge cannot fail to prove useful towards the discovery and proof of causal laws.

Lord Bacon's *tabulae praesentiae, tabulae absentiae*, and *tabulae comparativae*,[5] represent a somewhat crude attempt at such formulation. Sir John Herschel's *Preliminary Discourse on the Study of Natural Philosophy* marks an important step in the logical analysis of scientific procedure, and it was from this work that John Stuart Mill avowedly drew the four (or five) "methods" that have been ever since inseparably associated with the latter's name.[6] Mill's formulation of them, however, is somewhat clumsy, and is, besides, in many ways defective; and, while altogether overrating their value, he in part misunderstood their real scope and significance. Those mistakes of his have been corrected by subsequent logicians.[7] Most of what these rules contain is implied in what has been said in the previous section (240) concerning analysis and experiment; but what was there briefly outlined will be better understood by formulating and illustrating the various

[1] *op. cit.* ii., pp. 122-41.
[2] *ibid.*, pp. 140-1. *Cf. infra*, 245.
[3] *Cf.* also chaps. xxii. and xxvi.
[4] p. 408.
[5] *Novum Organum*, ii. *passim*, *Cf.* ADAM, *La philosophie de François Bacon* (Paris, 1890).
[6] *Cf. Inductive Logic*, III., viii. and ix.; JOSEPH, *op. cit.*, p. 397, n. 2.
[7] *Cf.*, for instance, the text-books of WELTON, MELLONE and JOSEPH; also Mr. LAURIE's articles in *Mind* (*N.S.*, vol. ii., 1893).

canons in question. They have been stated in a variety of ways, nowhere, perhaps, more clearly than in Dr. Mellone's *Introductory Text-book of Logic*,[1] whose treatment of the subject we purpose mainly to follow.

The first rule, which is called the METHOD OF AGREEMENT, or of *single* agreement, he states in this wise :—

"WHEN OBSERVATION SHOWS THAT TWO EVENTS ACCOMPANY ONE ANOTHER (EITHER SIMULTANEOUSLY OR IN SUCCESSION), IT IS PROBABLE THAT THEY ARE CAUSALLY CONNECTED ; AND THE PROBABILITY INCREASES WITH THE NUMBER AND VARIETY OF THE INSTANCES."

This rule is based on the principle that whatever can be removed or eliminated without interfering with the phenomenon is not causally connected with the latter ; and hence it is thus briefly expressed by Mr. Joseph :[2] "Nothing is the cause of a phenomenon in the absence of which it nevertheless occurs". It is a method of *observation* mainly, *i.e.* a rule to which we have recourse when we cannot experiment. Its chief utility lies in the fact that it *suggests* a causal connexion as an hypothesis for verification. What *appears* to be the sole invariable antecedent of a phenomenon probably contains the "necessitating and indispensable cause" of the latter ; though other things also may contain this "necessitating and indispensable cause". Hence, at best, this rule merely suggests (without proving) that our "sole invariable antecedent" is *a* cause (in the wider sense) of the phenomenon. And since it suggests (without proving) only a cause *in the wider sense*, it does get us over the difficulty arising from the fact that the same phenomenon can have a *plurality* of *such* causes. Briefly, it fails to prove a reciprocating causal relation because what we think to be the sole invariable antecedent (1) may not be so *de facto* (since another may be invariably present unknown to us) ; and therefore, possibly, (2) may not be relevant to the phenomenon at all (since the latter may be in reality due to the unknown antecedent) ; and (3) even though the *observed* invariable antecedent is *de facto* the one which, when present, necessitates the effect, it need not be *indispensable* to the latter ; for it may merely *contain* this indispensable element *plus* something irrelevant, while (for all we know) this indispensable element may be equally well supplied by each of the varying

[1] chap. ix. [2] *op. cit.*, p. 403.

agencies actually observed, or by quite other agencies, and under quite other conditions, than those under actual observation.

To take a simple illustration: "Suppose you mix three different kinds of poison [*B*, *C*, *D*] with water [*A*] and give it to three people: they all die, but you cannot argue that the water is the cause of death, though it is the only invariable antecedent".[1] You cannot even argue with certitude that the water is *a* cause of death. And why? Because you are not certain that it contains the "necessitating and indispensable" cause of death: there may be *another* "invariable antecedent" present in the three cases, namely, something (say *X*) contained in, and *common* to, the poisons, *B*, *C*, *D*, each of which therefore may be *a* cause; while the indispensable element, *X*, though not in *A*, is not eliminated by the absence of *C D*, or of *B D*, or of *B C*, but is present and operative in all three instances, *A B*, *A C*, and *A D*, first in *B*, then in *C*, then in *D*. From all of which we see that the causal hypothesis which this method can merely suggest, may conceivably be even a wrong hypothesis. Simple observation reveals to us only what are causes in the wider sense; and in the absence of further analysis the method of agreement may possibly eliminate many such causes successively, leaving the *indispensable* factor present every time, not in the observed "invariable" element, but in one or other of the "varying" elements: "If heat, for instance, is produced by friction, combustion, electricity, all these real causes would be eliminated by this method, for they are points in which the different instances of heat differ".[2] At best, then, this method can merely suggest, as a more or less probable hypothesis to be otherwise tested, the supposition that the observed invariable antecedent is, or contains, the cause.

The rule was entitled by Mill the "method of agreement," because, though the instances differ in details that are presumably irrelevant to the phenomenon, they *agree* in the *one* presumably *essential*, and therefore important, point. He formulated the rule in the following *way*:—

"*If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree, is the cause or effect of the given phenomenon.*"

He means, of course, "one circumstance in common" *besides the phenomenon itself*, which is common to all the instances. "Two" instances would give very little probability, for this depends on the number and variety of the instances. Of course, *if* we could be *certain* that the observed instances of any phenomenon had really "*only one* circumstance in common" we could

[1] *Palaestra Logica*, p. 110, § 338.

[2] MELLONE, *op. cit.*, p. 297.

infer with certitude that this circumstance is *a* necessitating cause, though not that it is the *only possible* (or *indispensable*) cause, of the phenomenon. But the "if" here points to a condition that is practically never fulfilled in the unanalysed data presented in our sense experience as the raw material for induction.

The following is one of Mill's own illustrations of the method of agreement: "For example, let the effect be crystallization. We compare instances in which bodies are known to assume crystalline structure, but which have no other point of agreement; and we find them to have one, and as far as we can observe, only one, antecedent in common: the deposition of a solid matter from a liquid state, either a state of fusion or of solution. We conclude, therefore, that the solidification of a substance from a liquid state is an invariable antecedent of its crystallization."[1]

The second method is called the METHOD OF DIFFERENCE, or of *single* difference. It is stated as follows by Dr. Mellone:—

"WHEN THE ADDITION OF AN AGENT IS FOLLOWED BY THE APPEARANCE, OR ITS SUBTRACTION BY THE DISAPPEARANCE, OF A CERTAIN EVENT, OTHER CIRCUMSTANCES REMAINING THE SAME, THAT AGENT IS CAUSALLY CONNECTED WITH THE EVENT."[2]

This rule is an application of the principle that whatever cannot be eliminated without interfering with the phenomenon is causally connected with the latter.[3] We saw that the method of agreement was a method mainly of *observation*, a method of *discovering* causal laws by *suggesting* these as hypotheses for verification. The present method is a method mainly of *experiment*, a method of *proving* causal laws by the *verification* of hypotheses already suggested. It compares a *positive* instance (in which the phenomenon occurs) with a negative instance (in which the phenomenon does not occur). It is only when we can produce the positive and negative instances by experiment, as immediately successive phases of the same general set of conditions and circumstances, that the rule can be applied with any considerable degree of success. And the reason is this: for the successful application of the rule we must be sure that there is *no other difference* between the two instances besides the presence of the supposed cause in the one and its absence in the other. But simple observation can rarely, if ever, guarantee a *certain* conviction

[1] *Logic*, III., viii., § 1. [2] *op. cit.*, p. 300.

[3] Mr. Joseph, emphasizing the fact that we are always led to the true cause indirectly, *i.e.* by eliminating what is not the cause, enunciates the present ground of elimination in this way: "Nothing is the cause of a phenomenon in the presence of which it nevertheless fails to occur,"—taking "cause," of course, as the strict, reciprocating cause (*op. cit.*, p. 404).

that two instances offered by nature are of this sort. It is only when we have thoroughly analysed all the surroundings of the phenomenon, when we *know* all of them, when we can control, modify, introduce, or remove, each of them separately, without disturbing the others, that we can be sure our two instances *have everything else in common*, and differ *only in the one essential respect.* This latter condition, we may remark, is why we call this rule the "method of *difference*".

In applying the rule experimentally it does not matter which of the two instances, the positive or the negative, comes first in order of time. Occasionally, we may find it more satisfactory—or, rather, less unsatisfactory—to produce the two instances simultaneously. "*E.g.* to try the effect of a certain manure on a wheat crop, you would not try it one year and compare the result with the year before, for the weather might be different. You would take two fields exactly alike and try the manure in one of them and not the other." [1]

For the most part, however, the instances are procured successively. The well-known coin and feather experiment will afford a simple illustration. It is supposed that the greater resistance of the air to the relatively larger volume of the lighter sorts of bodies is the reason why these fall more slowly than bodies of the heavier sort. This is the hypothesis for verification. To test it we contrive an experiment by means of the air-pump. Before exhausting the receiver we take the coin and feather and let them fall within the receiver: they fall in unequal times (as they would outside the receiver): the supposed cause and the effect are both present, the resistance of the air and the retardation of the fall of the feather. This is the positive instance. We next eliminate the supposed cause by exhausting the receiver of air: we let the coin and feather fall in the exhausted receiver, and we observe that the effect has disappeared; the feather is not retarded, it falls as quickly as the coin. This is the negative instance. Since, then, so far as we know, there was no change in the circumstances of the falling coin and feather, *except the removal of the air*, we conclude that the air *contained* (among other things, assumed to be irrelevant) *that element* (namely, *resistance*) which is the necessitating and *only possible* cause of the phenomenon *in the conditions of our experi-*

[1] *Palaestra Logica*, p. 110, § 341. *Cf.* MELLONE, *op. cit.*, p. 303; and JOSEPH, *op. cit.*, p. 519, for the limitations of such an experiment.

ment, and in all similar sets of conditions. Further experiment is hardly needed to convince us that it is the *resistance* of the medium that necessitates the retardation: not necessarily *atmospheric* resistance, but the resistance of any other gases that we might substitute for the atmosphere. Yet, strictly speaking, the single experiment with the atmosphere assures us only that the *air* is *a* cause of retardation in the fall of the feather.[1] But granted, now, that we have made, if necessary, such experiments with other gaseous media, and convinced ourselves that *in the circumstances of those experiments, and therefore in all similar sets of conditions*, the *resistance of a gaseous medium* is the *only possible* cause of the phenomenon, can we further infer, from such single experiments, that this is the *only possible* cause of the phenomenon, *absolutely and universally, i.e. in any and every conceivable place, time, state or condition of things, in the universe? No;* the method of single difference does not by itself really guarantee this further inference. What it brings to light as the "only possible cause" may perhaps be not yet sufficiently analysed, may perhaps contain "irrelevant matter" *plus* "the only possible cause"; so that this latter may perhaps be forthcoming elsewhere, in quite a different vehicle, other than that in which it was embodied in any of our experiments. Hence the method of single difference proves *a* cause; but of itself it does not strictly prove that this is the *only possible* cause. But the latter is the only kind of cause that excludes plurality; there may be a plurality of causes of the former kind. *Therefore, the present method does not of itself completely exclude the uncertainty which arises from the possibility of a plurality of causes.* In other words, the method of single difference does not of itself establish with certitude a reciprocating causal relation.

Another illustration of the present rule and its limitations, is afforded by experimental inquiry into the conditions for the propagation of sound.[2] It is supposed that the presence of some elastic medium, such as air (or any other gas), between the ear and the sounding body, is an indispensable condition for the propagation of sound. This hypothesis is tested by ringing a bell in the

[1] By substituting successively, for air, other gaseous media, having presumably only RESISTING POWER in common, as a factor relevant to retardation of the feather, and finding the retardation take place in every case, we satisfy ourselves, as far as we can, by this application of the *Method of Agreement*, that it is *resistance* that causes retardation.

[2] *Cf.* JOSEPH, *op. cit.*, p. 442.

receiver of an air-pump, first before exhausting, then after exhausting, the receiver. In the former (positive) instance the sound is heard, in the latter (negative) instance it is not. From this we conclude that the presence of air is *a sufficient condition* for the propagation of sound; not, however, that it is itself an *indispensable* condition, but only that it *contains* an indispensable condition—supposed to be *elasticity*. This latter supposition we may confirm by applying the method of agreement to a number of positive instances, each containing a different *elastic medium*. But even when this stage has been reached we cannot, strictly speaking, be certain that elasticity is the only *relevant* factor *common* to the media employed in the positive instances; nor, therefore, that it is a really indispensable factor in those instances. Nor further, even were we convinced that elasticity *was* one of the *indispensable* factors *in the conditions of our experiments*, could we, strictly speaking, conclude that an elastic medium is indispensable to the propagation of sound *in any and every conceivable set of circumstances*. For it might still be objected that perhaps in some totally different set of circumstances a medium with some other property instead of elasticity might be capable of propagating sound. Such an alternative possibility we might be disposed to regard as far-fetched; but the only way of *disproving* any such suggestion would be by obtaining a negative experiment in which elasticity would be absent and the other supposed property present, and in which *sound would not be propagated*. It will be seen that the experiments throughout this example do not purport to prove that elasticity is *the only* indispensable condition of a sound-propagating medium, but only that it is *an* indispensable condition. There might be *elastic* media which would *not* propagate sound, owing to the absence of other indispensable conditions for such propagation. The hypothetical ether, supposed to be the propagating medium of radiant heat, light, electric and magnetic influences, may, perhaps, be incapable of transmitting sound, even though it be elastic.

The *method of difference* has been stated in the following terms by Mill:—

"*If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former, the circumstance in which alone the two instances differ, is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.*"[1]

[1] *Logic*, III., viii., § 2.

Again he means, of course, "every circumstance in common save one," *and the circumstance of the successive presence and absence of the phenomenon itself*, which appears in one instance and not in the other. "It is scarcely necessary," he adds, "to give examples of a logical process to which we owe almost all the inductive conclusions we draw in early life. When a man is shot through the heart, it is by this method we know that it was the gunshot which killed him: for he was in the fulness of life immediately before, all circumstances being the same except the wound." He regards the method of difference as the *experimental* method *par excellence*, as the only one by which "we can ever, in the way of direct experience, arrive with certainty at causes."[1] He also gives the following example: "If a bird is taken from a cage, and instantly plunged into carbonic acid gas, the experimentalist may be fully assured (at all events after one or two repetitions) that no circumstance causing suffocation had supervened in the interim, except the change from immersion in the atmosphere to immersion in carbonic acid gas."

Mill is inclined to overrate the value of this method. He failed to see that of itself it does not enable us to analyse the phenomena under investigation sufficiently to reach the *one* necessitating and *indispensable* cause. We require something better than the method of single difference to carry us beyond that less perfect stage of science in which we must admit plurality of causes.

242. COMBINATION OF "AGREEMENT" AND "DIFFERENCE".—As long as our methods of analysis merely enable us to assert that something is *a* cause of a certain kind of phenomenon, we have to recognize that the latter may have *other* causes; and it may be reasonably objected that the one we allege to be operative in any given instance is not the one that is really operative there. We endeavour to remove this uncertainty by *combining the methods of agreement and difference.* This may be done either when we are obliged to have recourse to simple observation alone, or when we can make use of experiment. In the former case it may be well to call this combined method the DOUBLE METHOD OF AGREEMENT; and in the latter to call it the JOINT METHOD OF DIFFERENCE AND AGREEMENT. The former title will emphasize the fact that although there is an element of the method of difference present—inasmuch as we have two sets of instances, a positive set and a negative set—yet we have not the *essentials* of the method of difference, since we have to rely on simple observation which does not give us any pair of positive and negative instances differing *in one respect only;* while the method of agreement predominates inasmuch as it is applied to each set of instances successively. The latter title will emphasize the fact that the method of difference is applied experimentally to the matter in

[1] *ibid.*, § 3.

hand, and that we seek to make good its limitations by experimental observation of a number of instances—especially of negative instances—varying among themselves in accordance with the method of agreement.

The DOUBLE METHOD OF AGREEMENT is stated thus by Dr. Mellone[1]: "WHATEVER IS PRESENT IN NUMEROUS OBSERVED INSTANCES OF THE PRESENCE OF A PHENOMENON, AND ABSENT IN NUMEROUS OBSERVED INSTANCES OF ITS ABSENCE, IS PROBABLY CONNECTED CAUSALLY WITH THE PHENOMENON".

The two sets of instances must, of course, be drawn from the same field of investigation; they must be *in pari materia;* they will, therefore, have a great deal in common (the more the better); each negative instance will be so chosen as to resemble *as much as possible* some positive instance; if any such pair could be procured *with everything in common* (save the supposed cause), we should have the requisite data for the method of difference, and we might not have recourse to the present method at all: it is precisely because we cannot by simple observation procure two such instances that we must, as the next best course, apply the present method; and in applying it we select positive instances which vary as much as possible among themselves, and likewise negative instances which, like the positive ones, vary as much as possible among themselves. Of all these points we have a clear and easy illustration in the following example:—[2]

"A fever has broken out in a town; what is the cause? The patients vary in age, general health, circumstances, etc., but they are all supplied with milk from the same dairy. You suspect (by the method of agreement) that some taint in the milk has caused the fever. Suppose the dairyman pleads the 'plurality of causes'; *viz.* the possibility that some of the patients have got the fever by direct infection, others from bad drains, others from poor living, none from the milk. How would you answer him? You would see whether those who did *not* drink the milk from his dairy were also free from the fever. If you could say, 'here are a number of people living under much the same circumstances as the fever patients, some exposed to direct infection, some to bad drains, some to semi-starvation, but none of them drink the milk from your dairy and none of them have fever,' then the case against the milk would be much strengthened; because you could

[1] *op. cit.*, p. 306.

[2] Taken from the *Palaestra Logica*, p. 111, § 346. *Cf. infra*, 245.

point to instances in which the other *possible* causes of the fever had not produced it."

"You would not use this method if you could use Difference. But you can *neither*, if any one drinks the milk and gets the fever, be sure that the milk is the only new antecedent, *nor* find two people exactly alike in everything likely to produce fever except that one has drunk the milk and the other has not."

When we attempt to trace causal connexions between the phenomena which form the subject-matter of the social, political and economic sciences, we shall rarely find it feasible to apply the method of difference simply; we are obliged, therefore, to have recourse to the double method of agreement, or to "concomitant variations" (243), or "residues" (244).

Mill's formulation of this double method, which he called the *indirect method of difference*, or, also, the *joint method of agreement and difference*, is as follows :—

"*If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance; the circumstance in which alone the two sets of instances differ, is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.*"

This formula is vague, if not even misleading. *Two* instances in each set would be rarely, if ever, sufficient. Very rarely can the positive instances have "only one circumstance in common," or the negative instances "nothing in common save the absence of that circumstance".

Among his examples of this method is the following: "It appears that the instances in which much dew is deposited, which are very various, agree in this, and, so far as we are able to observe, in this only, that they either radiate heat rapidly or conduct it slowly: qualities between which there is no other circumstance of agreement, than that by virtue of either, the body tends to lose heat more rapidly from its surface than it can be restored from within. The instances, on the contrary, in which no dew, or but a small quantity of it, is formed, and which are also extremely various, agree (as far as we can observe) in nothing except in *not* having this same property.[1] We seem therefore to have detected the characteristic difference between the substances on which dew is produced and those on which it is not produced."[2]

Let us now see how the combination of difference and agreement is applied to data which we can control by experiment, in accordance with what we have called the JOINT METHOD OF

[1] *i.e.* in nothing *that is considered likely to influence the presence or absence of dew*: for, surely, being *in pari materia*, concerned with the same group of phenomena, the instances must have several other *positive* (though presumably indifferent) circumstances in common, and must also agree in the *absence* of several other presumably irrelevant circumstances.

[2] *Logic*, III., ix., § 3.

DIFFERENCE AND AGREEMENT. This is really the method which pushes experimental analysis to its farthest possible limits, and, by doing so, enables us to eliminate by degrees the misgivings that may still remain after we have applied the method of difference. These misgivings arise, as we saw, from the fact that the total cause, as determined by applications of the latter method, may not after all be *absolutely indispensable*; or, in other words, that somewhere or somehow other agencies might conceivably be substituted for it in whole or in part, and so produce the effect in the total or partial absence of this cause. Now the only way of allaying such suspicions is by conducting as many negative experiments as these suspicions demand, within what we consider to be a reasonable sphere of investigation: experiments in each of which our supposed cause is absent, and one or other of the conceivable alternatives present. Each such experiment, in which the phenomenon is absent, negatives or disproves the suggested alternative cause which has been introduced instead of the supposed real cause. This is, in reality, the method of analysis which we have already outlined (240), and its connexion with the process of verifying hypotheses by the *disproof* of alternatives, as set forth in a previous chapter (212), will be at once apparent; for it simply describes the manner in which that process of verification ought to be conducted. The following is the canon in which Dr. Mellone[1] embodies this method:—

"*When one phenomenon has been shown to be* THE CAUSE OF ANOTHER UNDER GIVEN CONDITIONS, *by the method of single difference; and when we fail to find or to construct any instance where the one phenomenon occurs without the other: then it is probable that the first is the* 'UNCONDITIONALLY *invariable antecedent' of the second—i.e. that the latter can be produced in no other way than by the former; and the probability increases with the number and variety of the negative instances all agreeing in the absence both of the effect and its suspected cause.*"

The phrase "unconditionally invariable antecedent" means the "sufficient (or necessitating) and indispensable cause," or again, the "reciprocating cause". Dealing with the conditions for the verification of an hypothesis, we saw that an hypothesis is rigorously verified when it has been shown to be the "only possible" one that will account for the facts (229); and the question now occurs: Does "only possible" mean "conceivable absolutely in the

[1] *op. cit.*, p. 309.

abstract," or rather, "the only one that the concrete facts of experience warrant us in regarding as any way plausible or deserving of serious consideration"? And the latter alternative is, of course, the true one. So, too, the practical question now arises: Over what extent of field must we conduct our negative experiments, or how many must we perform, or when should we be satisfied that our supposed cause is really the only possible, or indispensable, cause? And the answer will be: when we have allayed all reasonable fears that there may be other causes of the phenomenon; when we have examined and disproved all the alternatives that we think really worthy of consideration. The decision of this will obviously depend on the nature of the investigation, and on the knowledge, insight, and prudence of the investigator.

An instructive illustration of the value of the joint method is that furnished by investigations into the cause of fermentation.[1] "When sugar is changed into alcohol and carbonic acid in the ordinary alcoholic fermentation, the process is in some way related to the cells of the yeast plant. . . . For many years these minute organisms received little or no attention; but in 1838 Schwann, one of the founders of the cell theory, and Cagniard de la Tour, demonstrated the vegetable nature of these yeast cells, and showed that they grew and multiplied in saccharine solutions." The method of single agreement warranted the conclusion that they were probably *a* cause of fermentation; but not the conclusion that they were *indispensable* to fermentation. Liebig contended that they merely formed "a substance which by purely chemical action produces the chemical change called fermentation"—a substance which might conceivably be produced otherwise than by the action of living germs like the yeast cell. The hypothesis that such living germs cause fermentation not in this indirect way, but by such direct and immediate action that their presence is indispensable and has no possible substitute—this hypothesis had now to be tested. In other words, it had to be proved not only that living germs cause fermentation, but also that nothing else can. Symbolizing cause and effect by G and F, respectively, the two propositions "If G then F," and "If not G then not F," had to be established. The first of these propositions offered no difficulty: it was accepted as sufficiently established for the time by the method of single agreement. At all events, the *onus* of proving that the presence of living germs in fermentable materials need not necessarily cause fermentation, would have been rightly placed on the shoulders of any one who would venture to put forward such a contention (*cf.* 207). But to establish the second proposition a careful series of negative experiments had to be conducted: the real difficulty in the case being to get negative instances in which the complete *absence* of living germs would be assured beyond all reasonable doubt. The following were some of the experiments: (1) "Gay-Lussac showed that clean grapes or boiled grape juice, passed into the Torricellian vacuum of a barometer-tube, remained free from fermentation for

[1] *Cf.* MELLONE, *op. cit.*, pp. 298-9, 310-11.

any length of time ['If not A (air) then not F'], but that if a single bubble of air were admitted fermentation soon appeared ['If A then F']." This careful experimental application of the method of single difference merely proved that *something in the air actually admitted*, and therefore in all similar air admitted in similar circumstances, *causes fermentation*. The supposition that this "something" consists in living germs had yet to be proved by negative experiments that would presumably rid the air of such germs. (2) "Schwann repeated Gay-Lussac's experiment and showed that if the air were admitted to the vacuum through a red-hot tube then fermentation did not occur." This proved that the "something" could be destroyed by great heat, and so went to strengthen the probability of the proposition "If not G, then not F". (3) Still further probability was added by various experiments which went to show that a "temperature of from 20° C. to 24° C. was most favourable to" fermentation; "while the process was stopped at freezing point (0° C.) and again at 60° C.; and boiling destroyed it". (4) "Afterwards Helmholtz showed that oxygen produced by electrolysis in a sealed-up tube containing a boiled fermentable fluid did not cause fermentation." Here again, the probability of the proposition "If not G, then not F" is increased, inasmuch as the pure oxygen so obtained differs from ordinary atmospheric air in the absence of all extraneous organic impurities, living germs included. (5) "Hoffmann showed that air filtered through cotton wool was incapable of causing fermentation"—the air being presumably purified of all organic matter, including living germs, by such filtering. Therefore, again, the proposition "If not G, then not F" was corroborated. (6) Better than all the foregoing negative instances was the one secured by Helmholtz in the following application of single difference, with its *simultaneous* positive and negative instances: "He placed a sealed bladder full of grape juice in a vat of fermenting juice, and found that the fluid in the bladder did not ferment. Thus the cause of the fermentation could not pass through the bladder. If the fermentation were excited, as Liebig held, by a separate substance formed by the yeast cells, and presumably soluble, one would have expected it to pass through the wall of the bladder; but if the process were caused by the small yeast cells, then one can see why fermentation was not excited, as the yeast cells could not pass through the membrane." This experiment tended to disprove the hypothesis of a *soluble* substance as the immediate cause of fermentation. It did not, however, *prove* Pasteur's view, that fermentation is not a merely chemical process taking place outside the yeast-cells, but rather a vital or physiological process which takes place within them and transforms the assimilated sugar into alcohol and other products. So far as was yet known, fermentation might be due to the chemical action of some *insoluble* product of the vital functions of the yeast cell. Subsequent experiments did bring to light a substance of this sort, namely *zymase*, to the action of which fermentation seems to be immediately due: a substance which, however, can be produced only by the vital functions of living cells.[1]

[1] "In 1897 Buchner submitted yeast to great pressure, and isolated a nitrogenous substance, enzymic in character, which he termed 'zymase'. This body is being continually formed in the yeast cell, and decomposes the sugar which has diffused into the cell. . . . In this respect the plasma behaves in a similar manner towards

The following example[1] will prove instructive as showing that the method of difference often requires to be supplemented, and can be supplemented, by the experimental examination of *positive* instances; in other words, by the *experimental* application of the *method of agreement* to a number of positive instances; the object being to determine *what precise factor* in the previously *observed* "invariable antecedent" is really the factor that is relevant or essential to the effect. As early as the thirteenth century Roger Bacon had inferred, by the method of agreement, that the passage of light through transparent globes, prisms, crystals, etc., was probably connected causally with the production of "rainbow" colours. Four centuries later, Sir Isaac Newton commenced his investigation of the phenomenon by an application of the method of difference. "A beam of the sun's light, admitted through a small hole in an otherwise darkened room, produces on a screen a circular image of the sun (negative instance). But on passing the beam through a prism, the image becomes nearly five times as long as it is broad, and is coloured from end to end by a succession of vivid tints (positive instance). Hence *something in the glass* is the cause of the colours [assuming that no other factor was unconsciously introduced simultaneously with the prism]." But what was the "something"? Was it (*a*) the particular size of the prism? (*b*) the particular quality of the glass? (*c*) the particular position in which the prism was held? (*d*) the particular temperature of the glass? (*e*) the substance itself (glass) of the prism? No. It was none of all these; for Newton proceeded to vary all these, to eliminate them successively in a series of experiments, in all of which he retained what he himself suspected to be the essential factor, *viz.* the *prismatic shape* of the various transparent media which he employed. The instances were all positive instances, and the "prismatic shape" (of the medium) was common to all of them. "He eliminated this [presumably in every instance] by placing on the original prism a second one of exactly the same angle, but inverted, so that together the two prisms formed a solid with parallel surfaces"; thus securing in each case a negative instance, and making each experiment an application of the method of difference. He next conceived the hypothesis that white light is really composed of various primary rays which are subject to refraction in unequal degrees—from the red which is least, to the violet which is most, refrangible. This hypothesis he verified by passing each colour of the spectrum separately through a hole in the screen, and then through a second prism: the latter refracting each of the rays in different degrees without further decomposing any of them.

the sugars as does the living yeast cell. . . . According to Buchner the fermentative activity of yeast-cell juice is not due to the presence of living yeast cells, or to the action of living yeast protoplasm, but it is caused by a soluble enzyme. [But other investigators brought to light certain facts which] cannot be explained by the theory that it is a soluble enzyme which brings about the alcoholic fermentation of sugar. The remarkable discoveries of Fischer and Buchner . . . reconcile Liebig's and Pasteur's theories. Although the action of zymase may be regarded as mechanical, this enzyme cannot be produced by any other than living protoplasm."—*Encyclopædia Britannica*, eleventh edit., vol. 10, p. 276.

[1] From BADEN-POWELL'S *History of Natural Philosophy*, quoted by Dr. MELLONE, *op. cit.*, p. 301; *cf. ibid.*, p. 298.

243. METHOD OF CONCOMITANT VARIATIONS. MEASUREMENT. STATISTICS.—The methods we have examined so far are often called *qualitative* methods, as distinct from the present and the next following methods, which are often called *quantitative*. The former enable us merely to discover the presence, and to some extent the nature, quality, or kind, of the causes and effects investigated; the latter enable us to take advantage of exact *quantitative measurements*, to calculate the extent and intensity of the agencies with which we are dealing: an advantage which has contributed enormously to the progress of the physical sciences in modern times. These so-called quantitative methods do not, however, differ in principle from the methods examined; they, too, are practical plans for *eliminating* supposed irrelevant elements from the field of investigation.

The METHOD OF CONCOMITANT VARIATIONS, when applied to phenomena which we can merely *observe*, may be regarded as an improved application of the method of agreement; when applied to phenomena which we can *control by experiment*, it may be considered as a special and more exact application of the method of difference. Its canon is thus formulated by Mill:—

"WHATEVER PHENOMENON VARIES IN ANY MANNER WHENEVER ANOTHER PHENOMENON VARIES IN SOME PARTICULAR MANNER, IS EITHER A CAUSE OR AN EFFECT OF THAT PHENOMENON, OR IS CONNECTED WITH IT THROUGH SOME FACT OF CAUSATION."

In somewhat simpler terms: *If two phenomena always vary together, other circumstances remaining the same or varying independently, there is probably a causal connexion between the two phenomena.*[1] Where we have to rely on simple observation, the conclusion as to a causal connexion is only probable, because in such cases we can rarely if ever be sure that *all* other circumstances *do* remain the same, or vary independently. For the same reason, simple observation cannot of itself *assure* us whether the one phenomenon is the cause of the other, or whether both are joint effects of a common cause. For instance, though observation assures us that "the loudness of a clap of thunder varies with the intensity of a flash of lightning," of itself

[1] MELLONE, *op. cit.*, p. 312. Mr. JOSEPH states it thus: "Nothing is the cause of a phenomenon which varies when it is constant, or is constant when it varies, or varies in no proportionate manner with it" (*op. cit.*, p. 404). *Cf. ibid.*, pp. 517-18, where the author points to the difficulty of applying the rule in the social sciences, in which the discovery of reciprocating causes is practically impossible.

it cannot assure us that "neither is the cause of the other, both alike being effects of the electrical condition of the atmosphere".[1] But when we can experiment we may succeed in removing such sources of uncertainty.

The following examples will illustrate the *experimental* application of the method: (1) The first law of motion states that all bodies in motion tend to remain moving with uniform velocity in a straight line until acted on by some interfering force. How was this law proved? The method of difference could not be applied, for it is impossible to procure a negative instance, *i.e.* an instance with no interfering force and no retardation of uniform rectilinear motion. It was well known that all moving bodies are being constantly influenced by interfering forces of such a kind that *all* of these cannot be *totally* eliminated, *e.g.* gravity, friction, resistance of the atmosphere, etc. How, then, verify the hypothesis that retardation of motion is always due to such interfering influences? Obviously, by trying to *vary* the influence of these obstacles, to diminish it, for instance (since total elimination is impossible), and then see whether, by doing so, the phenomenon itself, the *retardation* of motion, would be diminished proportionately. And this is what Borda did in his experiments with the pendulum. The influence of gravity on the oscillation of a pendulum had already been calculated. If the first law of motion were true, an oscillating pendulum, uninterfered with by any other force than gravity, should remain oscillating indefinitely. Friction at the point of suspension, and atmospheric resistance, were the supposed causes of the retardation. "The simple oscillation . . . which in ordinary circumstances lasts but a few minutes, was prolonged in Borda's experiments to nearly thirty hours, by diminishing as much as possible the friction at the point of suspension, and by making the body oscillate in a space exhausted as nearly as possible of air."[2] The less of the supposed obstacles, the less retardation. "There could, therefore, be no hesitation in assigning the whole of the retardation of motion to the influence of the obstacles."[3] It was concluded, therefore, that in the absence of all interference moving bodies would continue to move with uniform velocity in a straight line. (2) Another simpler experimental example of the method may be exhibited by projecting a body successively, with the same initial velocity, along specially prepared surfaces

[1] FOWLER, *Inductive Logic*, p. 177. [2] MILL, *Logic*, III., viii., § 7. [3] *ibid.*

which vary in roughness, and measuring the rate at which increase of friction (the supposed cause) gradually retards the motion.

From those examples it will be seen that the present method is applicable to a class of experimental cases in which the method of difference proper *cannot* be employed: *viz.* cases in which the supposed causal agency or agencies are what are called "PERMANENT CAUSES," *i.e.* of such a kind that they cannot be *totally* eliminated from any experiment: friction, gravity, heat, electric and magnetic influences, etc. Two instances in which the phenomenon is present in a greater and in a lesser degree may be taken as representing the "positive" and the "negative" instance respectively (the presence of a phenomenon in a lesser degree being really the absence of a greater degree of that phenomenon); and in this way the present method may be regarded as a modification of the method of difference. It perfects the knowledge obtained by the latter method, in so far as this knowledge bears upon the quantitative proportion between cause and effect.

Numerous instruments for measuring are based upon the ascertained concomitant variations of certain natural causes and effects. The thermometer, for instance, depends upon the concomitant variation of heat and the expansion of mercury (or certain substitutes, such as alcohol) in volume; and the barometer on the concomitant variation of atmospheric weight or pressure and the height of a column of mercury (or other fluids) supported by that pressure.

While the method of concomitant variations is more exact than the method of difference, by enabling us to measure the quantitative proportion between cause and effect *within the limits of observation and experiment*,[1] at the same time any inference that the variation will continue at a regular rate *beyond observed limits* is, for the most part, hazardous and unreliable: inasmuch as, under changed conditions, other agencies may become operative.[2] When the concomitant variations of physical phenomena can be *exactly measured within certain limits*, and when we are sure that *all other circumstances are irrelevant*, the variations may be such as to enable us to conclude that the one phenomenon is the *total* and *indispensable* cause of the other within these limits, and that therefore it will continue such *beyond these limits*. Hence, although we can have no experience of perpetual rectilinear motion,

[1] *Cf.* VENN, *Empirical Logic*, pp. 419-20. [2] *Cf.* JOSEPH, *op. cit.*, 463.

or of the total absence of impeding forces, which is its essential condition, we nevertheless infer that, *if* this condition *were* fulfilled, the phenomenon *would* certainly take place. But where the variations within our experience do not reveal in the one phenomenon the *total* and *indispensable* cause of the other; where they do not assure us that in changed conditions no other agencies could possibly interfere: then, of course, we cannot infer an absolutely *universal* and *reciprocating* causal relation. For instance, "though solids and liquids diminish in bulk as heat is withdrawn, they do not diminish at such a rate as to suggest that there is a temperature at which they would vanish altogether".[1] Again, in regard to gases, it is calculated that "they diminish in bulk, when heat is withdrawn, at such a rate that they would vanish altogether at a certain very low temperature ('absolute zero'). But before reaching that point they liquefy. . . . We must not assume [therefore, without proof] that a variation will continue beyond observed limits. Water, to which you communicate heat, up to a certain point only gets hotter; when its temperature reaches 212° [F.] it boils."[2]

Applied experimentally, the method often enables us to establish "laws" in the sense of *exact quantitative statements* of the proportions in which certain factors are found to contribute invariably to a complex total process or phenomenon within the limits of a certain range of experience (227), without at the same time enabling us to *explain* the nature of the *causal* relation of those factors to one another, or to some other cause whether known or unknown. A good example of this use of the method is furnished by the experiments devised to determine the *rate* of the acceleration due to the force of gravity at the earth's surface.[3]

Let us now consider the method of concomitant variations in its application to phenomena which we can merely observe, without controlling experimentally. In the domain of physical phenomena it is particularly applicable to what are called PERIODIC CHANGES, *i.e.* certain regular changes of a phenomenon from a greater to a lesser extent, or degree of intensity, and *vice versa*. For instance, if we observe two neighbouring intermittent springs acting at regular intervals, the one, say, of an hour, the other of four hours, the latter discharging on each occasion about four times as much water as the former; we might infer that probably

[1] *Palaestra Logica*, p. 118, § 361. [2] *ibid.*

[3] *Cf.* FOWLER, *op. cit.*, p. 194.

they came from chambers the one four times as large as the other, each chamber being fed by the same stream. A better example is that furnished by the daily, monthly, and yearly variations in the motions of the tides. The cause of those tidal motions, with their periodic variations, had long been unknown, although some connexion of them with the moon had been suspected for centuries. It was only, however, when Newton formulated the law of universal gravitation, and when in the light of this law the combined action of the sun and the moon upon the waters of the ocean began to be studied, that scientists gradually discovered, in the relative positions and motions of the sun, moon, and earth, *a set of phenomena which varied concomitantly with the variations in the tides*, and which, by their variations, accounted satisfactorily for the latter.

When we have to rely on observation alone, the degree of assurance which the method gives us in any individual inquiry will depend on the likelihood that no other unobserved influences are relevant to the observed variations. In all such cases it may be regarded as a modified application of the method of agreement (single or double). The advantage it has over agreement is this: it enables us to see that the supposed causally connected phenomena are not merely present in a variety of different instances, but that they vary concomitantly in degree throughout these instances. This is better than observing merely the presence (or the total absence) of such factors in a number of instances.

The method is used extensively, by way of observation, in astronomy, in geology, and in the study of climatic phenomena.[1] But its use in the biological, social, political, economic, and commercial sciences, is perhaps still more extensive and important.[2] The extremely unreliable character of some of the inferences based upon it, is due to the fact that owing to the impossibility of sufficient analysis numerous influences really relevant to the varying phenomena remain undetected. Numerous sets of instances exhibiting concomitant variation of the degree of *development in "intelligence"* with the *weight of the brain*—and other sets exhibiting concomitant variation of the former with the *complexity of convolution* in the latter—have been observed and

[1] *Cf.* FOWLER, *op. cit.*, pp. 182-87.

[2] It is *not* the employment of this method, but rather the extensive use of the *argument from analogy*, in certain sciences, that has given currency to the descriptive title "comparative," in reference to such sciences as *e.g.* "*Comparative* Philology," "*Comparative* Anatomy," "*Comparative* Psychology," etc.

tabulated by scientists, in their investigations of the animal kingdom, man included; yet, for the reason just given, it would be rash to hazard an inference that there is any necessary or causal connexion between these varying phenomena in the departments from which the instances have been taken. But, on the other hand, the data to which we can apply the method are often such that it will yield highly probable, and even practically certain, conclusions.

The following example, from Adam Smith's *Wealth of Nations*,[1] illustrates the combined use of *Agreement* and *Concomitant Variations*. The lowness of money prices for goods in ancient times was regarded as due to the poverty and barbarism of the ancient peoples. The author undertakes to prove that low money prices are *not* caused by "poverty and barbarism," but may be due to "the barrenness of the mines supplying the commercial world with gold and silver": (1) "China is a richer country than any part of Europe, yet the value of the precious metals is higher there than anywhere in Europe," and the money prices therefore lower. Here we have "low money prices," without "poverty"; therefore the latter cannot be the cause of the former. No doubt, within the last four or five centuries Europe has grown more wealthy and its money prices have risen: which may account for the impression that low money prices indicate poverty. The increase in wealth has, however, no real connexion with rise in money prices. (2) Increase of wealth is due to "the fall of the feudal system and the growth of public security": influences present throughout Europe, except in Poland, where the feudal system still prevails (*an.* 1793) and which is still poor. But low money prices cannot be due to poverty, because in Poland, notwithstanding its poverty, "the money price of corn (its most important single commodity) has risen" just as in the rest of Europe. (3) In Spain and Portugal, which are also poor countries, money prices have risen as in the richer countries during those centuries. Hence poverty does not account for low money prices. (4) Money prices remained low in ancient times because the output of gold and silver from the world's mines was comparatively small, and these metals were therefore comparatively more precious. But since the discovery of America mines have multiplied: gold and silver have become more plentiful: the purchasing power of these metals has diminished: money prices have therefore risen, especially in Spain and Portugal, which, though poor countries, can command large supplies of these metals from the mines of their American colonies at the cost of a comparatively small expenditure of labour. Thus it was proved *inductively* that low money prices are due, not to "poverty and barbarism" but to the comparative "barrenness of the mines" and the consequent scarcity of the precious metals. The same conclusions could be proved *deductively* from the general principles that "a poor country cannot afford to give a comparatively *large* supply of labour or any other commodity in exchange for a comparatively *small* supply of gold or silver (*i.e.* a low money price)," and that "the purchasing power of gold and silver depends on the amount of labour, energy,

[1] Bk. i., chap. xi., vol. i., p. 365, 7th edit. 1793;—*apud* JOSEPH, *op. cit.*, p. 417.

expenditure of natural resources, with which these metals can be obtained, and will therefore diminish (*i.e.* money prices will rise) according as mines become numerous and fertile ".

In the sciences of observation, especially in the social, political, and economic sciences, the immediate data to which the present method is most fruitfully applied are not merely isolated facts as they come, more or less haphazard, under our notice. What is true of all the methods (240) is true of the present one: the raw materials of ordinary experience have to be *prepared* for its application. And here the preparation will consist in the careful compilation of *statistics.*

We are said to compile statistics when we count or compute the number of instances of the occurrence of a phenomenon—and if possible, also, the measure or degree in which it occurs in each instance—within any selected limits of time and space. Thus, if we measure the rainfall of each of the counties of Ireland for each month, or for each season, during a period of, say, five years, we are preparing, arranging, tabulating, the results of our observations in such a way as to enable us to suspect causal connexions, which might otherwise have remained undetected, between rainfall and other phenomena such as the succession of the seasons, the distribution of land and water in the country, the prevalent direction of the winds, the proximity of the ocean, the existence of mountain ranges, etc. It is mainly by bringing to light the existence of *concomitant variations* between phenomena, that statistics thus suggest causal connexions, or help us to test connexions whose existence may have been already suspected. Observations tabulated in this way enable us to compare the *relative frequency* with which different phenomena occur, and often to *measure* the *relative amounts* of these phenomena, within a given range of time or space. Uniform concomitant variations, brought to light in this way, suggest the existence of some law of causal connexion between the phenomena so varying—and may even convince us that the variations are not mere casual coincidences: may, in other words, *prove* the *existence* of *some causal law*—without at the same time enabling us to determine what *is* the total cause of the variation, or what *are* the partial causes, or any of them, in virtue of which the observed phenomena occur and vary concomitantly as they do.

Suppose, for instance, that intemperance among the poorer classes in a city is observed to vary concomitantly with the squalor

and misery of the overcrowded tenement lodgings in which they have their "homes": we may infer that such variation is not merely *accidental*, that it is *causal*, that it reveals existence of some law of causation; but we may not safely infer that either phenomenon is exclusively the *cause*—whether partial or total—and the other exclusively the effect; for there may be interaction: each may be a partial cause of the other, and each, or both, may be partly due to a combination of many other causes, such as defective early training, religious indifference, thriftlessness, absence of opportunity for healthy recreation, high rents, unemployment, excessive inducements to drink, insufficient control of the drink traffic, etc. Again, suppose "that statistics show a close correspondence between a diminution in convictions for drunkenness and an increase of money in saving-banks—whether in one town in successive years, or in different towns at the same time;—even if other things do not remain the same, we should be justified in concluding that both improvements, if not actually cause and effect, depended on some common cause, improvement in wages or education".[1]

244. METHOD OF RESIDUES. "CONJUNCTION OF CAUSES," AND "INTERMIXTURE OF EFFECTS".—From all that has been said, so far, about perceptual and experimental analysis, it will be evident that in all cases we endeavour to make use of whatever knowledge we already possess about the phenomena under investigation, and about the whole department of facts to which they belong. Such previous knowledge may help us in various ways—for instance, by suggesting hypotheses to be tested, and the various grounds of elimination to be applied successively in testing them. Now, there is a special sort of consideration which will help further analysis *when we already know a great deal*, comparatively speaking, about the nature and the exact measure of the causal agencies and effects which make up the whole sphere of facts under observation. It may, perhaps, for the sake of simplicity, be represented in this way. If, in a whole complex process, *abcdefgh*, we know already that the causal relations between *abcd* and *efgh*, between *a* and *e*, between *b* and *f*, between *c* and *g*, are all reciprocating causal relations; then we can infer a similar relation between *d* and *h*. Or, if *h* be absent from the whole complex event—that is, if there be nothing in the event to account for *d*: which, in all such cases, is described as a RESIDUAL PHENOMENON

[1] *Palaestra Logica*, p. 113, § 353.

—we can infer that there must be, connected with the whole event, something that causes *d;* and we can then proceed to search for this something in the surroundings of the event. No doubt, our knowledge of a complex event is rarely so perfect as this symbolism implies; the latter represents rather an ideal. But it expresses the sort of consideration embodied in the *Method of Residues.* It shows us, too, that the analysis here effected is *mental* rather than *actual*, and that *deductive* inference from our previous knowledge is the predominant feature of this analysis.

We may now state the rule or canon which is the ground of the analysis. It is formulated by Mill in this way:—

"SUBDUCT FROM ANY PHENOMENON SUCH PART AS IS KNOWN BY PREVIOUS INDUCTIONS TO BE THE EFFECT OF CERTAIN ANTECEDENTS, AND THE RESIDUE OF THE PHENOMENON IS THE EFFECT OF THE REMAINING ANTECEDENTS."

It is stated more briefly, thus, by Mr. Joseph:[1] "*Nothing is the cause of one phenomenon which is known to be the cause of a different phenomenon.*" To suit cases where no antecedent is forthcoming, within the ambit of the complex event itself, for an unexplained residue, Dr. Mellone[2] formulates the following rule, which becomes, in such cases, a "*finger-post to the unexplained*": "*When any part of a complex phenomenon is still unexplained by the causes which have been assigned, a further cause for this remainder must be sought*".

The method can be applied both to experimental cases and to cases of simple observation. As applied to the former it is a *quantitative* method, its value depending on the degree of exactness with which we can make use of measurement. It is extensively applied in this way to experiments in chemical analysis. The following example, from Jevons' *Elementary Lessons in Logic*[3] will illustrate the use of it. "Thus, the composition of water is ascertained by taking a known weight of oxide of copper [C], passing hydrogen [H] over it in a heated tube [T^1] and condensing the water [W] produced in a tube [T^2] containing sulphuric acid [in known weight, S]. If we subtract the original

[1] *op. cit.*, p. 404. His formula emphasizes the fact that reciprocating causes only are in contemplation. The author formulates (*ibid.* n.) other grounds of elimination applicable to causes that are non-reciprocating either because they contain too little (partial causes, *sine qua* conditions), or too much, for the effect. These are eminently instructive, *e.g.* that from Hume's *Treatise, etc.*, pt. III., xv.—"Where several different objects produce the same effect, it must be by means of some quality, which we discover to be common amongst them".

[2] *op. cit.*, p. 315.

[3] p. 254.

weight of the condensing tube $[T^2 + S]$ from its final weight $[T^2 + S + W]$ we learn how much water is produced; the quantity of oxygen $[O]$ in it is found by subtracting the final weight of the oxide of copper $[C - O]$ from its original weight $[C]$. If we then subtract the weight of the oxygen $[O]$ from that of the water $[W]$ we learn the weight of the hydrogen $[H]$ which we have combined with the oxygen $[O]$. When the experiment is very carefully performed, as described in Dr. Roscoe's *Lessons in Elementary Chemistry* (p. 38), we find that 88·89 parts by weight of oxygen unite with 11·11 parts by weight of hydrogen to form 100 parts of water." Thus we have:

Total weight (initial stage) $= [T^1 + C] + [T^2 + S]$
 " " (final stage) $= [T^1 + C - O] + [T^2 + S + W]$

Therefore the difference $[W - O]$ must represent the added hydrogen $[H]$. That is to say, we know how to account for the total weight at the initial stage; we then add one single antecedent, namely hydrogen; therefore the excess of the final weight over the initial weight must be due to the added hydrogen. From this we can understand why the present method has been described as a peculiar application of the method of difference.

It will also enable us to understand why the method is applicable only to a *homogeneous* intermixture of the effects of several co-operating causes, or to some *homogeneous aspect* of these effects, such as *weight* in the example given. Obviously, it is only when several causes act at once, and when we have before u the complex resulting effect of their co-operation, that the method can be used at all. But we have to distinguish two ways in which a number of causes may co-operate in the production of a total effect. This latter may be either of the *same kind* as the effects of the single causes acting separately would be; or it may be of a *different kind* from the single effects. In other words, any "CONJUNCTION OF CAUSES," *i.e.* joint action, simultaneous co-operation, of a number of causes, may be either a simple adding together of these causes, a "COMPOSITION OF CAUSES," to produce a "HOMOGENEOUS INTERMIXTURE OF EFFECTS"; or it may be a mutually modified co-operation of these causes, a "COMBINATION OF CAUSES," to produce a "HETEROGENEOUS OF HETEROPATHIC INTERMIXTURE OF EFFECTS". Chemical changes, for instance, are *heterogeneous* or *heteropathic* effects of the combining agencies which produce them. On the other hand, "if in one experiment friction, combustion, compression, and electric

action are all going on at once, each of these causes will produce quantities of heat which will be added together . . . We may call this a case of *homogeneous intermixture of effects.*"[1] And it is only to such cases the method of residues can be applied. "There cannot be a simpler case . . . than ascertaining the exact weight of any commodity in a cart, by weighing the cart and load, and then subtracting the tare or weight of the cart alone, which had been previously ascertained."[2]

The fact has been already emphasized, that it is impossible to regard processes of causation as made up of discontinuous factors (240); but we must, if we are to understand them, introduce discontinuity by the mental abstraction involved in analysis. So, when we do isolate them mentally, we must not forget that in the concrete actuality the factors are not independent of one another. It is this mutual interdependence and interference of causes that make inductive research so difficult. When causes "combine" to produce "heteropathic" effects, experiment alone will enable us to detect the nature of the separate influence of each, and the various kinds of effect each will produce in combination with others. When causes co-operate by way of simple "composition" to produce "homogeneous" effects, the action of any one "may be (*a*) augmented, (*b*) diminished, (*c*) diverted, (*e*) wholly counteracted, by that of another cause;[3] and all these various kinds of interaction may occur at the same time among the effects. Fowler observes truly that in every case *each cause produces its appropriate effect*, even though it may have disappeared wholly or partly in the total result. . . . When the full consequences of a Law are thus affected (modified or neutralised) by other Laws, it is called a *tendency*." A "tendency," therefore, is the action of a causal influence considered as impeded by an opposing causal influence.

The method of residues is employed extensively, by way of experiment, "in making allowance for the errors or necessary corrections in observations. Few thermometers are quite correct; but if we put a thermometer into melting snow, which has exactly the temperature of 0° Centigrade or 32° Fahr., we can observe exactly how much . . . we ought to add or subtract from the readings of the thermometer to make them correct".[4] Here the discrepancy between the actual level of the mercury and the point marked 0° C., or 32° F., may be regarded as a residual phenomenon, whose explanation is to be found in the erroneous graduation of the instrument.

[1] JEVONS, *op. cit.*, p. 252. [2] *ibid.*, p. 253.

[3] "Thus, for simple examples we may suppose a body (*a*) pulled by two forces in exactly the same direction; (*b*) pulled by two forces of different magnitudes in exactly opposite directions; (*c*) pulled in one direction by one force, and by another force pulled in a direction at right angles to the former; (*d*) pulled by two *equal* forces in exactly opposite directions, when no motion takes place".—MELLONE, *op. cit.*, p. 317, n.

[4] JEVONS, *op. cit.*, p. 253.

The use of the method of residues as a "*finger-post to the unexplained*," by the study of residual phenomena, has proved most fruitful in scientific research, and deserves particular attention. Applied in this way, it is obviously a method of *discovery* rather than of *proof*, a source of hypotheses rather than a means of testing them. In sciences like astronomy and chemistry, where exact calculations and measurements are extensively employed, very remarkable discoveries have been made by the application of it.

"Almost all the greatest discoveries in astronomy," writes Sir John Herschel,[1] "have resulted from the consideration of residual phenomena of a quantitative or numerical kind." The discovery of the planet Neptune by Adams and Leverrier in 1846 is a striking example. By calculating the effects of all known attractions on the planet Uranus, the path of the latter in the heavens was determined. This planet was actually found, however, to be sometimes before and sometimes behind the place calculated for it. It was concluded that the discrepancy, the "residual effect," must be due to the attractive influence of some unknown heavenly body. The search for this latter resulted in the discovery of a hitherto unknown planet—Neptune.

In the domain of chemistry, we may instance the discovery, by Lord Rayleigh and Professor Ramsay, in 1894, of a chemical element, which they called *argon*, present in the atmosphere in very minute quantities. "The investigation started from the detection of an unexplained residual phenomenon. Careful determination of density had shown that nitrogen obtained from various chemical compounds is of a uniform density, but that 'atmospheric' nitrogen is about ½ per cent. heavier".[2] In explanation of this residual phenomenon numerous hypotheses were advanced, and a long series of experiments culminated in the discovery of the new chemical element.

245. SCOPE OF THE "METHODS"; USE OF SYMBOLS.—The various modes of procedure that have been outlined under the title of "methods," are simply applications of various grounds of elimination, whereby we exclude irrelevant elements until the causal connexion alone is left. They *presuppose* the dividing up of our domain of inquiry into distinct factors, when mapping out the field for possible hypotheses and forming the disjunctive judgment among the alternatives of which we hope to find the cause: "x is caused by either a, or b, or c, or . . ., or m" (212). They

[1] *apud* MELLONE, *op. cit.*, p. 316. [2] *Cf.* WELTON, *op. cit.*, pp. 133-40.

then find their application, one or other or all of them, in excluding successively *a*, and *b*, and *c*, . . . until *m* alone is left: that is, in establishing minor premisses for our alternative major, "The cause of *x* is not *a*," "The cause of *x* is not *b*," etc., until we are thus enabled indirectly to prove that "The cause of *x* is *m*". In proving each minor premiss, by the application of the "methods," there is deductive reasoning from the hypothesis that is being tested. It is a mistake, therefore, to contrast the principles involved in these methods with the various principles or axioms of deductive inference, by describing those as "material" and these as "formal". Nor can the rules for elimination be called "inductive" in the sense that they enable us to generalize *directly* from the individual data of our experience, *independently* of hypothesis and deductive reasoning: for they do not enable us to generalize in this way. It was Mill's mistake to think that they do; though he seems to have felt the difficulty of such a position: for he admits the utility of hypothesis and deductive reasoning in what he calls the "deductive method";[1] and although he restricts the application of the latter to the more complex sort of phenomena, he is forced to admit that it is to this method "the human mind is indebted for its most conspicuous triumphs in the investigation of nature"[2]. Indeed, even the simplest phenomena of concrete nature are far too complex to admit of the *direct* application of any one of the "methods" to them. "The only true function of the methods is, indeed, given by Mill himself when, in speaking of cases in which there is an 'interference of causes with one another,' he says: 'The instrument of Deduction alone is adequate to unravel the complexities proceeding from this source; and the four methods have little more in their power than to supply premises for, and a verification of, our deductions' (III., x., § 3)".[3]

Attention has already been called (240) to the fact that the application of the so-called "inductive canons" supposes the materials already analysed and prepared, "that limited groups of antecedents and consequents, known to be causally connected, have been separated out for the purpose of 'inductive' inquiry, whose task is only to obtain simple causal connexions by eliminating some of the elements still left. . . . Whewell is, indeed, quite justified when he says: 'Upon these methods, the obvious

[1] *Logic*, III., xi., § 93. *Cf. supra*, p. 45.
[2] *ibid.*; *cf.* WELTON, *op. cit.*, ii., pp. 59, 83. [3] WELTON, *ibid.*, pp. 158-9,

thing to remark is, that they take for granted the very thing which is most difficult to discover, the reduction of the phenomena to formulæ such as are here presented to us' (*Phil. of Discovery*, p. 263)."[1]

The symbolizing of the methods by employing the letters of the alphabet to represent the various elements that enter into a process of inductive analysis has its advantages and its disadvantages. Its chief advantage is, perhaps, that it helps us to realize more clearly what we are aiming at in each of the methods. But, on the other hand, it is liable to create false impressions about the nature of the analysis symbolized: the impression that the work of analysis is simpler than it really is; the impression, too, that the elements with which we are dealing are distinct, isolated, independent, individual entities, like the symbols themselves. The system of symbols employed by Mill has an additional drawback: by employing corresponding capitals and small letters for the antecedents and consequents, he conveys the impression that nature presents us with corresponding connexions between the elements of our experience; whereas in reality these connexions have to be discovered by ourselves. The following are his formulæ for the various methods :—

Method of Agreement :—

A B C followed by *a b c*,
A D E ,, ,, *a d e*,
∴ A and *a* are causally connected.

Method of Difference :—

A B C followed by *a b c*,
B C ,, ,, *b c*,
∴ A and *a* are causally connected.

Joint Method of Agreement and Differences.—Mill himself gives no formula, but, following the lines of those just given, we arrive at some such scheme as the following :—

$$\left.\begin{array}{l}\text{A B C}—a\ b\ c\\ \text{A D E}—a\ d\ e\\ \text{A F G}—a\ f\ g\end{array}\right\}\text{ positive instances ;}$$

$$\left.\begin{array}{l}\text{B M N}—b\ m\ n\\ \text{D O P}—d\ o\ p\\ \text{F Q R}—f\ q\ r\end{array}\right\}\text{ negative instances ;}$$

∴ A and *a* are causally connected.

Method of Concomitant Variations :—

A^1 B C—a^1 *b c*,
A^2 B C—a^2 *b c*,
A^3 B C—a^3 *b c*,
∴ A and *a* are causally connected.

Method of Residues :—

A B C D is the *known cause* of *a b c d*
A ,, ,, ,, *a*
B ,, ,, ,, *b*
C ,, ,, ,, *c*
∴ D and *d* are causally connected.

[1] WELTON, *ibid.*, pp. 148-9.

It would be superfluous to show how far this abstract symbolism falls short of picturing the complexity of the concrete facts and processes it is supposed to represent. Nor is it of sufficient importance to call for any elaborate attempt at improvement. Logicians have, however, improved it in various ways, while pointing out its many shortcomings.

The separation of the elements into antecedents and consequents has to be effected by the investigator; and he must bear in mind that the actual influence of the cause is not prior in time to, but is simultaneous with, the actual production of the effect. To illustrate these points, we may instance Dr. Venn's manner of symbolizing the *Joint Method*:[1]—

"Surely what we want is something of the following kind. Let x be some phenomenon in regard to which eleven antecedents, *viz.* A to K are to be taken account of, and suppose that we have collected the following sets of affirmative and negative instances:—

Affirmative	*Negative*
A B C D E	B C F G
A D E F G	D E H I
A F G H I	F G J K
A H I J K	H I D E

"It is clear that A is the only element, of the given lot of eleven, which is always present in the former set of instances, and the only one which is always absent in the latter set. If we knew for certain that there could be only one cause, then clearly A is that one. So much indeed is established by the affirmative instances. What the negative instances do is to disprove one after another of the alternative causes other than A. It might be that A was not a cause at all, but that B, D, F, and H had respectively been at work in producing x in the four cases in question. The negative instances disprove this, however; and since they take account of every one of the ten letters,—or cause symbols,—B to K, and show that no one of these is operative, we are led to conclude that A alone is the cause which we are seeking."

A theoretical knowledge of the method of inductive analysis, by way of observation, experiment, and hypothesis, will help the scientific investigator in his work; but it will not determine for him which, or how many, of the rules of elimination he must employ in any given subject-matter, or in what order. This can be determined only by his own insight, prudence, and knowledge of the facts with which he is dealing. The progress of scientific discovery involves the use of all of them.

We may give here the following final example, which will illustrate the combination of three of the "methods" to form a series of positive and negative experiments for the purpose of verifying the hypothesis that life proceeds only from life, and never appears by *spontaneous generation*:[2] Let us assume that the reputed "spontaneous" production of minute living organisms is really due

[1] *Empirical Logic*, pp. 430-1.

[2] They are the experiments actually performed by Pasteur. *Cf.* JANET, *Traité de philosophie*, n. 146. Paris, Delgrave, 1879.

to the presence of living germs held in suspension in the atmosphere; that these germs find in fermentible liquids, or in decomposing organic matter, suitable media for multiplication. How are we to verify our hypothesis?

1. We expose to the air a number of vessls all filled with such liquids, and we find that wherever such germs can have access to them they ferment: the supposed "spontaneous" generation takes place. Here we have the *method of agreement*.

2. Next, we secure other vessels similarly filled and take care to seal them hermetically in the purest atmosphere available,—upon the Alps, for example, where there is little danger of encountering such germs. We lay the vessels aside, and there they remain indefinitely without any sign of fermentation—evidently free from any living organism. Here we have *negative instances* which *agree* in the absence of the supposed cause; and which, together with the positive instances in (1), illustrate the *combination* of *agreement* and *difference*.

3. We next expose to the air a variety of similarly filled vessels in a variety of different localities where the atmosphere is tainted in different degrees by the presence of such germs; for example, in cellars where the air is cold and still and all the germs probably settled, or far up on some high mountain where the air is most likely free from organic impurities, or in the hot, tainted atmosphere of a city where germs of all sorts abound, or in the moist shade of trees or shrubs, in the summer months, when the air is known to swarm with living microbes: we find the rapidity of fermentation, and the amount of living organisms produced in the liquids, to be in proportion to the degree of impurity of the atmosphere in each case. Here we have the *Method of Concomitant Variations*. From which experiments we infer that life is caused in all cases by the multiplication of some existing living thing, and, negatively, that non-living matter cannot give rise to life (*cf.* 229).

Living things, therefore, must be endowed with the *natural property* of reproducing their kind upon the earth. The *natural* origin of a new living organism is an already existing organism: *Omne vivum ab ovo; omnis cellula ex cellula*.

246. QUANTITATIVE DETERMINATION: MODES OF MEASUREMENT.—Very little reflection will show that the progress of discovery in the physical sciences has been proportionate to the degree in which exact measurement of phenomena has been found to be feasable.[1] The sciences of physics and chemistry have passed gradually from the study of qualities to the study of quantities: their laws are now not regarded as exact until they are stated quantitatively, or in mathematical formulæ. But knowledge of the quantitative aspect of things is not *all* knowledge of them; and, moreover, there are vast departments of facts whose quantitative aspect is of very minor importance to a scientific knowledge of them. Hence, the attempt made in the last century to reduce *all* the sciences to terms of mechanics (201, 217, 224,

[1] *Cf.* L. POINCARÉ, *The New Physics* (International Scientific Series), chap. ii.; JOYCE, *Principles of Logic*, pp. 363 *sqq.*; WELTON, *op. cit.*, pp. 160-5, 182-7.

229) only brought discredit on a procedure that is perfectly legitimate within proper bounds. What these bounds are, a glance at the nature of measurement, and its units and standards, will immediately disclose.

All measurement is *observation*—and something more. As observation, it is inseparably connected with *interpretation* of the data offered to us; and interpretation, being judgment, is subject to error. But measurement is more than observation; it is *comparison* of the observed fact with some other fact taken as *standard* or *unit*. And this comparison is not merely mental; it implies direct application of the unit or standard to the measurable magnitude, and *sense-observation* of the result of such application. The accuracy of measurement, therefore, ultimately depends on the acuteness of our powers of sense-perception. But these powers are trustworthy only within a certain range; and even within this range they are not infallible. There are, for all the senses, what are called *minima sensibilia:* objects so small that our senses are not sufficiently delicate to become aware of any smaller. No doubt, our powers of observation have been increased by such mechanical devices as the telescope and the microscope; and our powers of *measuring*, *i.e.* counting the application of a unit to a quantity, are aided by such delicate instruments as the micrometer screw, the electrometer, the bolometer,[1] etc. But, even still, there is a limit to the keenness of our sensibility; so that *every actual measurement is, normally speaking, only an approximation to the real state of the facts.* By accident, no doubt, a measurement may be just accurate; but, even when it is, we cannot be sure of this. So it is no exaggeration to say, with Jevons,[2] that "we may look upon the existence of error in all measurements as the normal state of things".

To fix upon certain fundamental and derived units for measuring various physical magnitudes, units of spatial extension (distance, area, volume), of time, of mass, of velocity or rate of motion, of acceleration, of mechanical energy, of temperature, of electromotive force, of strength of electric current, etc.; to devise means of applying these units with a greater approximation to

[1] "Langley's bolometer can detect a change of temperature of one hundred-millionth of a degree C." JOYCE, *op. cit.*, p. 364. So, too, in measuring mass, "Metrology vouches for the hundredth of a milligramme in a kilogramme; that is to say, it estimates the hundred-millionth part of the magnitude studied".—POINCARÉ, *op. cit.*, p. 33.

[2] *Principles of Science*, p. 357.

accuracy; and means of allowing for unavoidable deviations: such are the main functions of what is nowadays really a distinct science and art, known as *Metrology*.[1]

It would be beyond the purpose of a treatise on general logic to enter on the interesting but intricate questions involved in determining the various units of measurement. It will be sufficient to observe here that all actual measurement is *relative*, *i.e.* it is the perception of some magnitude, *not absolutely, but as compared with the unit magnitude;* that the unit magnitude itself cannot, as such, be measured; that for its constancy—on which its accuracy as a unit depends—our only guarantee is our own sense-perception, aided by whatever devices mechanical science can offer us; that these devices, though improving our powers of perception, do not make the latter perfect; while the devices themselves may contain errors, or sources of error, peculiar to themselves.

It is a problem of great practical importance, how to minimize the inaccuracies of measurement, and how best to make allowance for the residual error which cannot be eliminated. The needful degree of accuracy will, of course, depend on the nature of the case under consideration. In building an iron bridge the engineer must allow space for expansion due to heat; else the force of the expanding metal would burst the whole structure. For this purpose, however, he does not need to determine the coefficient of expansion of his materials with anything like the precision and exactitude required by the maker of a naval chronometer in regard to the compensation springs and balances for this article. But, in all cases, it is the ideal of the scientist to eliminate all error, as far as possible, from his calculations. How, then, is he to deal with the various inaccuracies incidental to measurement? In the first place, he must make allowance for them as far as he is aware of their existence. In physical experiments the possible sources of incorrect measurement are manifold: variations of temperature, friction, atmospheric pressure, electrical or magnetic conditions; defects in the measuring instruments, *e.g.* through want of rigidity in the telescope, through refraction in all optical instruments, through inaccurate adjustment in chronometers, etc. Then, besides all these, there are the sense-limitations of the individual observer, often involving deviations "in the same direction and to the same average

[1] Poincaré, *op. cit.*, p. 20.

amount"[1] from the normal, and hence called the "personal equation" or "personal error": *e.g.* "the inclination to record the passage of a star across the wires of the telescope a little too soon or a little too late. . . . The difference between the judgment of observers at the Greenwich Observatory usually varies from $\frac{1}{100}$ to $\frac{1}{3}$ of a second, and remains pretty constant for the same observers."[2] These various sources of inaccuracy can be to some extent detected, and their disturbing influence eliminated by calculation. But when all this has been done, when all possible precautions have been taken, it will still be found that in the case of delicate measurements a series of equally reliable trials, even when made by the same individual, will never yield exactly, but only approximately, the same results:[3] thus showing that, on account of *unavoidable* interfering influences, the *observed* or *registered* magnitudes are only approximations to the *true* magnitude.

Now it may be safely assumed, in the absence of any ground for suspecting the contrary, that the various unknown sources of discrepancy, in the results of such a series of measurements of any magnitude, *will tend to neutralize each other*, as being indifferent to excess or defect; that the true magnitude will, therefore, lie somewhere between the greatest and the least registered magnitudes; that by taking the *mean* of all the registered magnitudes we are approximating to the true magnitude; and that this approximation will be closer the larger the series of measurements which furnish this mean. From the assumption that the sources of error yield excess and defect indifferently in the individual measurements which make up the series, the inference is unavoidable that the oscillations on each side of the true magnitude will tend to balance each other in the long run: so that the mean will tend in the long run to coincide with the true magnitude.

There are two leading methods of determining the "mean" (of a number of measurements) which is most likely to approximate most closely to the true magnitude. The first of these—which is called the "METHOD OF MEANS" —is applicable only when we are engaged in measuring some *single magnitude*, such as the length of a line. It consists simply in taking the *Arithmetical*, or the *Geometrical*, *Mean* of the actual measurements. Usually, the arithmetical

[1] JEVONS, *Principles of Science*, p. 347. [2] *ibid.*

[3] A result which shows a *notable* discrepancy from all the others of the series should be rejected as being presumably vitiated by some *extraordinary*, though undetected, error in the measurement.

mean is taken. "When, however, as happens in a few cases, some condition is known to be operative which varies as the square of the distance, the Geometrical Mean[1]—i.e. $\sqrt{ab}$—gives the more accurate result. For example, if the true weight of an object is sought by weighing it successively in the two scales of an imperfect balance, as gravity is an operative condition, the true result will be the geometrical mean . . . though as, in small numbers, this differs but little from the arithmetical mean, the latter is generally taken as more easily calculated."[2]

The second method, called the "METHOD OF LEAST SQUARES," is applicable when, as is usually the case, the magnitude we are measuring is *compound*, or, in other words, involves measurements of two or more separate quantities one of which is some function of another. Our total measurement will, in such cases, be a combination of two or more distinct series, such as $[P_1 + P_2 + P_3 + P_4 + \ldots] + [Q_1 + Q_2 + Q_3 + Q_4 + \ldots] + [R_1 + R_2 + R_3 + R_4 + \ldots]$, where P is perhaps some multiple of Q, and Q perhaps some function, such as the square root, of R. Now, we cannot reduce these three series to one, so as to take the arithmetical or geometrical mean of the whole; because there is already a parity between the errors of the three series, and hence by squaring or otherwise altering the values of any single series, for the purpose of reducing it to terms of another, we alter the value of its errors as compared with those of the other series. We therefore have recourse to the Method of Least Squares, which rests upon this theorem: *That magnitude is most probably the true magnitude, which makes the sum of the squares of the errors in the actual measurements the* LEAST POSSIBLE. Such a magnitude can always be discovered, from the actual values, by an algebraic process. This method is really "an extension of the method of means, in that it indicates the most probable mean in cases which involve a plurality of arithmetical means".[3] The property of having the sum of the squares of the residual errors the *least possible* is always true of the arithmetical mean[4]; but it is also true of the mean magnitude of a *compound* series, a magnitude which cannot be obtained by the simple process of finding the arithmetical mean. The present method is, therefore, "the most general mode of finding the true magnitude from a number of divergent measurements; but when these measurements involve one magnitude only, the simplest mode of applying the method is to take the arithmetical mean".[5]

247. "EMPIRICAL LAWS" AND THEIR EXPLANATION: TRANSITION TO PART V.—In the present chapter an account has been given of the analytical process by which we seek to discover and establish *laws* by way of *hypothesis;* and the conditions requisite for the latter were outlined in the last preceding

[1] Rather than the Arithmetical Mean, $\frac{a+b}{2}$.

[2] WELTON *op. cit.*, pp. 184-5.

[3] *ibid.*, p. 186.

[4] For example, the sum of the squares on the residuals in the series 2, 5, 8 [$(5-2)^2 + (8-5)^2 = 18$] where the middle value is the arithmetical mean, is less than in the series 2, 4, 8 [$(4-2)^2 + (8-4)^2 = 20$) where the intermediate value is not the arithmetical mean.

[5] WELTON, *op. cit.*, p. 187.

chapter. We have seen, too, that all laws are embodied in *general* judgments, and that induction is a process of *generalization*, based on analysis of *individual facts*. We may distinguish, therefore, between knowledge of particular "facts" and knowledge of the "laws" reached by generalizing from these facts.

In regard to any individual fact, we may know simply *that* it *is* so, or we may know further *why* it *must be* so (*cur?* and *quomodo?*). We know the latter if we know the *causes* which produce the fact, and the *laws* according to which they produce it. But the same is true of the laws themselves. If we conceive a "law" simply as the *generalization* of a fact beyond the range of experience, simply as a statement that a certain kind of fact *did* and *does* and *will* happen, always and everywhere, in a uniform manner in similar circumstances, in a word, simply as an assertion that some observed uniformity of fact holds good generally, then, since "law" in this sense is merely *a wider fact*, we may know of it too, in turn, either *that* it is so, or we may know also *why* it is so.

Furthermore, confining our attention here to this conception of a law, our knowledge *that* the law is so, *i.e.* that the fact *does* happen uniformly beyond our experience as well as within our experience—this knowledge may vary from a very slender degree of probability to physical certitude. Our confidence in the universal truth of such a law, our belief in its reliability, will vary with the degree in which analysis of the facts points to its truth. (*a*) It may be a rough generalization from uncontradicted experience, based upon mere enumeration of instances, without any attempt at analysis, *e.g.* "All crows are black". (*b*) It may have been observed to hold good in *varied* instances, in accordance with the method of agreement, *e.g.* "All horned animals are ruminants," "All ruminants are cloven-footed". (*c*) It may be a tentative extension of some known law to a new set of cases by *analogy*, *e.g.* "All animals having their habitat in the Arctic regions are, like the polar bear, white-coloured". (*d*) It may be a *supposed* causal connexion, *partially tested* by the application of one or more of the modes of analysis already described (240-4), but not yet *fully verified*: an hypothesis in process of verification. Of this latter we have had numerous examples. Now, generalizations under any one of those various heads have this in common, that they are all *more or less probable*; but, since they are not certain, they *cannot be safely extended to cases that are not "adjacent,"*

i.e. similar to the observed cases. We hesitate to apply them to cases that differ considerably in their conditions from those we have observed. Every such "law" or "generalization" is usually called an "EMPIRICAL LAW," or an "EMPIRICAL GENERALIZATION". By these expressions we mean some observed uniformity of connexion between phenomena, or of the mode in which a phenomenon happens, which uniformity we *expect* to hold good always and everywhere, though we do not understand it sufficiently to *be sure* that it will hold good.

But suppose we have *verified* our hypothetical law by a sufficient application of the various grounds of elimination embodied in the "methods" already set forth; and that we are therefore *certain* that our established causal connexion will *de facto* hold good universally, that no other supposition would be consistent with the facts of our experience: we may still be unable to *explain why* such a connexion *must* hold good universally, or, in other words, to connect the law in question with other laws, and show how it is necessarily involved in, and derived from, these. The phenomena connected by such a law "we see to be connected, though how they are connected we know not,"[1] because we cannot *explain* the law that connects them. "They are connected for us empirically, that is, in our experience;"[2] they are connected for us *to some extent* even "rationally, that is, for our intelligence,"[3] inasmuch as we have convinced ourselves that the connexion is not merely a *contingent* connexion which occurs within our experience, but that it is in some sense a *necessary* connexion which must hold universally; but we have not a *full intelligence*, a *full rational knowledge* of the connexion, so long as we are unable to see *why* the law connecting the phenomena must be so. The law, though verified, is still for us an *isolated* law, not connected "rationally," "logically," "scientifically," "philosophically," with any wider laws; in other words, not "scientifically explained". Now, it is in accordance with fairly common usage to describe also such *verified* but *unexplained* laws as *Empirical Laws*, or *Empirical Generalizations*, like the probable, *unverified* generalizations described in the preceding paragraph.

It is evident, therefore, that laws may exist merely as empirical laws for a long time before they are verified, and again for a long

[1] JOYCE, *op. cit.*, p. 369. [2] *ibid.*, p. 370. [3] *ibid.*

time as verified before they are explained. When, at length, they are explained by deriving them deductively from wider laws, they are called DERIVATIVE LAWS, or SCIENTIFIC LAWS, simply. Thus, it is a function of science *to convert empirical laws into derivative scientific laws*, by connecting them deductively with more universal laws of causation.

But the "explanation" of these wider and more remote laws of causation themselves is a matter of greater difficulty. We have seen already in what a qualified sense we must understand the sort of "verification" that can be attained when there is question of such laws (228-33). The law of universal gravitation, the law of the conservation of energy, the first law of motion, the law of biogenesis ("*omnis cellula ex cellula vivente*"), etc., are regarded as "verified," merely because they explain facts more satisfactorily than any conceivable alternative (230), and "on so wide a scale that it is very unlikely that exceptions to them exist".[1] Very wide laws of this kind, "not yet *explained*"[2] themselves, but verified by their extensive power of explaining both narrower laws and individual facts, are called by scientists LAWS OF NATURE simply, in contradistinction to the narrower or derivative laws[3] just referred to. But those wider laws too call for "explanation".

The human mind is not satisfied to take these laws, whether wider or narrower, as mere actual uniformities. Some logicians seem to identify laws with facts:[4] but law is something more than fact: fact is contingent, at least all phenomenal or empirical fact;[5] whereas law involves the idea of a necessity of some sort, the notion of what *must be*, rather than of what *is*. And Mr. Joseph rightly says: "it must not be imagined that uniformity is the *fundamental* element in the causal connexion, but *necessity* or *law*".[6] The philosophical explanation of laws—that is, the sort of explanation ultimately attainable by the human mind—leads, therefore, inevitably to inquiries into the nature of the necessity attaching to "laws," "necessary truths," "principles," and "axioms"; into the ideally perfect form of scientific knowledge and scientific certitude; into the ideal conditions for "scientific explana-

[1] *Palaestra Logica*, p. 127, § 393. [2] *ibid.* [3] *Cf.* JOSEPH, *op. cit.*, pp. 380-1.

[4] The Empirical school of philosophers, of which Mill is a typical representative, must consistently do so. Dr. Mellone, though dissenting from this attitude, says "the truth is, to 'explain' a fact, in science, comes in the last resort only to this, that we show it to be part of a wider fact" (*op. cit.*, p. 319). The *scientist* may, perhaps, be satisfied with this; the inquiring human mind certainly is not: it wants to know further whether, or how far, or in what sense, every fact that is *must be so*.

[5] There is only one absolutely necessary fact, *viz*. the Necessary, Self-existent Being, God.—*Cf.* RICKABY, *First Principles*, p. 89.

[6] *op. cit.*, p. 376.

tion" and "demonstration". Although such inquiries have been to some extent anticipated in the various chapters of the present part of this volume, they call for a little more explicit treatment.

From another point of view, we may regard the attainment of *certitude* or *certain knowledge* as the goal of all scientific effort; and we may regard it as the practical aim of logic to prescribe the conditions for attaining to this ideal. But, in the progress which the human mind may make towards this latter, it may in some matters succeed in reaching *certitude;* in others it may reach only *opinion or probability;* and in others again it may fail altogether by falling into *error.* A consideration of these conditions, and their causes, will, therefore, form the subject-matter of the remaining portion of our general inquiry.

WELTON, *op. cit.*, bk. v., chaps. v., vi. and vii. JOSEPH, *op. cit.*, chap. xx., xxii. and xxvi. MILL, *Logic*, III., chaps. vii.-x. MELLONE, *op. cit.* chap. ix. VENN, *Empirical Logic*, chap. xvii. *Palaestra Logica*, part iii., chaps. iii., iv. and v. FOWLER, *Inductive Logic*, chap. iii. JOYCE, *Logic*, chaps. xx. and xxiii.

PART V.

THE ATTAINMENT OF SCIENCE AND CERTITUDE.

CHAPTER I.

SCIENCE AND DEMONSTRATION.

248. ELEMENTARY NOTIONS DEFINED: TRUTH, IGNORANCE, ERROR, EVIDENCE, CERTITUDE, OPINION, PROBABILITY, DOUBT.—The terms "true," and "truth," are applied to the things we know, to our knowledge of them, and to the language in which we express this knowledge. Truth as applied to *things* or reality, is called *real* or *ontological* truth. It is simply the things, or reality, as revealed or manifested to some mind, and thus related to that mind. It is identical with reality, and is the proper object of metaphysics or ontology. The truth of *language*, called "truthfulness," or "veracity," is the conformity of our language with our thought. It is ethical or moral in character; and the study of it belongs, therefore, to ethics or moral philosophy. The truth of *knowledge*, called *logical truth*, is *the conformity of the mind judging about reality, or of the mind's judgment about reality, with the reality to which the judgment refers.*[1] This knowledge and its truth are embodied in the mental act of judgment.

The term "knowledge" expresses that relation of the mind to its object (things or reality), of which everyone is conscious, but which is so simple, fundamental, primordial, that it does not admit of *definition* proper, though it may be psychologically

[1] "Veritas intellectus est adaequatio rei et intellectus secundum quod intellectus dicit esse quod est, vel non esse quod non est."—ST. THOMAS, *Summa Contra Gentes*, i., q. 5. *Cf.* ST. THOMAS, *In Met.*, iv., *lect.* 8: "Verum enim est cum dicitur esse quod est vel non esse quod non est. Falsum autem est cum dicitur non esse quod est aut esse quod non est"—which reproduces Aristotle's definition: "Τὸ μὲν γὰρ λέγειν τὸ ὂν μὴ εἶναι ἢ τὸ μὴ ὂν εἶναι ψεῦδος, τὸ δὲ τὸ ὄν εἶναι καὶ τὸ μὴ ὂν μὴ εἶναι ἀληθὲς," *Met.* iii. 7, ed. Didot. *Cf.* MERCIER, *Critériologie*, 5th edit. pp. 17-31; SENTROUL, *La verité et le progrès du savoir*, in the *Revue néo-scolastique*, May and August, 1911.

analysed and described. By "knowledge simply" *true* knowledge, or the possession of *logical truth*, is, of course, always meant. The *absence* of such knowledge *in a being capable of possessing it*, is called *ignorance*. Either the mind does not possess any ideas at all about the matter in question, in which case it is *absolutely* or *totally ignorant*, in a state of *nescience* regarding the thing; or, possessing some ideas about the thing, it does not know what is the proper relation to establish between these, and is thus *partially* ignorant, and in *doubt*.

The opposite of truth, or true knowledge, is *error*, or erroneous belief. Error necessarily implies the possession of *some ideas* about the object thought of, and is the *disagreement of the judgment* which the mind has formed about the thing, and to which it adheres, *with the thing or reality in question*.

When the mind adheres firmly to a judgment which it knows to be true, it is said to have certitude. *Certitude* is, therefore, *the fixed or firm assent or adherence of the mind to a truth, without any prudent fear of error*. It can be had about *immediately* evident judgments, or about those that are *mediately* evident, *i.e.* known by reasoning. The name *Science* is applied specially to knowledge of which we have *mediate* certitude; we are said rather to have *Intelligence* ("*Intelligentia*") or *Intellectual Intuition* of immediate, abstract first principles, and *Sense Intuition* of the immediate, concrete facts of sense: we "see" these rather than "learn" them. Mediate certitude presupposes, and rests ultimately on, certitude that is immediate.

Properly, therefore, certitude is a state of the mind, the quality of our mental assent to a judgment which we have formed, which is true, and which we know to be true. The object to which the mind assents in forming such a judgment is, as already explained (78, 80), the reality itself, seen and grasped by the mind through the relation established between the two aspects of that reality, represented by subject and predicate. The reality itself, thus looked at as the object of a true and *certain* mental judgment, is usually called a *certainty*, or an objective certainty (in addition to its still more common name, a truth); while the mental state is described as the state of *certitude*.[1]

When or how can we be certain that our judgment, our knowledge, is true?—that we possess the truth? When, on reflection, we find that the *evidence*—which is the cause or motive of our firm or certain assent—is fully

[1] *Cf.* CLARKE, *Logic*, p. 426; NEWMAN, *Grammar of Assent*, pp. 195-6.

sufficient to guarantee such assent. And what *is* this *objective evidence* that calls forth our assent? It is, in ultimate analysis, simply the objective relation between two aspects of the same reality, between subject and predicate, shining in clearly upon the mind, and grasped by the mind in forming the judgment, in interpreting the reality through this mental act. It is, therefore, simply the *manifestation of reality to the mind.* In other words, it is the real or ontological truth; it is *Being*, *apprehended* by the mind as its natural, proper object: "*Objectum Intellectus est Ens*" is the Scholastic formula which sums up this whole doctrine on the criterion of truth and the motive of certitude.

The state of certitude, alone, excludes all prudent fear of error. When the judgment connects two simple, abstract concepts, whose comprehension is perfectly clear, the relation of subject to predicate is seen to be *absolutely necessary* and *immutable*, so that the object of our thought could not possibly be otherwise without a contradiction in thought. With regard to such self-evident judgments—"*per se nota non solum in se sed quoad nos et omnes*"[1] there can be *no possibility* of error. Our assent is *compelled;* the evidence is *cogent.*

But when we pass to concepts that are more complex, and judgments arising out of them, the evidence for such judgments is usually not so clear; the relations are not so manifest; they need not *compel* our assent: we realize that, absolutely speaking, error is possible in their regard; and hence we are *free* to withhold assent if we choose, though at the same time we may see the evidence to be such that it leaves no ground or reason for a *prudent fear* of error.

Similarly, of the judgments which we form on the immediate testimony of our senses, and by which we interpret the facts of immediate sense experience, many are so evidently true that, although we might conceive ourselves to have been mistaken in regard to them, there is no *prudent ground* for any fear that we are mistaken. Again, the same is true, in its measure, of judgments for the truth of which we rely on the testimony of our fellow-men. In all such cases, *where prudent fear of error is excluded, and where the judgment to which we assent is actually true*, we have *certitude.*

When such fear is not *wholly* excluded by the grounds or reasons we have for our assent, the judgment is described as "*probable*": our attitude towards it is called "*opinion*":[2] the mind

[1] ZIGLIARA, *Logica*, (41), v.
[2] Or, also, "belief," in one of the many meanings of this term.

here inclines towards one of two contradictory judgments as true, but not so strongly as to exclude a prudent fear that it may be false and the other true. The one to which it inclines is said to be "probably" true. It will be *equally* probable with its contradictory, or *more* probable, or *less* probable, than the latter, according as the reasons for it are equal to, or less, or greater than, the reasons for the latter. Here our assent is not fixed or stable; it is provisional. *Opinion*, therefore, is *the provisional assent of the mind to one of two contradictory judgments, with more or less fear of error.* When the fear is so trifling as to be practically negligible, our assent is commonly described as *moral certitude.* When, on the other hand, our fear of error in assenting to a judgment is exceedingly great, our assent is called a mere *suspicion*, rather than an opinion.

When the mind is balanced, hesitating, wavering between two contradictory judgments, without adhering to either of them, it is said to be in "doubt," or "suspense": it suspends or reserves its assent. *Doubt*, therefore, is *the state of the mind suspending its assent for fear of error.* It is said to be a *negative* doubt if there are no reasons on either side, *positive* if there are equal reasons on both sides. Deliberate doubt always implies a judgment or judgments about the weight of the evidence for and against; together with the absence of any judgment about the main matter under consideration.

The chief attitudes of the mind towards a truth, are, therefore, *certitude*, *opinion*, *suspicion*, and *doubt;* the latter indicating the entire absence or suspense of assent.

But our estimate of the grounds of our assent may possibly be erroneous: we may be deceived into assenting with more or less firmness to a *false* judgment: and, hence, we can conceive degrees of assent varying indefinitely from that of *certitude* or absolute adherence to the *true* judgment, to that of the strongest adherence to its *false contradictory*. These two extreme states of mind are, of course, *contraries*. The latter state—firm assent to an erroneous judgment—has no special name other than that of *error*, a term which primarily means the disagreement of the judgment with reality. Conversely, the term *truth* is sometimes used to signify the subjective mental state of certain assent to a true judgment, rather than the conformity of the latter with objective reality. In these subjective meanings of the terms, truth is the *contrary* of error. For, regarded subjectively, simply as states of the mind, the state of *ignorance*—implying, as it does, either the absence of all ideas (ignorance, simply), or the absence of assent to any judgment about the matter (doubt)—is intermediate between the state of assent to a true judgment, and that of assent to an erroneous one about the same subject-matter.

But, regarded in their proper, objective sense, as characteristics of the judgment, *i.e.* of the relation between the mind and its object, logical truth and error imply *contradictorily* opposite relations of the mind to its object: that of positive conformity with the latter, and that of positive discrepancy or divergence from it. Granted some definite relation or other of the mind to the thing, or, in other words, some *judgment* about the thing, the opposition between these two kinds of relation is *contradictory* opposition: the relation is either one of agreement or one of disagreement; there is no mean. Every judgment has a contradictory; and of a pair of contradictories one must be true and the other must be false. We cannot have, as a mean between them, a judgment which is partly true and partly false.

This last statement seems at first sight entirely opposed to our ordinary habits of thought and expression. Are we not constantly describing statements we hear, as "*more or less* true," as containing "a certain amount of truth," or "a grain of truth and a large measure of error"? Are we not constantly distinguishing truth from error in the judgments formed and expressed in every department of experience? That is undoubtedly so; but what, in reality, does it prove? Merely that such statements are ambiguous, are open to a variety of interpretations, express or imply or suggest not one simple judgment, but many; and that, while some of these latter are true, some also are erroneous.

249. THREE KINDS OF CERTITUDE: METAPHYSICAL, PHYSICAL, AND MORAL.—Our assent to any judgment as true may be influenced, and often is, no doubt, largely influenced, by motives of a non-intellectual character, by our likes or dislikes, by our character and beliefs, by our feelings and emotions. Of these logic cannot take account, except indirectly: its main concern is with the *intellectual motives*, the *rational grounds* or *reasons*, of our assents. Assent is an intellectual act, and no assent can be rational without sufficient *rational* grounds. Moreover, it is our reason that must ultimately decide what weight of influence we may prudently and reasonably allow to non-intellectual motives in determining our assent in any given case. Logic, then, is concerned with the intellectual grounds of certitude, and these are all included in the term "*evidence*".

Now, broadly speaking, we may distinguish three great sources of evidence, and three corresponding kinds of certitude.[1] (*a*) *Analysis and comparison of abstract ideas*—involving some judgments that are immediately evident, and others mediately evident, inferred from the former—are processes which yield *metaphysical certitude.* (*b*) *The testimony of our senses*—involving judgments about the immediate data of our sense experience, and general

[1] *Cf.* TOOHEY, *The Three kinds of Certitude*, in the *Irish Theological Quarterly*, vol. iv., n. 15, pp. 254 *sqq.* (July, 1909).

judgments established by induction about the facts of sense—is a source of *physical certitude*. (*c*) The testimony of our fellow-men is, under certain conditions, a source of *moral certitude*. But evidence from all three sources combines and coalesces in the production of the greater part of the certain knowledge which is actually possessed by men generally.

The main characteristic of pure metaphysical certitude is this, that it is for the most part[1] confined to truths of the *abstract* order, to judgments about *possible* essences or objects of thought, judgments which abstract from the question of the existence or non-existence, occurrence or non-occurrence, of these objects in the sphere of *actual* reality. In other words, it concerns judgments, whether affirmative or negative, *in materia necessaria* (85-8); or again, judgments which affirm a necessary identity or a necessary incompatibility between the objective concepts compared; that is to say, judgments which have been described as *apodeictic* (89-90). And all such judgments, dealing as they do with the *necessary* implications of concepts, are the intellectual expressions of *laws*, and are, therefore, *universal* judgments—at least *potentially* universal.[2] All the truths of pure mathematics are of this order. The evidence is intrinsic to the truths themselves; and it is cogent, whether it be immediate evidence of axioms, or mediate evidence of conclusions deduced logically from such axioms.

Physical certitude is, first of all, the certitude we may have about the actual existence or occurrence of *concrete* facts of our own individual sense experience. These all occur in time and space. For present facts of actual sense experience we rely on the testimony of our senses; for certitude about *past* facts of our own individual experience we must rely on *memory*. But,

[1] Except where the concept of the subject involves in it the concept of actual existence. "We have an example of existence being involved in the idea when we mentally or orally affirm 'I exist'. I cannot even think 'I' without implicitly affirming that I exist; the very fact of saying to myself 'I' is an acknowledgment that I exist, for I cannot think 'I' without existing and being conscious of my existence. Hence to say 'I do not exist' is to be involved in a concrete contradiction, just as to say 'Two and two do not make four' is to be involved in an abstract contradiction; it is to deny in the predicate what is implicitly affirmed in the subject. . . . Hence it is that we are each of us metaphysically certain of our own existence, and the causal proof of the Being of a God which is based upon our personal existence leads to metaphysical certitude that God exists."—TOOHEY, *ibid.*, pp. 255-6.

[2] Their actual extension may include only a single subject, as, for example, when such judgments are formed about the thinking subject himself, or about God, the Necessary Being.

secondly, what about facts that fall *outside* our own individual experience, whether these be past, or present, or future? Apart from the moral certitude we may have of the occurrence of any such facts on the *extrinsic* evidence of human testimony or authority, can we have physical certitude about them? Obviously, they do not present their own *intrinsic* evidence to us *immediately*; but they may do so *mediately*, *i.e.* by way of some known and proved connexions between them and certain other facts which we have ourselves experienced directly and immediately. We may be certain, for instance, that, besides the bars of iron we ourselves have seen being elongated by heat, *other* bars of iron have been, or are being, or will be, likewise elongated by heat, provided that in such cases, past, present, or future, certain conditions be fulfilled. But, note that our certitude here is about the *conditional* occurrence of these other instances: we are certain that they have occurred, or do, or will occur, not absolutely, but *contingently* on the fulfilment of certain conditions—such for instance as the actual existence of other bars of iron and other sources of heat, the repetition in them of the physical conditions involved in the law that "heat elongates iron bars," the uniformity of the action of physical causes, the absence of interfering causes (224). Our certitude of their occurrence as *facts*, involves, and is based upon, our certitude of the universal prevalence and validity of *laws* within the domain of these facts. *Law* is the rational connecting link between experienced facts and unexperienced facts. Hence, our certitude about such unexperienced facts is not precisely certitude that they did, or do, or will occur; but, rather, that they *must* occur. It is a mediate, inferential certitude, based on prior certitude regarding the laws. Now, of the laws on which these inferences are based, some, no doubt, are absolutely and self-evidently necessary, such as the principle of causality; but others are inductively established laws, the necessity of which we have seen to be, in ultimate analysis, not an absolute, unconditional necessity, but rather a conditional necessity, contingent on the will of the Supreme Being who rules the whole created universe (224). This kind of certitude, which we thus obtain by induction from experienced facts, regarding the conditional prevalence of laws, and the conditional occurrence of unexperienced facts inferred from the latter, is likewise commonly recognized and described as physical certitude.

Moral certitude is the certitude of belief based upon authority;

and, in this sense, it may be had either for individual facts or for general laws. The certitude of historical knowledge is of this kind. But there is a sort of certitude, also called "moral," which is based upon intrinsic evidence. It is the certitude we have concerning (*a*) the generalizations we form, from our own experience, about the conduct and activities of *free* agents; and (*b*) concerning the individual facts, phenomena, or instances, to which we apply these generalizations. Such generalizations we call "moral universals," liable to exceptions; and their application to individual cases we know to be more or less precarious. Such is the certitude we have, for instance, about the truth of these judgments: "Men are naturally truthful," and, therefore, "A.B., who has no inducement or temptation to deceive me, is telling me the truth"; "Parents naturally love their children," and, therefore, "A.B. and C.D., whom I know to be good people, love their children"; "Men respect and protect the lives of their fellow-men," and, therefore, "My food at dinner to-day will not be poisoned". Finally, we may have, whether for facts or for laws, such a weight of cumulative evidence of various kinds as will warrant that very high degree of probability which is commonly called "practical" or "moral" certitude.

250. NECESSARY TRUTH OF METAPHYSICAL LAWS; CONTINGENT TRUTH OF PHYSICAL LAWS AND FACTS.—There is an inclination among modern philosophers and scientists not to recognize as a "scientific law" any general formula or statement *to which we can conceive an exception.* We should, they think, so formulate our laws, by finding out accurately, and expressing hypothetically, all the conditions for the truth of these, that they may admit of no exception as stated. But when we pass from the abstract world of mathematics, and from the first principles of thought and being in logic and in metaphysics, where the mind abstracts from the concrete reality and marks out clearly for itself the grounds for its judgments;[1] when we come to complex and concrete reality as revealed in the physical world through sensation, and try to grasp the laws according to which phenomena take place, it is not so easy to apprehend *all* the causes and conditions of their appearance, and so to embody these in our formulæ that the latter will be true *by an absolute necessity of thought.*[2] Of course, if such care be taken, our judgments in all

[1] *Cf.* WELTON, *Logic*, ii., p. 202; MELLONE, *Introductory Text-book of Logic*, pp. 265-70.
[2] WELTON, *ibid.*, p. 205.

departments of human knowledge will be of the same inviolable necessity, but they will be almost[1] *all abstract and hypothetical.* If we make provision for all possible conditions, including the Divine Will, and also human free-will, for physical and moral facts, then our universal judgments in these departments will be as necessarily true as our metaphysical judgments. The propositions, "*If the natural causes of the planetary movements continue to exist and to act as heretofore, uninterfered with by a higher Power*, the sun will rise to-morrow," and "*If the inhabitants of the town A are exactly of the same mind and character as those of the town B*, they will make an equally gallant defence when attacked"—are as necessarily true as "*If a triangle be inscribed in a semicircle*, it will be right-angled". [2] All three alike are embodiments of the principle of identity in this way: "If a given cause or reason be sufficient to produce or account for a given fact or truth, it will produce or account for this absolutely, continually, universally". And in so far as they embody this principle they share in its absolute necessity.

But, then, they are only *hypothetical* as regards the verification of their antecedents (223), and in the first two judgments this verification involves more than it is given to man to fathom *in regard to any future cases of them:* for they deal, not with *abstract, possible* objects of thought, but with *concrete, actual* things and events. Our assurance about the categorical proposition that "The sun will rise to-morrow" is not absolute or necessary, but contingent on our certitude that the causes of the planetary motions will continue to exist and to act unimpeded as in the past; and since all this is dependent on the free will of the Creator, our certitude about it cannot be absolute.[3] In the second case, similarly, our ignorance as to any future fulfilment of the antecedent is increased by the intervention of the free will of man. Hence it is that our knowledge of the continued occurrence or recurrence of *concrete, existing* phenomena, whether physical or moral, is hypothetical and contingent: the permanence of the grounds on which they are based is hidden from us. I am not *compelled*, therefore, to assent to the categorical proposition that "The sun will rise to-morrow". To have *cogent* evidence either for or against it would imply on my part a *certain* knowledge of the Divine Will. From which it follows that man can never

[1] *Cf.* p. 215, n. 1.
[2] *Cf.* JOSEPH, *op. cit.*, pp. 507-8.
[3] *Cf.* JOYCE, *Logic*, p. 237.

analyse the conditions or antecedents of a concrete, actual fact or phenomenon, sufficiently to be compelled by the evidence to pronounce *categorically* that it *will recur*.

Truths for which we have necessary or cogent evidence—"necessary" or "metaphysical" truths—are all *abstract:* they formulate relations between aspects of reality *apart from their existence or happening.*[1] Even when categorical in expression, as *e.g.* "Triangles inscribed in semicircles are right-angled," they are *conditional in thought* (134-5) as regards the actuality of their antecedents. The logical necessity of these relations between subject and predicate is the necessity by which any abstract object of thought is identical with itself, with all that the mind apprehends in it. It is a necessity that pertains to the abstract essences of things. So long, therefore, as we deal with purely abstract judgments, such as those of mathematics, we can analyse the grounds for the abstract relations we establish, and can see these to be cogent. Such truths are called "necessary," because they express essentially or intellectually necessary relations between abstract objects of thought. But when we deal with generalizations about concrete, existing things, beyond actual sense experience, our analysis must always leave an unknown and uncertain residue, that, namely, which is the ground for the persistence—through the changing conditions of time and space—of the elements about which we are thinking. For truths, therefore, which imply the actual existence, beyond experience, of the elements of reality to which they refer, the ground or evidence can never be cogent, can never *necessitate* our assent. These are called, and rightly called, "*contingent*" truths, because they make such mental assertions about things as will hold good only *if those things persist* in the concrete existence and activity with which we know them to have been endowed within our experience.

Of course, if we express this assumption about their persistence, in formulating "a general judgment concerning natural phenomena"[2]—that is, a judgment affirming their universal repetition throughout all time and space whenever and wherever *all the conditions of our hypothesis are verified*,—we

[1] They are "altogether independent of any *physical process.* In some cases we see that certain concepts, statically considered, stand in a relation of identity (or difference) under pain of a contradiction in terms. . . . In other cases a causal relation is involved in the very nature of the abstract concept, apart from any dynamic efficiency."—JOYCE, *op. cit.*, pp. 238-9. *Cf.*, however, p. 215, n. 1.

[2] WELTON, *ibid.*, p. 205.

are, *eo ipso*, making our law as abstract, hypothetical, and necessary as any law of mathematics ; and we can say of it, when thus formulated, "that if it is once true, it is always true, and that so far as it is true it is necessary in that system of reality "[1] : for all conceivable reality is subject to the laws of thought, and, therefore, to the principle of identity.

In this sense—and it may be well to call attention to it here—*every* judgment, even the particular, elementary interpretation of a sense experience, such as "It rains," is *necessarily* and *universally* true : in the sense that if true at all it is always and everywhere true (80). If it rained at a particular time and place, it is true throughout all time and space that it did rain then and there. The laws of thought are thus involved in all intellectual judgments, and it is just because they are that all truth—all true judgments—can be described in a real, admissible, and intelligible sense, as "necessary". This necessity is purely logical, *i.e.* it belongs to the mental act of judgment, and virtually amounts to this, that if the judgment is true it cannot be otherwise than true.

Such necessity supposes and is dependent on a mental analysis and comparison of certain elements or aspects of *reality*. It is not, therefore, a merely *subjective*, psychical necessity ; for the mind may abstain altogether from analysing the elements of reality in question, or pronouncing any judgment upon them. There is no necessity about the actual occurrence or existence of such a mental process : it need never have taken place. But, granting that the process of analysis has taken place, it will lay bare the grounds for the judgment formulated. If this be about abstract objects of thought the objective grounds will be, or may be, cogent ; if it be an attempt to affirm a general law about the actual happening of phenomena, the grounds for a categorical statement will not be cogent, for, to use the words of Green, "any proposition about a natural phenomenon is true of it *only under conditions of which we do not know all*, while a proposition about a geometrical figure . . . is true of it under conditions which we completely know ".[2]

The general propositions in which natural or physical laws are formulated *do not usually express*, but rather abstract from, the one all-important condition on which their absolute truth depends, the positive Will of the Creator ; and hence we speak of their necessity and universality—or their necessary and universal truth, which we call "physical"—as being not "absolute" but "hypothetical," "contingent," *i.e.* dependent on the Free Will of the Creator.[3] To the ordinary formulæ, therefore, which express the "laws" of "Physical Nature" we can conceive exceptions. The Author of Nature can derogate from them, *i.e.* He can so alter or interfere with the conditions of the existence and activity of created agencies that our formula, which did not count on such interference, may be, in a particular case, inapplicable. If we formulated our law so as to include, and take account of, such interference, the law thus hypothetically formulated could not be disturbed by such intervention. This is why logical, metaphysical, and mathematical laws or truths are unchangeable : because they deal with hypothetical relations which obtain, by a necessity of thought, between abstract objects that are

[1] WELTON, *op. cit.*, p. 205. *Cf. supra*, p. 218.
[2] *Phil. Works*, vol. ii., pp. 249-50, *apud* WELTON, l.c. (italics ours).
[3] *Cf.* JOYCE, *Logic*, p. 237.

considered apart from all actual existence of the concrete things in which they are realized (206).

The general truths of physical science, on the other hand, and of the sciences which deal with phenomena dependent on the free activity of man, are not *inviolable* in their necessity, or absolutely all-embracing in their universality. They are, nevertheless, truths to which we can give a *certain*, *i.e.* a steady, firm assent, because we know that the Divine interference with natural agencies, being controlled by Divine *Wisdom*, will not be arbitrary or capricious ; and that although men are free, and thus masters of their own acts, they act, as a rule, not capriciously, but in accordance with their *common* nature, as men. Hence we have grounds for "generalizations" or "laws" that are *physically* or *morally* universal, and for assenting to these laws, and to their applications, with *physical* or *moral* certitude.

We can be certain that physical laws are *necessarily* true in this *hypothetical* sense, *i.e.* that they *must* hold good *contingently* on the Divine Will and Wisdom ; and that no other necessity attaches to themselves or their applications besides the consequent necessity of obeying the Divine *Fiat* (219). It is in vain that scientist or philosopher, agnostic or monist, endeavours to attach an *absolute* or *essential* necessity to the domain of *contingent existential* fact. Physical laws have only a hypothetical necessity ; and this necessity receives a rational explanation only in the philosophy of theism which recognizes the universe as a contingent reality, freely created, conserved, and governed, by the Power and Wisdom of a Necessary Being. The absolute, logical necessity, claimed for physical laws by the philosophy of monistic idealism, has no ground in common sense or everyday experience, and remains enshrouded in a mist of mystery (224).[1] The world of sense experience furnishes adequate grounds for proving the existence of a Necessary Intelligence and Will, distinct from itself. The monism of Hegel and his followers interprets this same world as the purely intellectual manifestation of a necessary, self-existent mind or idea. In its exclusive attachment to the conceptually *abstract*, *universal*, and *necessary* relations, established by our intellects between the objects of our thought-processes, monism loses sight of the other great aspect of reality—its phenomenal aspect, reality as revealed to our senses. But in the light of sense experience we are forced to believe that what actually exists is not abstract but concrete, not universal but individual (6), not unique but manifold. Of these things or realities our intellects can gain true, though inadequate, knowledge by the system of *universal* thought-relations which it

[1] This applies equally to the "mechanical" necessity ascribed by Empiricists to the processes and laws of nature. It is no *ultimate* explanation of this necessity to say that it is "mechanical". If all nature is merely one vast machine or mechanism, who made it? The necessity we ascribe to the course of *actual* nature in time and space is not the necessity we ascribe to abstract judgments about possible essences: it is not purely intellectual. The only immediate source it can have is our experience of the order, regularity, uniformity of all nature, compelling us to interpret the latter as a *Cosmos*, as the work of an *Omnipotent Will* directed by *Supreme Wisdom*. The only necessity for which we can rationally account in actual nature is that by which it pursues the course marked out for it by the Divine *Fiat*. To say *as a last word* about the course of nature that it is "mechanical," is scarcely any better than to ascribe it to mere chance, or to pronounce it an insoluble enigma.

establishes between *abstract aspects* of these things, and whereby it interprets the latter. No doubt, in formulating such relations our intellects are guided by certain absolutely immutable and necessary principles called laws of thought; we cannot *think* reality except according to these laws. But these laws themselves — the principles of identity, contradiction, etc. — called "*formal*" because they are standards to which all valid thought must conform—are not mere innate, subjective, empty intellectual grooves, themselves devoid of *material content*, mere forms with which thought clothes or *invests* all its material; they too are *material* and have content, because they too are formed by the intellect operating on the data of sense experience. It is only the intellectual faculty of acquiring them that is innate and prior to all individual experience.

These abstract relations, grasped by intellectual thought, continue to grow in complexity under the reasoning power of the mind, and the more complex of them are *explained* by referring them to the less complex from which they were deduced; the less complex are a *causa cognoscendi* as regards the more complex. But we must not forget that this is all in the order of *abstract thought*, or imagine that *concrete reality* is also a system in which the simpler element is the *causa essendi* of the more complex. To conceive the real world as nothing else than a system of logically reasoned, abstract relations, regarded or thought of as objective, is either to ignore the evidence of our senses altogether and regard concrete sense-phenomena as *unreal*, or else to *impose upon* the physical world revealed in the data of sense experience, as the only laws that govern it, certain relations that are considered as a subjective product of pure thought independent of sense experience, and which are on that account regarded as of absolute necessity—that is, whose violation would be unthinkable. This is simply to ignore the concrete for the abstract, and to reduce reality to a *subjective creation of the mind*. It gives a fictitious sort of objectivity to that "mental construction" which it calls "the world" or "reality," and is calculated to convey an erroneous impression about the "necessity" of the laws that govern the physical universe. To conceive the latter as a closed system of activities every fact or phenomenon in which will be "explained" by establishing *between it and the whole* certain relations of an absolutely immutable character, and to regard nothing as "scientifically known" or "explained" except in so far as it can be shown to be subject to such absolutely necessary relations, is to impose gratuitously a metaphysical or absolute necessity on the activities of physical nature, to accept a one-sided, abstract, unreal conception of the universe, to narrow "scientific" knowledge arbitrarily to the domain of *pure abstraction*, and to regard the totality of *things in the concrete* as *scientifically unknowable*.

No doubt, if we had an ideally perfect scientific knowledge of any phenomenon or fact, we should know that "it could not possibly be other than"[1] God sees and wills it to be. But no human mind ever has known, or ever can know, any *concrete fact or phenomenon* in that way. It can know thus only the *abstract relations* which *itself discerns* in phenomena. It can know, too, that the things

[1] *Cf.* ARISTOTLE, *Anal. Post.*, i., 2; WELTON, *Logic*, ii., p. 188; *cf. supra*, 224, p. 98. The only *actual* reality that "could not possibly be other than it is," is the Self-existent Reality of the Necessary Being. And the nature of that Being cannot be known directly or intuitively by the human mind.

from which it abstracted the data for these relations exist in conformity with these relations, *granted that as a fact they exist*: but that such things *must* exist by any necessity arising from *thinking* them in the abstract, is utterly unwarranted and untrue. The reality of the phenomena revealed to us in sense experience as constituting the physical universe does not contain or show forth any *absolute* necessity for its existence as a concrete system; though it gives rise to a *hypothetical* necessity *in the relations by which we conceive it in the abstract*. The intellect derives from sense experience, by the process of abstraction, the (abstract) elements which it compares. From the fact that those elements are apprehended by the intellect in the abstract—free from their conditions of concrete existence in time and space as phenomena of sense experience,—the relations between them, logical, mathematical, and metaphysical, are likewise apprehended as fixed, static, unchangeable. But, while we can understand that those necessary and universal relations apply to reality *as thought of in the abstract* by the intellect, that is, to the elements abstracted from sense experience, we must also remember that the intellect can discern, between those same elements *as given in sense experience*, other relations—not logical, or metaphysical, or mathematical, but *physical*—relations that are not necessary in the sense that any modification of them would be unthinkable, but which are stable, nevertheless, and permanent, in the hypothesis and in the measure that certain influences, to which their concrete existence in time and space is subject, will not interfere with these relations.

There is no need to dwell any longer here on the erroneous conceptualism of Hegelian idealists regarding the relations between thought and reality, between abstract and concrete, between the object of the intellect and the object of sense. Nor is it necessary to do more than merely indicate that the only satisfactory way of grasping and reconciling the terms of those relations is to be found in the Scholastic doctrine of *Moderate Realism*: The object of the intellect, while it is apprehended formally as abstract and universal *only* by the intellect, is nevertheless really and fundamentally present and inherent in the concrete object of sense: "*Universale est formaliter in mente et fundamentaliter in re*": "The universal exists *formally as such* only in the intellect but it *has a foundation* in the thing (of sense experience)".[1]

251. ARISTOTLE'S IDEAL OF "SCIENTIFIC" KNOWLEDGE.—We have frequently referred already to discussions as to the nature and grounds of the "necessity" attaching to *scientific truths* or *laws*, and to the distinction between abstract, necessary truth and concrete, contingent fact (*cf.* 220, 223). Those inquiries naturally lead up to the question: What, then, is the ideally perfect form to which our knowledge, our interpretation of universal experience, aims, or should aim, at attaining? This problem involves an analysis of many fundamental logical concepts—especially those of *Deduction*, *Induction*, *Demonstration*, *Explanation*, and *Science*—and may, indeed, be regarded as the "most difficult

[1] *Cf.* JOYCE, *Principles of Logic*, pp. 132-6, *supra*, pp. 107-8.

of logical questions ".[1] Apart from the deeper philosophical aspects of the problem, which belong rather to epistemology, there is the purely logical aspect which arises from its bearings upon logical method. And here, perhaps the principal difficulty in treating it arises from the fact that the theory as to what constitutes the perfect form of knowledge or science has been worked out from two distinct standpoints. Aristotle's conception of science is built up from the standpoint of deduction, and worked out in connexion with his doctrine on scientific *Demonstration.* The modern conception of science is closely allied with induction, and is set forth under the theory of *Scientific Explanation.* These two views of the ideal of science are not mutually opposed, but, when rightly understood, rather supplement each other. We shall outline each of them successively, and endeavour to compare them.

About the reasoning processes which enter into all scientific research, three distinct questions might be asked: (1) Are our conclusions validly derived from our premisses? (2) What sort ought our premisses to be, if our knowledge is to be the *most perfect* attainable? (3) By what sort of processes do we reach, and establish or justify, the premisses we *actually* make use of? Aristotle's three treatises on inference—the *Prior Analytics*, the *Posterior Analytics*, and *the Topics*—are devoted to these three questions respectively. The first examines the *formal* validity of inference; the second and third examine the conditions for its *truth*. We argue sometimes from abstract, self-evident, necessary principles; sometimes from generalizations—reached through observation, induction, or authority—not certain and necessary like the former principles, but only contingent and probable. In the *Topics* Aristotle dealt with these less perfect sources of knowledge by "analysing and formulating the actual procedure of his contemporaries; he did not, upon the whole, go ahead of the science, the disputation, the rhetoric and the pleadings of his day ".[2] Roughly speaking, he covered the ground that would be covered nowadays by the logic of induction and probability. The knowledge which he conceived to be the ideal of perfect knowledge was that derived by the syllogistic, synthetic method of reasoning from necessary axioms like those of mathematics; and in the *Posterior Analytics* he inquired into the nature of *demon-*

[1] JOSEPH, *op. cit.*, pp. 343, 349, 487. [2] *ibid.*, p. 348.

strative premisses and the conditions of the *demonstrative syllogism* which is productive of the highest form of knowledge—*Science.*

Not all knowledge that is certain is *scientific* in Aristotle's sense of this term. *Science* he describes as *knowledge of a thing through its cause;* and an adequate knowledge of the cause, he adds, will enable us to see *that the thing cannot be otherwise than it is.* [1]

Science, then, in the strict Aristotelean meaning of the term, is apparently confined to a knowledge of things "that cannot be otherwise," *i.e.* of abstract, metaphysically necessary truths, truths that are *in materia necessaria*, that may not be denied without violating some law of thought or involving some contradiction.[2] In ordinary modern usage, the term *science* includes our certain knowledge of another vast body of truths, endowed with a necessity of an inferior kind, not absolute but hypothetical, contingent, physical. In a still wider sense, it embraces our knowledge of moral, social, historical truths, etc., whose necessity is based on the stability of the laws that govern human intercourse: truths about which we have moral certitude.

252. NATURE AND CONDITIONS OF DEMONSTRATION.—Observing, in the next place, that we get science by *Demonstration*, Aristotle describes the latter as a *syllogism that engenders science;* [3] and he then proceeds to lay down the conditions for a cogent demonstrative, or apodeictic, syllogism: science, or demonstrated knowledge, must be inferred "from premisses that are true, ultimate, immediate, better known than, prior to, and causes of, the conclusion; from premisses that are the proper principles of the demonstrated truth: for without such the syllogism will not be demonstrative, or productive of science".[4]

In the *first* place the premisses must be *true;* for, "though formally a true conclusion may be got from false premisses, the error still infects the mind, and will lead to a false conclusion

[1] Ἐπίστασθαι δὲ οἰόμεθ' ἕκαστον ἁπλῶς . . . ὅταν τήν τ' αἰτίαν οἰώμενθα γιγνώσκειν δι' ἣν τὸ πρᾶγμά ἐστιν, ὅτι ἐκείνου αἰτία ἐστι, καὶ μὴ ἐνδέχεσθαι τοῦτ' ἄλλως ἔχειν—*Anal. Post.*, i., cap. ii., 1. In the term "cause" Aristotle here includes the four great classes of cause, formal and material, efficient and final.—*Cf. Anal. Post.*, ii., cap. x. [xi.], 1.

[2] *Cf.* DE MARIA, *Logica* (2nd edition), pp. 258, 259, 269; ZIGLIARA, *Logica*, (41).

[3] Φαμὲν δὲ καὶ δι' ἀποδείξεως εἰδέναι. Ἀπόδειξιν δὲ λέγω συλλιγισμὸν ἐπιστημονικόν. *ibid.*, i., cap. ii., 3, 4.

[4] Εἰ τοίνυν ἐστὶ τὸ ἐπίστασθαι οἷον ἔθεμεν, ἀνάγκη καὶ τὴν ἀποδεικτικὴν ἐπιστήμην ἐξ ἀληθῶν τ' εἶναι καὶ πρώτων καὶ ἀμέσων καὶ γνωριμωτέρων καὶ προτέρων καὶ αἰτίων τοῦ συμπεράσματος· οὕτω γὰρ ἔσονται καὶ αἱ ἀρχαὶ οἰκεῖαι τοῦ δεικνυμένου. Συλλογισμὸς μὲν γὰρ ἔσται καὶ ἄνευ τούτων, ἀπόδειξις δ' οὐκ ἔσται· οὐ γὰρ ποιήσει ἐπιστήμην, *ibid.*, 5, 6.

somewhere".[1] The mind which sees the true conclusion *only as in the false premisses*, and as *following from* these, can be scarcely said to possess "truth," or "conformity with reality": unless, indeed, when it drops the false premisses and assents to the conclusion absolutely; and even then it cannot have *certitude* about the truth of the conclusion, any more than it could about the truth of the *false* premisses from which it inferred them. Besides, the aim of demonstration is to derive a true conclusion from true premisses—the natural source of such a conclusion.

Secondly, demonstration must rest ultimately on *first* truths or *principles*, *i.e.* truths which are not themselves demonstrable, but *immediately evident* or *self-evident*. Such principles are called *axioms*. The concepts embodied in them are so simple that the relations expressed between these concepts are immediately apprehended by the intellect, without recourse to any simpler concepts as middle terms. Were the mind incapable of assenting with absolute certitude to such indemonstrable, self-evident truths, no certitude and no science would be possible. For either the premisses by which a scientific conclusion is established are immediately evident, or their truth has been established by antecedent premisses; about which latter the same question arises. But such a series of conclusions and premisses cannot stretch back indefinitely, for if it did the certainty of any one link could never be established. And if the series is finite, some of its members must be first and indemonstrable. These, moreover, must be self-evident; if they were not, no conclusions from them could be evident, or therefore certain or scientific. Such self-evident axioms are found involved in all the special sciences.[2] They are called *first* principles:[3] not in an absolute sense, but relatively to the conclusions they generate in the science that employs them.

Each science has *its own* first principles—"generating" principles as they are called. But then, also, human knowledge can be unified. Sciences of a lesser scope are subordinate to those of a wider scope, and borrow their initial notions and principles from the latter.[4] And hence the notions that are "absolutely first" are those investigated in the science which

[1] JOSEPH, *op. cit.*, p. 342 n. *Cf. supra*, 148.

[2] Some sciences may take as their principles truths established as conclusions in other sciences.

[3] A *principle* is that by which a thing *exists* or *comes into existence* (*ontological* principle); or *comes into our knowledge* (*logical* principle).

[4] *Cf.* JOSEPH, *op. cit.*, p. 359, n. 2.

Aristotle and the Scholastics call *Philosophia Prima*, *first philosophy*, or *general metaphysics*. Such, for example, are the notions of *thing*, *being*, *essence*, *existence*, *negation*, *distinction*, *change*, *potentiality*, *actuality*, *accident*, *substance*, *cause*, etc. These notions cannot be *defined*, properly speaking; though they may be explained, and the mind thus aided to distinguish and compare them. The self-evident judgments which formulate mental relations based upon them are, in the stricter sense, "*first* principles"; and in the strictest sense of all we describe as "first" those best known principles of *contradiction*, *identity*, and *excluded middle*, which, in the domain of logic, are seen to be *regulative laws* to which all demonstrative reasoning and all consistent thinking must conform, rather than principles which—like those of the special sciences—enter themselves as premisses into our reasoning processes.

Thirdly, the premisses must give the *cause* of the conclusion. Not only ought the *knowledge* of the premisses to produce the *knowledge* of the conclusion *in our minds*, in *our* "logical" order—that is true of all reasoning,—but the premisses, to be *strictly demonstrative*, ought to reveal to us the *real*, *ontological* cause of what is announced in the conclusion: understanding "cause" in the comprehensive sense in which it includes the formal, material, and final causes, as well as the efficient cause. While Aristotle rightly emphasized the importance of formal and final causes in science, modern logicians lay stress on the rôle of the efficient cause (216-18). The Aristotelean doctrine on causal demonstration is very clearly expressed by Dr. Mellone as follows:—

"Consider the premise 'if anything is *M* it is *P*'. Regarded as a logical proposition, in the formal sense, it states that the antecedent is the *reason* of the consequent: looked at in reference to the real world, it states that *M* is the *cause* of *P*; it implies that we have discovered a law of causation in Nature, and *M* is the cause in question. Now when the syllogism is changed from the hypothetical to the categorical form, *M* becomes the *middle term*:—

Hypothetical	*Categorical*
If Anything is *M* it is *P*,	All *M* is *P*,
S is *M*;	*S* is *M*;
∴ *S* is *P*.	∴ *S* is *P*.

Hence Aristotle says τὸ μὲν γὰρ αἴτιον τὸ μέσον (*An. Post.*, II., ii., 2): 'the middle term expresses the cause'. We may therefore say with Ueberweg (*Logic*, § 101): the worth of the syllogism as a form of knowledge depends on the assumption that general laws of causation hold in nature, and may be known; and that syllogism has the greatest scientific value in which the mediating concept (the middle term), by which we know the truth of the

conclusion, expresses the real cause of the fact stated in the conclusion. This is essentially the Aristotelian doctrine." [1]

Fourthly, the premisses must be *really prior* to the conclusion, inasmuch as the middle term of a demonstration is the "cause," and this is naturally prior to its effect. When there is question of a physical *efficient* cause, which produces its effect by means of *motion* or physical *change*, the cause must precede the effect *in time*, because motion, being successive, involves *duration*.[2] But it is not essential to a *real* or ontological principle that it be *prior in time* to what proceeds from it. For instance, the formal and material causes, which are the real, intrinsic, constitutive principles of any material being, need not necessarily exist prior to the being that is constituted by their union: the human soul may, at one and the same instant of time, be created and united with the material principle to form the human individual: but each principle is prior *logically*[3] and *naturally*—" prius *ratione* et *natura*" : λόγῳ ἢ φύσει πρότερον—to that which is constituted by their union.

Fifthly, the premisses must be *better known*, or, rather, *more knowable* or *intelligible*, than the conclusion. The aim of all inference is to lead us from the better known to the less known or to the unknown. But demonstration leads us from the real principle or cause to that which proceeds therefrom; and how can the former be more "known" or "knowable" than the latter? Aristotle explains how this is to be understood, by his famous distinction between the order of *nature* or *reality* and the order of *our experience*: "*Prior* and *more knowable*," he writes,[4] "may be understood in two distinct ways: *in nature* and *in our experience*. What are prior and more familiar *for us* are what lie nearest to sense-perception; whereas what are prior and more intelligible *simply* are the things which lie more remote from sense. Now

[1] MELLONE, *op. cit.*, p. 252; *cf. ibid.*, pp. 258-9.

[2] *Cf.* ST. THOMAS, *Quaestiones Disputatae: De Potentia*, xiii.

[3] The "logical" order referred to here is not the order in which our minds *acquire* knowledge, the order of *experience*; it is rather the order in which we understand things to be related in the *real*, or natural, or ontological order. *Cf. infra*, pp. 229, 231-2.

[4] Πρότερα δ' ἐστὶ καὶ γνωριμώτερα διχῶς· οὐ γὰρ ταὐτὸν πρότερον τῇ φύσει καὶ πρὸς ἡμᾶς πρότερον, οὐδὲ γνωριμώτερον καὶ ἡμῖν γνωριμώτερον. Λέγω δὲ πρὸς ἡμᾶς μὲν πρότερα καὶ γνωριμώτερα τα ἐγγύτερον τῆς αἰσθήσεως, ἁπλῶς δὲ πρότερα καὶ γνωριμώτερα τὰ πορρώτερων. Ἔστι δὲ πορρωτάτω μὲν τὰ καθόλου μάλιστα, ἐγγυτάτω δὲ τὰ καθ' ἕκαστα· καὶ ἀντίκειται ταῦτ' ἀλλήλοις.—*Anal. Post.*, i., cap. ii., 10. *Cf.* JOSEPH, *op. cit.*, pp. 354 *sqq.*; WELTON, *op. cit.*, ii., p. 33; MERCIER, *Logique*, pp. 287-90.

what are most remote are mainly universals, while what are nearest are singulars: things mutually opposed to one another." This distinction between the order of our experience and that of reality is an important one. Acquaintance with "particular" *facts* of sense experience is the beginning of all our knowledge; while its ultimate goal is an intellectual understanding of the "universal" or common natures, the unifying specific types, or forms, or essences (εἶδος), which are the source of their uniform activities, which reveal to us the *laws* of their nature, and form the ground of all our universal or scientific judgments concerning them, including, of course, the consequent scientific (demonstrative or explanatory) understanding of the (contingent) particulars as embodying these (necessary) universals. When we have reached this understanding of the "kind" or "species," we have the key to all scientific knowledge of the individuals in which the "kind" or "species" is embodied. But how do we reach such knowledge of the universal type?

Having in mind such an abstract science as geometry, Aristotle answered, and rightly, that we get our definitions, which are the mental expression of this knowledge, by *abstracting* concepts from sense-data, *comparing* those concepts, and *seeing intuitively* self-evident relations arise between the latter.[1] Sense experience is necessary, no doubt, in order to furnish us with the concepts of the common natures or essences, but these latter are not themselves empirical facts; they are not *sensible*, but *intelligible*; they are apprehended in the sense-data by intellectual abstraction and intuition; and what is true of those abstract objects of intellectual thought is *necessarily* and *universally* true of the concrete particulars which embody them; while there is, besides, in each *particular*, empirical fact, much that is *contingent*, and, therefore, scientifically unknowable.[2] Thus, according to Aristotle, "all science is of the necessary and universal"; while, at the same time, science gives us genuine knowledge about the particular phenomena of our sense experience, inasmuch as these latter are embodiments or realizations of universal natures or essences.

253. Restricted Scope of Aristotelean "Science".—The explanation just outlined is quite satisfactory in its application to abstract metaphysics and mathematics: the ultimate laws conceived by the mind in thinking about real being—laws of thought, as they are called—and also the fundamental

[1] *Cf.* Joseph, *op. cit.*, p. 355; Windelband, *History of Philosophy*, pp. 136-43.
[2] Windelband, *ibid.*, p. 143.

principles of magnitude and multitude in geometry and mathematics, are reached in the way indicated by Aristotle. But have we the same sort of unerring intellectual insight into "the essence of gold or of an elephant or of a tortoise "[1] as we have, say, into the essence and definition of a triangle? Aristotle seems to have regarded it as impossible for the human mind to attain to "absolutely necessary" truths about the essences of the multitudinous phenomena that form the subject-matter of the special sciences; and in his *Topics* he has indicated various means of reaching and testing those less perfect and less reliable generalizations with which, in the absence of "absolutely necessary" truths, the human mind must rest content.

He was right in recognizing that we cannot have the same "absolutely necessary" truth about concrete existing facts or phenomena as we have about abstract, possible essences; for the actual realizations of these latter are contingent, not necessary. I see that "two and two *must* be four" because it is intrinsically and absolutely self-evident, and no conceivable sort of experience could contradict it; I see that "heat *must* elongate iron" because actual experience forces me to believe that heat and iron are *de facto* so constituted. Aristotle recognized only truths of the former class as principles of demonstration and science in the stricter sense; and these we accept "because our intellect assures us of their truth"[2]. In a somewhat wider sense, however, he admits "science" of that which, though not "absolutely necessary," nevertheless holds good "always, under certain conditions," or "for the most part".[3] This is the domain of those sciences whose laws are established by induction. Now, it has been thought that Aristotle claimed we should have the same sort of intellectual assurance for these "laws" as we have for abstract, self-evident axioms, before the former can be recognized as principles of demonstrative science. He seems to Mr. Joseph to demand the "*ipse dixit* of an incommunicable intuition . . . as a means whereby we are to establish the most important of all judgments, the general propositions on which the sciences rest".[4] But Aristotle could hardly have failed to see that we have no such intuition of what are now commonly known as inductive laws.[5] It was on that

[1] JOSEPH, *op. cit.*, p. 356. [2] *ibid.*, p. 357.
[3] *Cf.* WINDELBAND, *op. cit.*, p. 143. [4] *ibid.*

[5] Such laws differ manifestly from geometrical axioms in this, that they are not, like the latter, seen to be true by an absolute, intrinsic necessity arising out of the very nature of reality as conceived in the abstract by the intellect; and Aristotle surely knew this to be true of all laws established merely by induction, of all generalizations from experience. The evidence for the truth of these lies in our experience of contingent fact: they are seen to be true by a necessity which is *contingent* and *hypothetical*. Empirical fact forces us to assent to them as true. Such facts might conceivably have been otherwise; but, being what they are, the only intelligible interpretation we can put upon them involves our acceptance of the inductively established law as being true *de facto*. If we are asked why *do we believe* such a law to be true, we answer that facts force us to believe it. If we are asked further why *is* it true, or why *are* the facts (which force us to believe it) such as they are, we cannot answer that the law *must* be true, or that the facts *must* be so, by an absolute, inviolable necessity of their very nature, in the same way as "two and two *must* be four" by an absolute necessity arising from the nature of numbers conceived in the abstract. We can only answer that the facts are so, and that consequently the law is true, because the Creator of the actual universe has made the universe so, and not otherwise. *Cf.* 224, 255.

account he excluded them from the domain of demonstrative science proper; thereby, perhaps, unduly narrowing the scope of "scientific" knowledge. For doing so, we may, in the words of Mr. Joseph, "say this much in his favour. Such an intellectual apprehension of the necessary truth of the principles from which demonstration is to start forms part of our ideal of knowledge;[1] doubtless it seldom enough forms part of the actuality. But Aristotle idealized; he spoke of what, as he conceived, science in the fullest sense of the term involved, and forgot to state, or failed to see that the sciences did not realize it."

There seems, however, to be a more serious deficiency in this Aristotelean conception of science. We can understand well enough by means of it how a body of *abstract* truths like those of mathematics—truths about a system of *possible, ideal essences*—may be derived from ideally necessary axioms and principles, of the widest extension and the simplest or poorest comprehension. But does it enable us to understand how the existing things and concrete facts of the whole actual universe, ourselves included, are derived from, and dependent on, their *real causes?* It does not: a chain of abstract demonstrative reasoning in geometry is hardly an adequate representation of the form in which the human mind possesses a synthetic or philosophical knowledge of the actual universe. In the former the "first principles" are abstract and self-evident, and the mediating concepts or middle terms, which are the "causes" of the successive conclusions, are poorer in their comprehension or fulness of meaning than these latter; whereas in a synthetic knowledge of the concrete, actual universe, the existence of a First Principle or First Cause, on which all else depends, is not self-evident, but must be proved *a posteriori* by the principle of causality; and, furthermore, the First Cause, and all created subordinate causes which serve as middle terms in our synthetic explanation of actual facts, must be richer in comprehension than these latter, inasmuch as they must contain in themselves all the perfections of the latter; and, finally, the laws according to which these causes act in the production of the actual course of nature, or order of the universe, must be established by induction. No doubt, Aristotle realized the force of *a posteriori* proof, and utilized it to establish the existence, wisdom, and perfection, of an immovable Prime Mover of the universe.[2] No doubt, also, he propounded the doctrine of moderate realism, which alone makes scientific knowledge of concrete facts possible—the doctrine that the reality revealed to the intellect in abstract thought is embodied in the concrete data of sense.[3] But his theory of demonstrative science, which sets forth the connexion of self-evident abstract principles with their conclusions as a representation of the

[1] "With this proviso," the author adds, "that for perfect knowledge all the parts of truth ought to seem mutually to involve each other. In mathematics, where alone we seem to achieve this insight into the necessity of the relations between the parts of a systematic body of truth, we find our theorems reciprocally demonstrable; and if twice two could be three, the whole system of numerical relations would be revolutionized . . ." (p. 358, n.). A system of truths reciprocally demonstrable in this way, may, perhaps, be allowed to be the ideal of a science of *abstract, possible essences*. But it certainly has never been proved to be our ideal of *human science* (in the sense of *certain knowledge*) of the *concrete, existing* things that make up the *actual universe*.

[2] *Cf.* De Wulf, *History of Medieval Philosophy*, pp. 39-41.

[3] *ibid.*, p. 37; Windelband, *History of Philosophy*, p. 139.

real order, appears to be too narrow and one-sided; nor does he show clearly how it is to be connected with our inductive and *a posteriori* reasoning from the facts of experience, so as to help us in building up a synthetic world-view, or philosophy of the universe as a whole. He was right in holding that our scientific knowledge of any individual fact could never surpass what is deducible from the specific type embodied in it, or reach to the *individual*, accidental, determinations of the fact (συμβεβηκότα). But he is not clear as to how we reach the necessary and universal judgments which give us knowledge about *real* essences, outside the domain of the purely abstract sciences. If he demanded for them the "*ipse dixit* of an incommunicable intuition"[1] he demanded what people generally will say is not forthcoming. He does seem to have held that such principles are *not* reached by any process of inductive generalization, or *a posteriori* reasoning from experience. And yet, outside the limited domain of abstract mathematics and metaphysics, where we have self-evident intuitions of possible essences, the only means we have of discovering and establishing scientific truths, *i.e.* necessary and universal truths, about the actual world, are induction and *a posteriori* reasoning.

254. PRINCIPAL KINDS OF PROOF.—(*a*) *Causal or "A priori" Proof; Proof of Fact or "A posteriori" Proof; "A Simultaneo" Proof.* Besides strict *Causal Demonstration*, by which we know anything *scientifically* through a knowledge of all its causes, and of the way in which it is produced by its causes (ἀπόδειξις διότι: *demonstratio propter quid:* proof which shows the causes of anything),[2] there is a sort of demonstration called *Proof of Fact.* (ἀπόδειξις ὅτι or εἰ ἔστι: *demonstratio quia*), which gives us certitude *that* a thing is so, without explaining to us why it is so. It falls naturally into a syllogism in the first figure, and differs from the demonstrative syllogism proper only in this, that it has for middle term not a "cause"—which is prior to the conclusion in the real order, φύσει or λόγῳ πρότερον — but some "effect" or otherwise connected fact which, though not really prior to the conclusion, is prior to it in our experience (ἡμῖν πρότερον), and is for us a sure evidence (τεκμήριον[3]) of the truth of the conclusion. Thus, when we argue from an effect that the supposed cause is such or such, our argument will be cogent if we know that no other cause could account for the effect in question. This is often possible, as, for instance, in the diagnosis of a disease by studying the patient's symptoms. So, also, when, in virtue of the principle of causality, we argue from the existence of an effect to the existence of an adequate cause, we are making valid use of

[1] JOSEPH, *op. cit.*, p. 357.
[2] *Cf. Anal. Post.*, ii., cap. x. [xi.], 1, where Aristotle enumerates the four causes.
[3] *Rhet.*, i., c. ii., 16, 17. Aristotle mentions this proof in connexion with the *Enthymeme. Cf.* JOSEPH, *op. cit.*, p. 323 n.; *supra*, 234-5.

this "proof of fact". Such are the proofs by which Aristotle and the Schoolmen have argued from the existence of motion, causation, and contingent being, to the existence of an immovable Prime Mover, a First Cause, a Necessary Being, distinct from the universe.

Strict or causal demonstration is also commonly known as "*a priori* proof"; while "proof of fact" is known as "*a posteriori* proof". Causal proof is called "*a priori*" because it proceeds from what is naturally or really prior, to that which is naturally or really posterior. And since the effect is naturally or really posterior to the cause, an argument which proceeds from effect to cause is called "*a posteriori*".

When the middle term is really neither prior nor posterior to the conclusion, when the passage of inference is from one of two concomitant connected facts, or abstract aspects of reality, to the other, the argument is called an "*a simultaneo* argument". The great historic example of this is the argument by which St. Anselm (1033-1109) sought to prove the existence of God from the notion we have of His infinite perfection. In the domain of induction, *a pari* arguments, arguments from *Example*, and from *Analogy*, are in a certain sense "*a simultaneo*"; while in the domain of deduction, proofs that are based upon reciprocal properties and relations (spatial or numerical) might also be regarded as "*a simultaneo*" (258).

(*b*) *Indirect Proof* or *Reductio ad Impossibile*.[1] The forms of proof already examined prove the truth of their conclusions *directly*. Where this cannot be done, it may be possible to show *indirectly* that a judgment is true, by showing that if it were false and its contradictory true, something impossible, absurd, or self-contradictory would follow. This method of establishing a truth by disproving its contradictory, is obviously less satisfactory and less scientific than direct proof; for it does not give the mind any insight into the positive, intrinsic causes or reasons why the established proposition is really true. Nevertheless, it is of great importance as a path to certain knowledge, and it is used extensively in every department of research. It is, as we saw, the sort of consideration underlying inferences in the second figure of syllogism (169). We have seen, too, that laws are discovered and verified inductively by disproving alternatives through the

[1] εἰς τὸ ἀδύνατον ἀπαγωγή.—*Cf. Anal. Prior.*, i., c. xxiii. [xliv.], 2; *supra* 169. For a different use of the term ἀπαγωγή *cf. Anal. Prior.*, ii., c. xxvii. [xxv.].

application of arguments in the second figure of syllogism (209, 212). Hence, it is by this process of indirect proof we know that inductively verified laws are *de facto* true, even though we may not be able to *explain* the latter, or show *why* they are true (247).

In dialectical discussions, when one of the disputants makes use (provisionally, and without necessarily assenting to them himself) of premisses admitted by the other, in order to disprove the latter's main contention, the former is said to be arguing "*ad hominem*" or making use of the "argumentum *ad hominem*". This latter expression is, however, also used, in quite a different sense, to designate a special form of the fallacy known as *Ignoratio Elenchi* (275, *A*, *a*).

(*c*) *Pure*, *Empiric*, and *Mixed Demonstration*. This is a division of direct or ostensive proof, and is based upon the nature of the judgments which are employed as premisses. *Pure* demonstration is that into which none but metaphysically necessary judgments enter (85-90, 198). It is exemplified in abstract metaphysics and mathematics. *Empiric* demonstration is that in which the premisses are synthetic judgments, truths of fact, inductively established generalizations, as in the physical sciences. Such demonstrations must, of course, *conform* to the *a priori* and absolutely necessary laws of thought; and each step or syllogism must contain at least one universal premiss, endowed with *some* degree of *necessity*; but these need not be *analytic* or *metaphysically necessary* propositions (198).

Mixed demonstration is that which contains both pure or abstract, and empirical or concrete premisses. The major states some metaphysically necessary principle; the minor asserts an empirical application of it; and the conclusion infers some consequence categorically. Such, for example, is the line of proof by which the existence of God, as the uncaused First Cause, is established: A series of efficient causes, directly subordinate to one another in their activity, cannot exist without an independent and uncaused First Cause. But we see in the world concrete examples of the existence of such series. Therefore an independent First Cause exists, namely God.

This is, perhaps, the most important of all forms of proof. By means of it we can apply the rational principles of abstract thought, the great necessary truths of the ideal or conceptual order, to the concrete facts and data of our sense experience. And it is only by such a process of proof we can infer from "the things that

are made"[1] the supreme truth of God's existence, which is the goal of all philosophy. Mixed proof, in this application of it, whereby we ascend from the knowledge of effects to the knowledge that there must exist a cause adequate to account for them, is obviously *a posteriori*. It is not to be confounded with ordinary scientific induction, which gives us *physical* certitude about the laws of created causes (229-33); for, based as it is on the principle of causality and the immediate data of consciousness (249), it gives us metaphysical certitude of the existence of a First Cause.

(*d*) *Circular* or *Regressive Demonstration.*—Reason ascends (by induction, or by *a posteriori* demonstration) from effect to cause, and then descends again to explain the former by the latter. Its movement is first *analytic*, then *synthetic* (202). It thus completes a sort of circle or regress, returning in a certain sense to its starting-point. Such a complete process is called *circular* or *regressive* reasoning. This is the natural path of valid thought in the discovery and proof of truth. Hence it must not be confounded with the fallacy known as the *vicious circle* (the *circulus vitiosus* or *petitio principii*). The latter is an attempt to prove a premiss by means of the conclusion which that very premiss is employed to establish[2] (198). But regressive demonstration sets out inductively from some fact or phenomenon, whose *existence* is certain, though its *nature* or *cause* is only vaguely conjectured as an hypothesis; and it returns from that nature or cause to the existing fact, only when it has established the former, and reached such a knowledge of it as explains the fact or phenomenon in question.

255. Demonstration and Scientific Explanation. "Popular Explanation".—What strict Aristotelean *demonstration* is to the deductive sciences, that *scientific explanation* is to the inductive sciences. We are said rather to demonstrate a "truth" and to explain a "fact," but the difference is only a verbal one. A "truth" is a judgment that is in conformity with some reality which it purports to interpret; the judgment itself is the logical truth, the reality is the ontological truth. The reality itself has various names: "being," "thing," "event," "fact," "phenomenon". The judgment which asserts that a thing "is"

[1] Rom. i. 20.

[2] The premisses of a valid demonstration must be known otherwise than through a knowledge of the conclusion itself; they must be known from an independent source.

or "exists," that a fact "takes place," that an event "happens," is said indiscriminately to assert a "truth" or a "fact". The expressions "That is a *fact*," and "That is *true*," are synonymous. Now, to give a *causal* demonstration (a *demonstratio propter quid*—διότι) of the truth of such an assertion is evidently the same as to explain fully why and wherefore the thing or event or phenomenon exists or takes place as it does—to "show" or "demonstrate" it, in and through its connexions with all its causes.

Nevertheless, the term "demonstration" seems by preference to be applied to the process by which we connect abstract truths with their first principles; and "explanation" to the process by which we connect the concrete existence and happening of things and events with their causes, and so come to understand the modes in which, or the laws according to which, they are produced by those causes. We may know that the three angles of a triangle are equal to two right angles without knowing *why;* and we may know that ice begins to form on the surface of a pond and not at the bottom, without knowing *why*. To answer the first "why" is to *demonstrate* a theorem in geometry; to answer the second is to *explain* a phenomenon in physics. To demonstrate truths is simply to show their connexion with simpler truths which we already understand, and ultimately with *first principles:* to show *how* they are involved in the latter, to harmonize and fit them in with that part of our knowledge to which they are logically or rationally akin. To explain facts is simply to show *why* they happen, *how* they occur, *how* they are connected with their *causes*, what these causes are, and what are the *laws* according to which they bring those facts about. We demonstrate consequent by antecedent until we reach *first principles;* we explain effects by causes until we reach *remote causes*, and, ultimately, the One, Uncreated *First Cause*.

It is in this discovery of causes, and of the laws to which their activities conform, that scientific explanation essentially consists. We "explain" a fact or phenomenon when we show it to be an instance of the application of some law. But this "law" itself may be only a general statement of the uniform occurrence of the fact in certain definite circumstances (247); and if so, the "law" itself needs explanation, suggesting as it does a distinct "why?" of its own. And so we try to explain the law itself in turn. This we do either (1) by showing that it expresses the combined application, simultaneous or successive, of certain

other already known laws.[1] For instance, the law that describes the path of a projectile as a parabola expresses the combined effects of the initial motion and of gravitation, acting simultaneously. The law that rocks and mountains are disintegrated by frost succeeding rain, is an expression of the joint effect of causes acting successively, each according to already known laws of its own. Or (2) we may be able to connect the law in question with other known laws showing them to be all special applications of some wider and more general law. Thus, the law of gravitation connected and "explained" the laws of falling bodies and the laws of the revolving planets. We have already met numerous other examples of explanation; and when dealing with hypothesis and causation we discussed the nature of the limitations within which phenomena can be "explained".

It is sometimes stated that Aristotle's conception of science is entirely different from the modern conception. But—apart from the fact that inductively established laws and their applications, which are nowadays universally regarded as "scientific," would not be so regarded by Aristotle—the difference really lies only in the terminology. He conceived science, after the manner of geometry, as starting from the *definition*, which reveals the essence of the "kind," and demonstrating the properties derivable from the latter. His theory of the specific type or form, as embodied in the individuals and forming their essence, was copiously illustrated by examples drawn from the domain of biology. But it is not so easy to distinguish the attributes that are *essential* to an organic type from the *properties* of the latter, as it is to distinguish between the essence or *definition* of a triangle and its *properties*. And the same difficulty prevails in all the sciences which deal with concrete, actually existing things. Hence, in these sciences, "for definition such as we have it in geometry, we must substitute classification; and for the demonstration of properties, the discovery of laws. A classification attempts to establish types; it selects some particular characteristics as determining the type of any species. . . . It will be the description of the type, drawn up on such principles as these, that will serve for definition".[2] Obviously, there is no change of ideal in substituting classification for definition; our aim in classification is to reach definitions of *real* kinds of things. So,

[1] *Cf.* MELLONE, *op. cit.*, pp. 328 *sqq.*; JOSEPH, *op. cit.*, pp. 474 *sqq.*
[2] JOSEPH, *op. cit.*, p. 89. *Cf. supra*, 47, 66.

too, the "discovery of laws" according to which natural agencies co-operate in maintaining the order of nature, is, *eo ipso*, the "demonstration of properties" which characterize those agencies. Mr. Joseph points to what is rather a contrast of terminology than a conflict of views when he says[1] "Science seeks to-day to establish for the most part what are called 'laws of nature'; and these are generally answers rather to the question 'Under what conditions do such and such a change take place?' than to the question 'What is the definition of such and such a subject?' or 'What are its essential attributes?'" He seems to think that the contrast lies in the different manner of putting the problems: "though it is possible to bring many scientific investigations to-day under one or other of the types of question which Aristotle says we inquire into, yet looking to his examples, one must confess that (as is natural) he puts the problems of science to himself in a very different manner from that in which scientific men put them now".[2] But the difference lies rather in the nature and scope of the problems themselves: the progress of discovery since the days of Aristotle has inevitably given rise to scientific problems of which he could not have had even a suspicion. And Mr. Joseph admits that it is "more in respect of the problems to be answered than of the logical character of the reasoning by which we must prove our answers to them, that Aristotle's views (as represented in the *Topics*) are antiquated".[3]

It will serve to bring out more clearly the nature of scientific explanation, if we contrast it briefly with the process of *Illustration*, which sometimes gets the name of "*Popular* Explanation". While the former enables us to understand things by what is *naturally prior* to them, the latter helps us to take in and realize new facts by what is *prior in our experience* to these latter. All progress in knowledge must be from the better known to the less known: *ignotum per ignotius* is not an aid to knowledge, but an impediment.

Since, however, explanatory principles are more remote from experience than familiar facts, we often have to try to take in

[1] *op. cit.*, pp. 358-9.

[2] *ibid.*, p. 359, n. 1. He refers to *Anal. Post.*, ii., c. i. 1: τὰ ζητούμενά ἐστιν ἴσα τὸν ἀριθμὸν ὅσαπερ ἐπιστάμεθα. ζητοῦμεν δὲ τέτταρα, τὸ ὅτι, τὸ διότι, εἰ ἔστι, τί ἐστιν. From the context these four would appear to be two alternative ways of asking (1) whether a thing is, and (2) why it is. In other words, it distinguishes two kinds of proof, namely, proof *of fact*, and *causal proof* or strict demonstration. *Cf.* 254.

[3] *ibid.*

some strange fact, or new generalization, before we understand the principles which really explain it. And we are enabled to take it in if it is described for us in terms of some already known, familiar facts. Such descriptions, by means of rough analogies and illustrations, are very extensively employed in all attempts at popularizing the truths of science, and bringing them somehow or other within the mental horizon of the man in the street or the youthful learner. And such descriptions are sometimes called *individual*, or *subjective*, or *popular* "explanations," because they are addressed to individual minds that are not yet capable of understanding the *real* explanation of the matters so described.

To borrow an example quoted by Professor Welton from Clifford's *Lectures and Essays*:[1] "It is [a popular] explanation of the moon's motion to say that she is a falling body, only she is going so fast and is so far off that she falls quite round to the other side of the earth, instead of hitting it; and so goes on for ever". This does not give us the *why* or *wherefore* of the fact: it is not a scientific explanation: We cannot *understand the moon's motion scientifically* until we are able to refer it to the laws of motion and gravitation. "But it is no [popular] explanation to say that a body falls because of gravitation. That means that the motion of the body may be resolved into a motion of every one of its particles towards every one of the particles of the earth, with an acceleration inversely as the square of the distance between them. But this attraction of two particles must always, I think, be less familiar than the original falling body, however early the children of the future begin to read their Newton." Therefore the latter explanation is not "*popular*"; but it *is* "*scientific*"; it is an explanation by principles and causes and laws which are in themselves prior to the concrete facts (*priora in se, natura sua*), though not more familiar to us (*priora et notiora quoad nos*).

256. LIMITATIONS OF SCIENTIFIC EXPLANATION.—If we regard scientific explanation as the process of bringing particular facts under inductively established laws, and of unifying these separate, isolated laws by bringing them in turn under still wider and remoter inductive generalizations, then there arises this peculiar difficulty: that we are explaining particular facts and narrower laws by an appeal to wider ultimate laws which do not themselves admit of a similar explanation. We saw something analogous in examining demonstration: it also rests ultimately on principles that are themselves *indemonstrable*. But then, these latter are self-evident, whereas the widest generalizations of induction are not self-evident; and hence Aristotle would not recognize the knowledge based on these as "scientific" in the strict sense. But such knowledge is now universally regarded as scientific; and rightly so, for these ultimate inductive laws have sufficient evidence of their truth in the facts of experience. They are not intrinsically and immediately evident like the axioms of geometry, but they are believed to be true because we see that they alone are compatible with the facts of experience, or, at all events, that they furnish us with the most satisfactory explanation we can find for the facts of experience (221, 230). We

[1] pp. 102-3, *apud* WELTON, *op. cit.*, ii., p. 190.

see intuitively that the abstract principles of logic, metaphysics, and mathematics *must be true* absolutely, because reality as conceived in the abstract by the intellect, or, in other words, the abstract, possible essences of things, are seen by the intellect to involve necessarily the truth of those principles. We do not see in this intuitive manner that our widest inductive generalizations—such as the law of gravitation, or the uniformity of nature—are true in this same absolute sense; for they are not. But we see that they *are*, *de facto*, hypothetically and contingently *true*, because our sense experience of concrete, contingent facts forces us to admit their truth, and would not be explicable, or intelligible, or rational, on any other hypothesis.[1] We see why *we must believe them* to be true, namely, because the facts of our experience are what they are. But it may be said that this knowledge is not *explanatory*: that it does not show us why the laws in question *must be true*. And this may be admitted; for no contingent, hypothetical laws, however wide, can offer an ultimate explanation of concrete facts. Laws are but the expression of the *modus agendi*, the manner of acting, of causes or combinations of causes. And we shall not have fully explained any concrete fact in the universe until we know why the agencies of nature act according to those widest laws—why, for example, matter gravitates, or why life comes only from life, or why natural causes act uniformly.

"We may point to facts," writes Mr. Joseph,[2] "from which it follows that we must believe a proposition; but we do not thereby explain the proposition, It is the thing believed, and not our believing, which must be shown to follow, if we are to say that we are finding an explanation." But "the thing believed" is the proposition. And unless the proposition itself were seen by us to follow from our previous interpretations of experience, neither would our belief in it follow from these. We give our assent to the *existential* propositions embodying the facts, because we have the testimony of our senses that the facts *are* so. And we give our assent to the laws because we see that the facts involve these latter. The next question, about both facts and laws, is, *Why* are they so? Or, to apply the same question to one great assent underlying all inductive inference: We believe that physical agencies are uniform in their activities because *they are* uniform, but *why are they* uniform? Philosophers differ in their answers to this ultimate question because they differ in their views as to the nature of reality as a whole. The sufficient reason which satisfies all theists, why this and all facts are so—the one which they consider the *only* true reason—is that the Will of God has made them so. In reaching

[1] Mr. Joseph says of *logical* principles that "every explanation must be consistent with them but they will not themselves explain anything" (*op. cit.*, p. 466). But if the principles of abstract (geometrical) *magnitude* and *number* "explain" or "demonstrate" the abstract conclusions derived from them in geometry and mathematics, so do the principles of pure *thought* and *being* "explain" or "demonstrate" the abstract conclusions of logic and metaphysics. But by "Explanation" he understands here *proximate* or "*scientific*" explanation, as opposed to *ultimate* or "*philosophical*" explanation; for he says: "In all explanations, our premisses are 'special' or 'proper' or scientific principles" (*ibid.*); though he goes on to raise distinctly ultimate or philosophical questions in discussing "Explanation". In accordance with our view of logic, as concerned not merely with "scientific" but also with "philosophical" thought (202) we take the term "Explanation" in its fuller and deeper sense.

[2] *op. cit.*, p. 466, n. 1.

this position we do not rely on induction alone; we reach a stage at which we must substitute the simple *a posteriori* argument from effect to cause. This we do when we pass from the *special sciences*, which deal with the *proximate causes* of *limited groups* of phenomena, and the *proximate* principles of special departments of knowledge, into *philosophy*, which aims at offering an ultimate explanation of all truths and of all things—as far as the human mind can attain to such,—by tracing all truths and all things to the One Divine Being who is the First Principle of all truth and the First Cause of all created reality (232).

257. AN ERRONEOUS VIEW OF EXPLANATION.—In contrast with this *theistic* view of Explanation—as a knowledge of things through their causes, terminating ultimately in the recognition of a Supreme First Cause, the Deity, on Whom the universe of sense depends—we have the *Hegelian*, *Idealistic* view of Explanation as the knowledge of things *through their relations to other things*, terminating in the conviction that all are parts of one systematic, self-existent, self-explaining whole.[1]

Writers of this school *identify* "reality" with "thought," and endeavour to show that "things" are "sets of unalterable relations" established or constituted by "mind". The effect of this attitude on their logic is to extend the necessary and universal relations which we institute between our abstract concepts, to what we call the concrete world of phenomena: in other words, to assume or postulate that the world of our experience is governed by the same necessary laws as govern our necessary judgments (215, 224). To suppose, thus, that everything which actually *exists* or *happens* does so by the same necessity by which whatever happens has a cause, by which a thing is what it is, by which two and two are four, etc., is to confound the actual with the possible, the existent with the merely thinkable, the physical or moral necessity which governs those things and occurrences that are dependent on the Divine Will and on human free will with the logical and metaphysical necessity which characterizes the relations established by our thought between abstract, possible essences. Hence these authors set up the strict Aristotelean concept of science—the knowledge by which we know that a thing "cannot be otherwise than it is"—as the ideal of *all* science, even of physical and moral phenomena; whereas it really applies only to those sciences which yield metaphysically necessary judgments about abstract objects of thought considered by the mind in a "possible" state, *i.e.* as apart from actual existence and free from all change. Professor Welton, for example, lays down as a "postulate of knowledge" in regard to the actual world, that we must assume its "every detail, even the smallest, as so determined by conditions that, under the circumstances, it *could not possibly be other than it is*. That the given is *necessary* is an assumption without which it would be helpless to attempt to explain it, for all explanation resolves itself into ascertaining the *exact conditions* by which the given is determined. *When the conditions of every detail of a phenomenon are so fully and exactly known that not only a phenomenon of this general character, but just this very phenomenon, with exactly these details, and each in exactly this amount, must follow from those conditions and from those only, then that phenomenon is fully explained.* Doubtless, in the vast

[1] WELTON, *Logic*, ii., ch. vii., § 159; *cf.* JOYCE, *Principles of Logic*, pp. 248-51, 338.

majority of cases such thoroughness . . . is not attained. . . . But the ideal of explanation is the same : it is thorough in so far as the given can be shown *to be the necessary consequence of certain definite and necessary conditions.*" [1]

The "necessity" referred to here is that which characterizes science in the strict Aristotelean sense, whereby we are said to know a thing *scientifically* when we know that it "cannot possibly be other than it is". To demand such "necessity" as a hall-mark of all scientific explanation is tantamount to a declaration that we can have no scientific knowledge of the concrete, actual, existing world at all, but only of the abstract, possible objects of thought conceived by our own intellects.

Father Joyce, in his *Principles of Logic*,[2] contrasts this [Idealistic] account of explanation, as that of the *part* by the *whole* of an *organism*, with the Scholastic [Theistic] account, as that of *effect* by *cause* in an *organization.* Of course, the Idealist conception of the universe as one in *nature* or *being* (*Monism, Pantheism*), and not merely one in *order* (a "*Cosmos*" distinct from the Infinite Being who created it ; *Dualism, Theism*), and of individual "things" and "events" as not *really distinct* from one another, but as made up of groups of "relations" conceived by that one Mind, which *is* the world —such a conception is entirely erroneous.[3] But its evil effect on the doctrines of "Explanation," "Demonstration," and "Science" is to narrow these concepts unduly by setting up for them too exacting and even impossible ideals, rather than, as Father Joyce states, to make them purely provisional and involve them in an endless regress. Even in the Scholastic view of explanation there is a true sense in Mr. Bosanquet's remark that "nothing can be known rightly, without knowing all else rightly" ;[4] for, all research into the ultimate reason of logical first principles and other axiomatic *truths* leads us ultimately to the Divine Intellect ; and all research into the ultimate causes of *existing things* leads us ultimately to the Divine Will ; and we take it that Divine Wisdom has so planned the created cosmos, and interrelated its parts, that the whole might be understood in the part, and the part in the whole, if these were known "rightly," *i.e. comprehensively ;* but to know them thus would be to see into the *Fiat* of the Divine Will, which is proper to God alone : the only "*must*" the only "necessity" there can be in *actual* things and events, past, present, future, is that they *must* be as God freely wills them to be (224).

Since the immediate causes of any individual phenomenon depend on remote ones, and these on remoter ones still ; and since in this way no individual phenomenon in nature is isolated, but each is bound up with the others : a *full and complete* knowledge of any one would necessitate a like knowledge of all nature. If, therefore, the latter were regarded, according to the Monistic view, as a closed system subject to absolute logical necessity or determinism, and if we were certain of the truth of this view—as we are of its falsity,—we could entertain hopes of a complete and perfect knowledge of all reality ; and our knowledge of physical causation would be an *absolutely certain* knowledge of an *absolutely necessary* relation between phenomena. Few, however, have the hardihood to put forward such a claim. "As the universe is a systematic

[1] *op. cit.*, vol. ii., pp. 188-9 (italics ours). [2] pp. 338-9.

[3] A trenchant and destructive analysis of these Neo-Hegelian views will be found in Professor Veitch's *Knowing and Being* (Blackwood, 1889).

[4] *Logic*, p. 393, *apud* JOYCE, *ibid.*

whole," writes Professor Welton, "[the totality of the conditions of any concrete phenomenon] is, in its primary meaning, that whole system. In this sense an ultimate analysis is obviously impossible. . . ."[1]

If, however, the whole physical universe and all its activities be regarded as contingent, and dependent on the free creating and conserving influence of a Supreme, Self-existent, Necessary Being, distinct from this universe, then, obviously, our certitude about these activities and their laws cannot be *necessary*, *absolute*, *metaphysical*, but only *contingent*, *conditional*, *physical*.

Mr. Joseph[2] seems to think that all our scientific knowledge rests ultimately on certain assumptions, or "maxims," or "anticipations,"—such as our "notion of what a rational universe should be," and our "belief that the universe is rational," and our belief in the "uniformity of nature,"—which are neither self-evident nor capable of deductive explanation on the one hand, nor "derived from experience" on the other (231). If our assent to such fundamental truths, no matter by what name we call them, is thus in no way rationally explicable or justifiable, the science that is based upon them ceases to be rational too, inasmuch as its foundations are insoluble enigmas. But the human mind has never acquiesced in any such ultimate avowal of its own impotence. It claims—and it is right in claiming, for it really possesses—the power to justify its assent to these foundations of science, by connecting them rationally with the truth of God's existence. And this truth it undoubtedly derives "from experience"—that is, from experience as revealed to the senses and interpreted by reason. For all facts, including the existence of God, the First Fact, experience, in this full sense, is our ultimate court of appeal.

So, too, the truth of the uniformity of nature (224) is not independent of experience in the same way as mathematical axioms are; though this seems to be the way in which Mr. Joseph regards it.[3] Not only do we derive from experience the concepts involved in it, as indeed he admits,[4] but the truth itself, referring, as it does, not to the abstract, conceptual order merely, but to the concrete, existing order of things, is grounded in, and confirmed by, experience. Not that we can ever positively call it into doubt: to do so would be as fatal as to doubt seriously the capacity of the mind to attain to truth: it is one of those principles the truth of which we must postulate or assume *provisionally* from the start: an assumption which experience *justifies* afterwards, by illustrating the success of these principles rather than by any formal demonstration of them.

258. DISCOVERY AND PROOF OF TRUTH BY INDUCTION AND BY DEDUCTION.—The question is sometimes asked whether logic ought to concern itself with laying down canons for the *discovery* of truth as well as for the *proof* of truth (210). There can be no doubt that it ought. And as a matter of fact it always does; nor is this surprising when we remember that although "discovery" naturally precedes "proof" or explanation, yet we can scarcely be said to have "discovered" a truth fully, *i.e.* to have mastered it mentally and made it our own, until we have connected it rationally with the rest of our knowledge, and seen its relations to kindred truths, and their bearing upon one another—in a word, until we have "explained" or "proved" it scientifically.

[1] *op. cit.*, ii., p. 119.
[2] *op. cit.*, p. 469.
[3] *Cf. op. cit.*, pp. 506, 510, 511.
[4] *ibid.*, p. 511.

We have already compared deduction and induction as methods, or lines of direction, according to which progress in knowledge may be made (213). Let us now compare them briefly from the standpoint of their material contents, inquiring how truth is discovered and proved in each.

The problem of discovering and proving a general truth or law by *induction* may be stated in this way: Given that in a particular case (or cases), *S*, is observed to be connected with *P*, find *whether* and *why* "*All S's are P*," or, *discover* and *prove* that "*All S's are P*". And the problem should be regarded as fully solved only when we can assert that "All *S*'s will be and must be *P*, *because*, or *provided that*, or *as long as* (*a*) they will be *M*, and (*b*) *M* will be *P*; *and in no other conditions or circumstances*". This solution supposes that in *M* we have reached the ground or reason which not only *necessitates* the connexion between *S* and *P*, but which *alone* can necessitate this connexion—on the assumption that the course of nature is not miraculously interfered with. It supposes that we are able to overcome and remove all indefiniteness from the conditions, to eliminate "plurality of antecedents (140) or causes" (221) by discovering among all the possible groups (each of which was regarded as a distinct and separate "antecedent"), the one *necessitating* and *indispensable* factor which was common to, and operative in, all of them, and in virtue of which all of them *necessitated*, though no one of them was *indispensable* to, the given consequent.

Let us see, in the next place, how the problem of discovering and proving a truth, whether general or particular, by *deduction*, may be stated. What exactly is the nature of the mental process by which truths are discovered and proved "deductively,"—in geometry, for example? It would certainly be an inadequate and misleading statement of the deductive method to represent the problem of deduction as: "Given a certain antecedent, or certain premisses, find the consequent or conclusion." In discussing the *nature and characteristics of inference* (197, 198) we saw that the real difficulty of discovering and proving new truths deductively lay rather in *discovering proper antecedents*—fresh and fruitful combinations of old truths for the formation of new sets of premisses,—than in the comparatively simple task of detecting the new consequent or conclusion in the newly formed premisses, and formally inferring it therefrom. If the general *inductive* problem might be stated: "Given one of the multitudinous facts of sense experience, which make up the physical universe, discover the causes and laws by which it happens," the general *deductive* problem might be stated: "Given a knowledge of certain, necessary, self-evident principles, discover all the truths involved in them".

"Deduction and Induction," writes Dr. Mellone,[1] "are not two different and independent kinds of reasoning. The real process of thinking is *the same in both*—*i.e.* to find a place for some fact as a detail within a system [*cf.* 212, 213]. In the case of syllogistic deductive reasoning our 'system' is partly known beforehand, in the form of a general law under which the fact or detail is brought. We start, having *in our hands* the common thread which unites the various facts. But in Inductive reasoning we have to *find* the common thread. We [*a*] start with certain kinds of facts which occur together in our experience. We [*b*] assume that there *is some* principle which unites them; and our object is to read out of these particular details the general law of

[1] *op. cit.*, p. 383.

their connection, and [*c*], if possible, to *explain* this connection by further connecting it with other laws: and this is to connect facts and laws into a systematic whole."

With some qualifications, this presents the case correctly. We may admit that in both induction and deduction the common aim is to *systematize;* but it is more. We must know the *contents* of the system before arranging them; the mental process of advancing in knowledge, whether inductively or deductively, involves *discovery* as well as *proof*. We start, in deduction, with the knowledge of a number of general laws, but without a knowledge—or even a suspicion, at first—of any of the consequents that may be evolved out of them, and made to serve in turn as proximate antecedents for further consequents. We have "in our hands" not any common thread which we see or know to be the means of uniting further truths or facts, but rather with a whole tangled skein of endless threads, growing into and out of one another, and leading outward and onward we know not where. The complicated details involved in them it is the duty of the deductive scientist to bring to light; and he does this by obtaining successive intuitions of new relations in the domain of abstract thought which he has under consideration—for example, by intuitions of spatial or quantitative relations in the study of geometry or algebra.[1]

The first step in the discovery of a new truth, "*S* is *P*," by deduction, seems to be (*a*) the observation of some case or cases in which *de facto* "*S is P*," or (*b*) the occurrence, to our minds reflecting on known truths, of some such connexions (between these latter) as *suggest* to us the possibility that "*S as such is P*". It might, for example, have been (*a*) the *actual measurement of a few instances* that first led to the surmise that "The right-angled triangle *as such* has *always* and *necessarily* the square on its hypotenuse equal in area to the sum of the squares on its sides". Or (*b*), the theorem might have first suggested itself to a geometrician from some speculations of his, some mental connexions he established by pondering on the concepts and truths he already possessed about triangles, squares, etc. In either case the process, so far, would seem to correspond to the inductive scientist's initial observation of certain facts, leading him to suspect, and to conceive as an hypothesis, the truth of the judgment "*S as such is P*". Neither the deductive nor the inductive investigator may ever have experienced a single *actual* case of *S* being *P*: but only some other truths or facts relative to *S* and *P*, which might have suggested that there is perhaps a necessary or causal connexion between the latter.

Of course, if the deductive inquirer, in his meditations, actually hits upon some notion (*M*), which reveals to him a *necessary* connexion between *S* and *P*, he simultaneously *discovers* and *proves* this latter connexion. As soon as he realizes *simultaneously* in consciousness the truth of the two judgments, that "*M as such is P*" and "*S as such is M*," he instantly discovers, and simultaneously *demonstrates* (nay, *discovers by demonstrating*, by explaining its *how* and *why*) that "*S as such is P*" (197, 198). In such a case, the conception of the new truth as an hypothesis is simultaneous with its verification as a truth, and with its *demonstration* through its "causes".

Similarly, were the inductive inquirer to discover, among observed facts relating to *S* and *P*, some agency (*M*) whose operation he now realized to be

[1] *Cf.* JOSEPH, *op. cit.*, p. 505.

such as would, in the circumstances, *necessitate* S being P, he would, *eo ipso*, have both *discovered* and *proved* S to be P, even though he had never yet witnessed a single actual case of S being P. Such a mode, however, of simultaneous discovery and proof, is rare in the inductive sciences, though it represents, perhaps, what *more usually takes place* in direct, progressive, deductive lines of demonstrative inference (187, 188).

However, it does not *always* happen, even in deduction, that the investigator thus simultaneously suspects, verifies, and demonstrates or explains, a new truth. Let us, therefore, pursue the case in which the geometrician, for instance, after finding by measurement that the square on the hypotenuse of a given right-angled triangle was equal in area to the sum of the squares on the sides, *supposes* this to be always and necessarily true of all right-angled triangles, without yet *knowing* either *that* it is, or *why it* is, true. He sets about *verifying* and *proving* his supposition—not by carefully measuring a number of other right-angled triangles, but rather by reflecting that *if* the general proposition is true there must be a discoverable reason *why* it is true, a reason which will *demonstrate* its truth. Hence he proceeds to seek for something (M) in the nature of S and P—of the squares on the hypotenuse and sides of a right-angled triangle—in virtue of which S and P *necessarily* involve each other. This M he must discover *by reflection on what he already knows* about triangles, squares, etc., by *analysing* and comparing his notions, by making mental experiments, as it were, with a view to *eliminating* from all the judgments which he finds crowding in upon his mind around "S is P," those that are unessential to the latter, until he succeeds in explaining or demonstrating to himself that "S is P" by connecting it through the medium of M with truths he has already established for certain, and so, ultimately, with first principles.

Thus, here too, as before, he *verifies* the new judgment "S is P," or *discovers* that it is always true *de facto*, by *demonstrating* it, by showing it to be *necessarily involved* in already known and proved truths. In induction, on the other hand, a general law is often discovered and verified long before it can be explained (247). Apart from this point, however, there is a certain analogy between the process just outlined and the experimental testing and verification of an hypothesis in induction. Both are illustrations of what the Schoolmen called "*inventio medii*," the process of finding a "middle term" of proof (167, 197). In induction, having observed S to be *de facto* connected with P in some case or cases, our task is to find out whether the connexion is a necessary, causal connexion, or only an accidental, contingent one ; and we prove that it is causal by showing (if we can) that something essential to S, namely M, is necessarily or causally connected with P. But how do we satisfy ourselves here that the M which connects S with P is itself necessarily connected with P? In other words, how do we know that our premisses (especially our major premiss, "M is P") are necessarily and universally true? *Not* as in the deductive sciences, by showing our premisses to be either intrinsically self-evident principles or else derived by demonstration from such principles, *but*, as we saw in dealing with the verification of hypotheses (229-33) by convincing ourselves, *through observation and experiment*, that *nothing else* but the truth of those premisses *is compatible* with the conclusion which we know to be true as a fact. In other words, we have to convince ourselves, by inductive investigation, that *those pre-*

misses alone account for the conclusion. It may be that we do not yet understand *why* they must be true, not having so far found an "explanation" or "proof" of them, but the facts revealed in the conclusion force us to believe that they are *de facto* true; and that they alone are true compatibly with these facts. The establishing of such reciprocal relations between "cause" and "effect," between "antecedent" and consequent," is an ideal at which induction aims (212).

The attainment of this ideal involves a higher degree of precision in these latter concepts and terms than they possess in ordinary thought and language. Hence, while the canons of formal inference, following the wider usage which is consistent with *plurality* of "causes," or "reasons," or "antecedents," forbids us to infer the antecedent from the consequent, induction aims at reaching such an exact knowledge of the causal or rational relation, and its terms, as will make the relation reciprocal, and so enable us to infer from consequent to antecedent as well as from antecedent to consequent (221). We have seen already that induction does not often realize this ideal of knowledge; but at all events, starting from facts as consequents, it establishes the truth of causal laws as antecedents, by this kind of consideration: that these antecedents are true because they account satisfactorily for their consequents, and are the only ones that account for them so far as observation and experiment can inform us. And inductive science has no other and no better "explanation" to offer for its ultimate generalizations than the superior success of these latter in accounting for the facts of our experience (230): "Many explanations are put forward," says Mr. Joseph,[1] "which do not appeal only to principles already known, but have it as their avowed object to *prove* one or more of the principles which they employ. Explanation then figures as an instrument of induction; and J. S. Mill spoke accordingly of a 'deductive method of induction,' and rightly attributed great scientific importance to the process which he called by that name. No better instance . . . can be given than the familiar instance of the . . . theory of gravitation . . . [which Newton] proved for the first time by his use of it in explanation."

We have now to note that deductive science does not rely on any consideration of this kind for the proof of *its* antecedents: it does not prove its antecedents by showing that they alone can account for the consequents inferred from them. It proves them by deriving them from self-evident first principles. And as a matter of fact the antecedents so proved are, perhaps for the most part, *not* the *only* antecedents from which the consequents actually inferred can be derived. In the demonstrative proofs by which conclusions are derived from first principles in the deductive sciences, the antecedents are regarded as *sufficient* grounds, but not necessarily as the *only* or *indispensable* grounds, of the consequents (213). This is noteworthy—that in order to establish a conclusion deductively by a "causal" demonstration (252), it is required indeed that the middle term give some "cause" which will *necessitate* the conclusion, but not that it give the *only* or *indispensable* "cause" of the latter. In other words, a causal proof is recognized as a strict demonstration even although it does *not* give the *only possible cause* of the conclusion: for one and the same conclusion there may be a *plurality of antecedents*, a plurality of distinct lines of demonstration, each connecting the conclusion in some

[1] *op. cit.*, p. 477.

special way of its own with first principles. Each of these antecedents gives a "cause" of the conclusion, *i.e.* something which is nearer to first principles, and more evident, than the conclusion itself; or, at least, something which helps us to understand *why* the conclusion is true, by showing forth some intrinsic connexion of the latter with the whole system of reality to which it belongs. In proportion to the depth and clearness of our insight into this system, we shall be able to see the mental, rational relations that obtain among those antecedents themselves, and between these and their antecedents and consequents. When these relations are *reciprocal*, the antecedent cannot be said to give a "cause" of the consequent (254, *a*). "Still," writes Mr. Joseph,[1] "the reasoning is deductive, since our premisses display to us the rational necessity of the conclusion, and do not leave it resting on a mere necessity of inference"; that is, on the necessity of an inference which is "based on an appeal to facts which might conceivably have been otherwise".[2]

In the mathematical sciences, where we deal with conclusions arising from self-evident intuitions of *Quantity* ("magnitude" and "multitude"), truths are often reciprocally related, so that the distinction between antecedent and consequent practically vanishes. Not only can we, having demonstrated a truth, A, from first principles, use it in turn to demonstrate another truth, C, but we can embrace the alternative of demonstrating C from first principles and then using it to demonstrate A. If we know two such propositions to be reciprocals, *i.e.* such that the truth of either involves the truth of the other, and if, further, we know one of them to be true, we can prove the truth of the other by showing that *the latter as antecedent proves the truth of the former as consequent.*

"Thus in proving a theorem, or solving a problem which is supposed to be set before us, we take the result provisionally for granted as a starting-point, and say; If this be true then would that, and if that be true so would some other; and so on, until we come to some already recognized truth. The fact of being led back to this point establishes the conclusion".[3] The process might be symbolized thus: "If Z then Y, if Y then X, . . . if B then A; but A; therefore Z". The validity of this inference depends on the assumption that the truth of the consequent involves that of the antecedent: an assumption not guaranteed in *ordinary thought*, of which alone the formal canons of inference take cognizance. It is, however, often admissible in mathematics "where most of our propositions are of the nature of equations, rather than ordinary predications,"[4] and are, therefore, reciprocal and simply convertible.

259. MORAL CERTITUDE IN THE "HUMAN" SCIENCES.—In the foregoing sections (249-58) we have been considering the sources, conditions, and limitations of those sorts of "scientific" knowledge about which we can have metaphysical or physical certitude. But there is also a sort of knowledge which is rightly called "scientific," about which we have *moral certitude*. Among the many more or less closely allied meanings of "moral certitude," we may distinguish these three: (1) firm or certain assent to

[1] *op. cit.* p. 505, n. 2.
[2] *ibid.*, p. 410, n. 1.
[3] VENN, *Empirical Logic*, p. 369.
[4] *ibid.* p., 370.

general truths (and their applications) concerning the facts or phenomena exhibited in the conduct of men, considered as free moral and social agents; (2) firm belief based on the authority of human testimony concerning matters of fact beyond the range of personal sense experience; (3) a very high degree of opinion, practically amounting to a firm or certain assent, based on cumulative evidence. With this latter we shall deal in the following chapter.

Understood in the first sense, moral certitude is a genuinely firm or certain assent to general truths established by induction. It is the sort of certitude that prevails in regard to the subject-matter of the ethical, social, political, and economic sciences. The laws established in these sciences are based upon the undeniable existence of uniform tendencies implanted in human nature. These propensities are variously described as "moral" laws; "moral," or "human," or "rational," or "social" "instincts"; "natural bias"; "inclinations of free agents," etc.[1] They are quite compatible with the existence of free-will. They are only *conditionally* necessary, *i.e.* their applications to particular cases hold good only on the condition that in those cases man will not run counter to the dictates of his rational nature—as he may do, and is free to do, absolutely speaking. But this foreseen contingency does not diminish the firmness of our assent to these "moral" laws, any more than the foreseen contingency of a miracle interferes with our assent to "physical" laws. Nor is there any reason why the mere absolute possibility that a man may act "unnaturally," "abnormally," "unreasonably" in a particular case, should destroy our moral certitude that he will not do so, by producing in our minds a "prudent fear" that he will; just as the bare, absolute possibility of a miracle does not destroy our physical certitude about particular applications of physical laws. Provided we see no *special* reason to expect something abnormal in a particular case, we are certain that the law will apply. "When we assert that certitude shuts out all doubt and obviates all danger of a mistake, we have reference to well-founded, prudent, rational doubts, and to the danger of error truly such; and not to unfounded, foolish, irrational misgivings, and merely fantastic, imaginary perils. These latter are to be scouted and disregarded, and hence cannot destroy our firm adherence to the truth. As regards danger of error in particular [cases] . . . *danger* signifies exposure to

[1] *Cf.* Rother, S.J., *Certitude* (St. Louis, 1911), pp. 12-17, 40-70.

imminent or threatening evil; and, I think, it will be conceded by all that no risk is run, no chances are taken, if in reliance on the physical laws and moral instincts, I rest assured, for instance, that the solid oaken boards of my room, on which I am standing, will not be suddenly turned into thin air, but will continue to support me; or that a gay young student, who whilst boating with some of his friends has fallen overboard, will not refuse to grasp the oar held out to him." [1]

260. BELIEF ON AUTHORITY.—The moral certitude which we possess about truths of fact on the testimony of our fellow-men,[2] is based upon two of the "moral laws" referred to in the previous section, namely, (1) that men can attain to a certain knowledge of the facts of their experience, and (2) that men are naturally truthful in communicating such information to their fellow-men. In order to give a certain assent to any statement or proposition on the motive of authority, we must be sure that this authority is endowed with two qualifications: (1) knowledge ("*scientia*"), and (2) truthfulness ("*veracitas*"): that our authority is not deceived, and is not deceiving us. Manifestly, there is often very large room for the exercise of prudence, discretion, and judicious discrimination, in convincing ourselves of the presence of these two necessary conditions. And hence the practical impossibility of drawing any sharp line of demarcation between the evidence that will produce the firm assent of strict moral certitude, and the evidence which will guarantee only that very high degree of probability which is commonly described as "practical" certitude, or "moral" certitude in the wider sense of this expression (233, 249).

The knowledge which is based upon human testimony as to matters of fact is often described as "belief," in contradistinction to "science". There are undoubtedly grounds for the distinction. The motive for the assent which is called "belief" is *extrinsic* evidence: the testimony of a witness is somethiug extrinsic to the truth to which he testifies; whereas the motive for "scientific" assent is *intrinsic* evidence: it is something that is understood or seen *in* the truth or fact itself. Again, science is universally understood to be a knowledge primarily of *general truths* or *laws*, and of facts only as embodying and exemplifying these; whereas

[1] ROTHER, *op. cit.*, pp. 48, 49; *cf. ibid.*, pp. 52-54.

[2] Assent to truth on Divine Authority is called *Supernatural Faith*. Before assenting to a truth revealed by God we must be certain (*a*) that God exists, and (*b*) that He has revealed the truth in question. These previous assents are called *preambula fidei*, preliminaries of faith.

belief on authority is primarily and mainly an assent to individual truths of fact, irrespective of the laws embodied in these latter. But from those points of difference it must not be inferred that "scientific" knowledge, simply as such, is either more certain or more important than knowledge of facts on human authority. It would be a serious mistake to think that it is. From the general standpoint of philosophy, that is, from the standpoint of man's general outlook on the world and life, and on his own nature, destiny, and place in the universe, there are individual historical facts that are incomparably more momentous than whole bodies of "scientific" knowledge. The great group of facts comprised in the establishment of the Christian religion nearly two thousand years ago, is bound up with truths of greater import to men than, for instance, all the laws of the science of mechanics. "Scientific" knowledge is not, therefore, merely as such, more important than the knowledge known as "belief". Neither is it always and necessarily superior to the latter from the point of view of certitude, or firmness of assent. The character of the mental state, as revealed in consciousness, is undoubtedly not the same when I assent to the truth that "The three angles of a triangle are equal to two right angles" as when I believe that "The Liffey is not flowing backwards to-day," or that "Great Britain is an island"; but my assent is "firm" or "certain" in all three cases.

Human testimony can never, of course, be an *ultimate* motive of certain assent. Such assent, given on the authority of such testimony, must be always preceded by two other assents—to the two judgments, "My informant is not deceived," and "He is not deceiving me". Now, if we assent to these judgments, or either of them, on the authority of *some other* testimony, the same two conditions have to be verified in regard to this latter testimony. Hence, under pain of being involved in an endless regress, we must ultimately have *intrinsic* evidence for the presence of the two requisite qualifications in some testimony of the series.

Furthermore, in matters of *science*, where the truth can be ascertained by rational investigation into the intrinsic evidence, the right and proper certitude to seek is *scientific* certitude. But the authority of a teacher or master is not a motive that can produce certitude of this kind. It is an *extrinsic* motive, which can produce certitude *of belief;* and hence we cannot appeal to it in *scientific research.* In science, where our aim is to reach *scientific* certitude, such an appeal is the weakest of all arguments. This

has been the uniform teaching of Scholastic philosophers at all times.[1] They have taught that human authority in matters of science is only a "confirming criterion" of truth and certitude. And it is a commonplace truth of experience that when an individual investigator brings to light some new conclusion by a long line of laborious and sustained reasoning, the fact that other investigators have reached the same conclusion, and assent to it as true, corroborates his own findings and strengthens his belief that his reasoning has been sound and logical.

But because Scholastic philosophers, being for the most part Catholics, have always taught that God has revealed to man truths of supreme importance, and has established on earth an institution endowed with infallible authority to teach those truths; and because, too, they have rightly insisted on the undeniable fact that men can and do reasonably hold, with a certitude that is based *not upon intrinsic evidence, but on the authority of their fellow-men*, the vastly preponderating bulk of their certain knowledge: Scholastic philosophy has been wrongly and unjustifiably accused of opposing the progress of science by setting up human authority in the place of man's natural reason[2] (201). It is only ignorance of the real teachings both of Scholastic philosophy and of Catholic theology that could have originated and perpetuated such charges. Even the uneducated Catholic knows that faith in revealed truth—or, indeed, in any truth that is believed on authority—ought to be reasonable, that blind faith is unnatural to a reasoning being and derogatory to the dignity of his nature, that his faith would be irrational were he not convinced with certitude that the channel, through which he has received what he believes, is a reliable one.

Scientific certitude is, of course, desirable, about matters which can be known scientifically. But even about these how infinitesimally few there are, comparatively speaking, who are in possession of really scientific certitude! It is not scientifically certain assents, but beliefs based on authority, that shape the conduct of men's lives. The multitudes of mankind are influenced and led by the authority of the few; and no less in the twentieth than in the fifteenth, or tenth, or fifth, centuries. The masses may transfer their allegiance from leader to leader, but they will ever be led by some *authority* or other—as those are, nowadays, who proclaim in the name of modern science that reason is at last emancipated from the shackles of authority and will henceforth bow in reverence to science alone![3]

[1] "Locus ab auctoritate quae fundatur super ratione humana est infirmissimus."—ST. THOMAS, *Summa Theologica, I., Q. I., art. viii., ad* 2.

[2] For an account of a recent attempt to revive those calumnies, *cf.* art. "Philosophy and Sectarianism in Belfast University" in the *Irish Theological Quarterly*, October 1910 (reprinted, Dublin 1910); *The Value of Scholastic Philosophy* (Report of Privy Council Investigation. Dublin: Catholic Truth Society, 1910); O'KEEFFE, art. *Scholastic Philosophy*, in the *Irish Ecclesiastical Record*, April, 1911. For a fuller treatment of the real attitude of Scholastic philosophy towards authority at all times, we may refer the reader to De Wulf, *Scholasticism Old and New* (2nd edit. 1910), pp. 53-75, 190-200: *History of Medieval Philosophy*, pp. 109-113, 173-177, 206-212, 348-358, 403-406, 501-505.

[3] Mr. Balfour shows very clearly, in his *Foundations of Belief* (P. III., ch. ii.)

261. HISTORICAL SCIENCE AND CERTITUDE: ITS CRITERIA AND SOURCES.—The *writing* of history has for its aim to give us a faithful, vivid, and instructive picture of the past; and this is an *art* rather than a science. But it presupposes the accurate determination of past events, the detection of the combined agencies that culminated in those events, and the discovery and illustration of the human forces, instincts, tendencies or "laws" —social, racial, religious, political, economic, etc.—according to which those various agencies have co-operated. Now all this belongs to *historical research*, which is obviously a *science*, as being concerned with the elucidation of such laws. And when the historian seeks to synthesize these laws, to explain them deductively by connecting them with the more fundamental motives, impulses, and instincts of man as a social being, he is labouring in that domain which has been rightly called the *philosophy of history*.

In the work of historical research, the investigator is assisted by a body of *practical canons* or *rules of method*, the formulation and study of which constitute the *methodology* of his science. The part of logic which deals with general rules of method (Part IV. of the present treatise), applicable to *all* branches of investigation, might be described as "*general* methodology"; the supplementary canons arising from the special application of these to the particular subject-matter of any individual science, form the "*special* methodology" of that science. Since it is the tendency of logic nowadays to expound the general rules of method with somewhat too exclusive reference to the *physical* sciences (201), a bare outline, at least, of the principal rules or canons employed in arriving at truth on *human testimony*, cannot be considered out of place here.[1]

what a preponderating share of our assents is due to authority, and how very little is the ripe fruit of personal reflection. Not merely the prescriptions of domestic or civil or religious authority, but the current ideas of the age and country in which we live, the ever-changing phases of "public opinion," the popular worship of some hero of the hour, or some fashionable theory: all these influences envelop us in a sort of "psychological climate" or "atmosphere," which moulds and colours our beliefs and convictions, and which none of us can possibly escape.

[1] For fuller treatment of the subject (under the Scholastic title "*De arte critica*") *cf.* ZIGLIARA, *Logica*, (60), (61); HICKEY, *Summula Philosophiae Scholasticae*, i., pp. 257-75 (*Editio altera*, 1911); RICKABY, *First Principles of Knowledge*, pp. 377-90. Among writers who deal *ex professo* with historical criticism, the following may be consulted: DE SMEDT, *Principles de la critique historique* (Liége, 1883); IDEM, *Introductio generalis ad historiam ecclesiasticam critice tractandam* (Ghent, 1876); LANGLOIS and SEIGNOBOS, *Introduction aux études historiques* (3rd edit. Paris, 1905); FREEMAN, *The Methods of Historical Studies* (London, 1886); DELEHAYE, *Les legendes hagiographiques* (2nd edit. Paris, 1906; English tr. also

Criteria of human testimony. By testimony (*testimonium*) we mean the sensible or visible manifestation—oral, written, or otherwise—of one's knowledge to others. If the knowledge communicated be *doctrinal* or *scientific*, its communication constitutes what is called the *teaching office* ("*magisterium*"), and the communicator is called a *teacher* or *master* ("*doctor*" or "*magister*"); if it is knowledge *of fact*, its communication may be called *narration* ("*narratio*"), and he who communicates it a *witness* or *narrator* ("*testis*," "*narrator*").

The *authority* ("*auctoritas*") of the person or persons communicating the knowledge, is, of course, the value of their testimony as a motive for assenting to what they teach or narrate. The firmness of our intellectual assent or belief should be in proportion to the ascertained value of the testimony; if it is in excess of this, it is not belief but *credulity*. Our assent may, therefore, vary from mere opinion or probability in some cases, or practical certitude in others, to strict moral certitude, which latter may sometimes be as firm in its own order as physical or metaphysical certitude in theirs.

Quite a number of the practical rules laid down for testing the *knowledge* and *truthfulness* of the source of our information are obvious dictates of plain common sense. For example, in regard to the *knowledge* possessed by our witnesses or narrators, (*a*) we prefer, *ceteris paribus*, the testimony of an *eye-witness* to that of one who testifies on *hearsay*; (*b*) that of a contemporary (with the facts) to that of a subsequent writer, (*c*) that of a number of independent (or, better still, mutually hostile) witnesses, to that of interdependent or co-operating witnesses influenced by the same point of view. Furthermore, we must (*d*) ascertain from every available source, and take into account, the attitude of the narrator in regard to the events narrated, and everything that bears upon his powers of observation: whether he is prudent and painstaking, or credulous, or imaginative; whether he is influenced by *unconscious* prejudice or attachment to a special "point of view," or by an apologetic purpose. It is extremely important, if extremely difficult, to distinguish between the facts narrated and the personal views, or theories, or

published); IDEM, *Hagiography* (art. in the *Catholic Encyclopedia*, Vol. VII.); KIRSCH, *History* (*ibid.* with authorities there referred to); CAUCHIE, *Introduction à l'histoire ecclesiastique* (Louvain); O'MAHONY, *The Alleged Epoch in Historical Criticism at the Close of the Seventeenth Century*, in the *Record of the Maynooth Union*, 1911.

hypotheses, with which the writer colours his presentation of the facts: these are liable to be distorted in the presentation—particularly in "books which treat of comparative mythology, comparative religion, the origin of social institutions, and such matters, in which documents are scarce and obscure, or written in a language ill-understood, while inferences are often more marked by ingenuity than conclusiveness"[1]. Finally, to mention one other point out of many, (*e*) we must be influenced by the nature of the facts narrated, in determining whether the writer or witness may not have been deceived: strange, unusual, unexpected, wonderful (alleged) events, will naturally require to have been submitted to closer and more careful scrutiny than ordinary facts, before they can reasonably claim our assent. Here, the character, education, and beliefs of the witnesses, the age and circumstances in which they lived, the object with which they wrote, the amount of imaginative embellishment with which it was understood in their time that narratives of events might be clothed, their general conception of what constituted "history," are all considerations of paramount importance. They are called for chiefly, of course, in the domain of religious history, and are concerned mainly with the miraculous.

That miracles are possible, this is not the place to prove. We have rather merely to point out that it is *a flagrant violation of logical method* to dismiss all narratives of the miraculous from human history as untrue and incredible on the ground that "miracles are impossible," as long as the latter contention remains unproven. To assert *a priori*, as something self-evident, that "miracles are impossible," is an excellent example of the fallacy known as *undue assumption of axioms* (275, *A*, *c*). There has prevailed for the past few centuries a rather superficial rationalism which cannot, or will not, see that its gratuitous rejection of the miraculous involves it in this fallacy. Granting that miracles are possible, the reality of any individual alleged miraculous fact must stand or fall with the evidence adduced for or against it. Modern research in the domain of historical criticism has established many invaluable canons for the better appreciation and understanding of miracle-narratives in ancient and mediaeval religious literature: canons which enable us to appraise more accurately the historical worth of what is contained in such documents, and to understand better their scope and import. It is all-important, for instance, to realize that exact fidelity to objective fact, exclusive of all apologetic purpose, and rigorous verification of sources, in the writing of history or biography, are comparatively modern requirements, which were not demanded in ancient times, or in the Middle Ages, and need not, therefore, be expected in the writings that have come down to us from those periods.[2]

Next, in regard to the *truthfulness* of our authorities, we

[1] RICKABY, *op. cit.*, p. 382. [2] *ibid.*, pp. 383-5.

have to consider, for instance, (*a*) whether these were *consciously* prejudiced by apologetic purpose, or personal views or motives, to such a degree that they may have misstated or misrepresented the facts; (*b*) whether the facts and circumstances were such that the writers *could* have deceived their contemporaries and posterity had they tried to do so; (*c*) whether the character of the writers, as known from all available sources, was truthful and upright, or dishonest and unreliable.

It will be plain that some considerations render testimony suspect without enabling us to trace its unreliability definitely to the ignorance, or definitely to the untruthfulness, of its source. Such, for instance, is the inference we may sometimes be forced to draw from the silence of all other witnesses in regard to an alleged fact recorded by one or a few,—the *argumentum ex silentio*, as it is called. When can we argue that some allegation of fact is untrue, because, though the alleged fact is vouched for by one or a few writers, it is not mentioned by any other contemporary writer? Manifestly, not always; but only when the alleged fact is such that had it really happened it would certainly not have been left unrecorded by all other writers of the time. But of this it is for the most part very difficult to be sure. There are many instances in which such "arguments from silence," after being held for ages as conclusive, have been themselves discredited by subsequent authentication of the supposed discredited facts. This should make us cautious and moderate in the use of such inference.[1]

Sources of historical science. The human records or remains from which we derive our knowledge of the past, may be divided conveniently into three great classes: (*a*) *monuments* of every kind; (*b*) *documents* written or printed; (*c*) *oral tradition.* By *monuments* we understand works of art constructed to commemorate an event—coins, statues, columns, temples, etc. Their significance and value, as evidences of historical facts, belong to the science of *archæology*.

Written or printed *documents* are our most valuable source of information about the past. The interpretation of ancient written documents (*manuscripts*) belongs to the science and art of *palæography*. Before an ancient book or manuscript can be used as a reliable source, we must be sure that it is *authentic* or *genuine.* This implies (1) that it really comes from the author

[1] RICKABY, *op. cit.*, pp. 386-7.

to whom it is ascribed, or belongs to the epoch and country to which it is assigned; (2) that it has not been interpolated or corrupted by the interference of other hands. The proper ascription of a document is ascertained partly by *external*, and partly by *internal* evidence. Evidence of authorship, or of time or place of composition, is said to be external when derived from sources other than the document itself. The chief external evidence in most cases will be found in what tradition, oral or written, has handed down in each case. If tradition, whether oral or written, ascribes the composition of a work to a certain author, time, or place, *and if there is no special reason to doubt the reliability of its source*, we can be morally certain that the tradition is correct: for the simple reason that men are naturally truthful—"*nemo gratis mendax*,"—and that human testimony under due conditions is a sufficient criterion of truth. If, for instance, we find all contemporary and immediately subsequent writers, who refer to a work, ascribe it to a certain author, this may in itself suffice to settle the question. But, on the other hand, it may perhaps merely create an initial presumption in favour of the alleged authorship, and form a basis for further critical inquiry. This is true especially of all works that come down to us from the centuries prior to the invention of printing—which dates from the end of the fifteenth century. The origins of all traditions as to authorship, previous to this latter date, call for careful scrutiny; for in those earlier ages it was comparatively easy to give currency to writings by falsely representing them as the works of eminent authorities; and this facility was largely availed of.

Where external evidence is doubtful or conflicting, or where there is no such evidence at all, the *internal* evidence, derived from the work itself, may prove to be very valuable. Internal evidence must, of course, be very conclusive before it can upset a strong, uniform, and apparently well-grounded tradition as to authorship; but as a matter of fact it has exploded many such traditions; and it is constantly furnishing conclusive settlements to disputed questions regarding the provenance of documents. Internal evidence is of various kinds: (*a*) a comparison of the *style* of the work in question with that of other works known for certain to come from the supposed author, will be of more or less assistance in determining authorship; (*b*) references, in the document, to events the dates of which are known, will help to fix the date of composition (the possibility and the fact of *prophecy*,

in religious literature, being dealt with as in the case of miracles); (*c*) the consistency or inconsistency of the contents of the document with other certain works of the supposed author will also help to decide; and, in regard to manuscripts, (*d*) the form of the handwriting, modes of contraction, formation of the letters, etc., will often enable us to fix with sufficient accuracy the century, or country, or both, in which the manuscript was written.

By the *integrity* of a document we mean its fidelity in representing what the author actually wrote. This connotes absence of all later interpolations in the case of *original* manuscripts; and, furthermore, freedom from copyists' errors in the case of *copies*. Where the original written text is in existence, and known for certain to be such, it is, of course, the best evidence of what the author wished to express. But where, as in the case of all ancient works, we have only copies, and comparatively late copies, these invariably differ more or less among themselves. In all such cases, we aim at re-constituting the original text, or getting as near as possible to the latter, by collating all the best and oldest extant manuscript copies. This is only one of the wide fields of investigation that form the domain of historical criticism.

We have set down *oral tradition* in the third place as a source of historical or moral certitude. About the first beginnings of human institutions we have only oral traditions to rely on. When the art of writing was invented, these began to be committed to documents; and, many centuries later, were embodied in printed literature. Authentic documents can only testify to the existence of an oral tradition down to the date of composition of such documents; and it is with the value of the oral tradition itself we are concerned here. Now, oral tradition can be a sufficient motive for moral certitude. It is only, of course, in regard to facts of very great moment to some nation, or section of the human race, or to the race as a whole, that we actually have uninterrupted oral tradition. Again, such tradition can assure us only about the *substance* of such facts, not about *minor details;* in regard to these latter, oral tradition naturally and inevitably fluctuates: the very nature of this mode of narrating and transmitting knowledge from generation to generation involves imperfections and limitations of this kind. Furthermore, even in regard to the substance of the fact transmitted, the tradition must be not only *continuous*, reaching back to the fact itself, but also *widespread* (in a country, or continent, or throughout the

world, as the case may be), and *uniform* or *self-consistent* (while it may vary, even to inconsistency, about details). Our evidence that a tradition has been *continuous*, that it *reaches back to the fact itself*, will be sometimes mainly negative: lying in our inability to trace its origin to any subsequent source. But for the continuity and genuine origin of many traditions we can have valuable positive evidence: the existence of written documents, or of monuments, whether national, popular, tribal, military, or religious, testifying to the continuity of the tradition back to the time of the fact which it enshrines. The most reliable evidences to the origin and continuity of any form of social custom, or religious belief, are, of course, those that come from documents and monuments that have been brought into existence by the people, or institution, or society, conforming to such custom, or professing such form of belief. There are, perhaps, no oral traditions more strongly confirmed and corroborated by such evidences than those of the Catholic Church. We can sometimes prove that a given belief really had its origin at a certain time, and in certain facts, by proving that such belief could not have originated in a spurious tradition of later origin, or, in other words, that such a tradition could not have arisen in such a deceptive manner. This form of proof is known as the *argument from prescription.*

The proof that oral tradition can produce moral certitude is based upon the principle that men are naturally truthful, and upon the fact that in certain cases it is morally impossible that ignorance or deception could have intervened. The cases in point are those of remarkable public facts of great moment to whole peoples or nations, or to the human race as a whole. To admit that men generally could either be mistaken themselves, or mislead all their contemporaries and all posterity in regard to such facts, would amount to a practical denial that man is capable of attaining to any truth.

ROTHER, *Certitude: a Study in Philosophy*. HICKEY, *Summula Philosophiae Scholasticae*, vol. i., pp. 143-70. TOOHEY, *The Three Kinds of Certitude*, Irish Theol. Quarterly, July, 1909. RICKABY, *First Principles*, pt. i., chaps. iii., iv. and v.; pt. ii., chaps. vii. and viii. WELTON, *op. cit.*, ii., bk. v., chap. vii. JOYCE, *Principles of Logic*, pp. 132-6. JOSEPH, *op. cit.*, chaps. xvii., xxiii., xxv. MERCIER, *Logique*, pp. 286-93; *Critériologie Générale* (5me edit.), pp. 5-35. MELLONE, *op. cit.*, pp. 252, 258-9, 326-32, 382 *sqq*. WINDELBAND, *History of Philosophy* (Eng. tr.), pp. 132-54. DE WULF, *History of Medieval Philosophy*, pp. 39 *sqq*.

CHAPTER II.

OPINION AND PROBABILITY.

262. NATURE OF PROBABILITY: CUMULATIVE EVIDENCE: "PRACTICAL" CERTITUDE.—When the evidence, whether mediate or immediate, on which our assent to a judgment is based, *excludes all prudent fear of error*, our assent is firm; and this firmness of assent is termed *certitude* (248). When a normally constituted mind would not deem the evidence sufficient to exclude all such fear from its assent, the latter is called *opinion*. The intellectual motives, or reasons, which produce such assent, are described as "probable evidence". In the presence of such evidence we cannot be *sure* that our judgment is true, that it represents, even partially, the reality as it is; but we can recognize in the judgment a certain degree of *verisimilitude*: that it is more or less *like the truth*, or *likely to be true*; or, again, a certain degree of *probability*: that the judgment is more or less *probable*, *i.e. capable of being proved*, established, demonstrated, later on; and we will therefore give—or at least ought to give—to such a judgment a degree of assent proportionate to the weight of the evidence. While, therefore, opinion is the subjective mental assent, we may define the *probability* or *verisimilitude* of a judgment as *the degree of likelihood, estimated from the evidence in favour of the truth of the judgment, that the latter is really true.*

Probability, therefore, is not the "approach" or "approximation" of a judgment to truth, to conformity of the mind with reality, in the sense that *the judgment itself is more or less true*, but only in the sense that the evidence pointing to its truth is more or less strong.[1] The judgment itself, as a mental unit, a mental product, giving *one* representation of the reality, must be either true or false all the time: it cannot be *more or less*; either the mind in assenting to the judgment is in conformity with reality or it is not. There are not, properly speaking, any grades or degrees of con-

[1] *Cf.* ZIGLIARA, *Logica*, (42); *supra*, 248.

formity: conformity is not divisible. When, therefore, a judgment or proposition is spoken of as "more or less true," we must bear in mind the correct meaning of such language: that there is in the judgment a true sense and a false one, that these can be distinguished or separated, that *this* part is true and *that* false, and that the original composite judgment, in which the unanalysed, or insufficiently analysed, concepts and judgments were united into one mental relation between subject and predicate, was *false, taken as a whole*, putting the mind that formulated it out of harmony with reality (248).

Opinion, therefore, is the result of objective evidence which is not grasped or comprehended subjectively with sufficient clearness to guarantee a certain assent. If objective evidence, which is really sufficient for moral certitude, is grasped with sufficient clearness, but is undervalued, underestimated by the mind that receives it, it will not actually exclude the fear of error from such a mind, though it would from a normal mind. The subjective element is thus seen to be a factor in determining those varying grades of assent; but logic deals only with the objective element, and its effect upon the normal mind, leaving the study of the subjective factors to psychology.

Neither can logic furnish any calculus for determining the precise degree of mental assent proportionate to a given weight of probable reasons. Mental assent is a vital, conscious act, entirely beyond the scope of measurement in terms of any mechanical or quantitative units. It is the objective factors, which constitute the "probable evidence," that are alone in any way amenable to what has been called the "Logic of Chance" or the "Logic of Probability" (264). The degree of assent with which the individual will respond to probable evidence, and allow the latter to influence his actions and shape his conduct, is a matter of personal prudence and discretion.

Probable evidence, as we saw, may vary in strength from the extremely vague and doubtful indications that give rise to a mere suspicion, to the very solid and substantial motives that produce the sort of assent described in common language as "moral certitude." Hence, as there are degrees in the positive firmness of a *certain* assent, so there are also degrees in that of a probable one. The question may therefore be asked: Can opinion pass gradually into certitude, a probable judgment into a judgment certainly true, a probability into a certainty? Or, the question

may be put in this way: If we are to understand by *cumulative evidence*, one form of which is *circumstantial evidence*, a more or less considerable collection of reasons or motives, all of which point towards the truth of the judgment, and each of which, taken by itself, would only create a suspicion of that truth, can such a cumulation of reasons ever produce certitude? It is a matter of experience that they often *do* produce an assent which excludes all prudent fear of error, and which falls, therefore, within the definition of certitude. Such assents are commonly described as *morally* certain.[1] They constitute the vast majority of the assents upon which man's ordinary activity in life is based. Nor is there any reason why we should deny to them the quality of certitude. No doubt, the evidence for them is not of that cogent character which excludes even the *abstract possibility* of error, as it is in the case of metaphysical certitude. But it can exclude all *prudent fear* of error: so that in the concrete circumstances it is *morally impossible* that we should be deceived. And if we find ourselves in this state of mind, *after having exercised all due care and caution in weighing and analysing the evidence, we have certitude.*

It has to be remembered, however, that in applying the "moral laws" or "generalizations" already mentioned—such, for instance, as that "man is naturally truthful"—to *individual* cases, our assent must be for the most part qualified by some measure of reserve. It will be a "practical" certitude: an assent upon which a prudent man would act. But this means, after all, that the assent is in practice equivalent to a certain assent, though it is not really a certain assent. For, even in the concrete circumstances we *may* be deceived. If, for instance, it is an assent on the authority of a fellow-man, it may happen that, notwithstanding his well-known prudence and care in observing facts, he was in some unaccountable way deceived just on this occasion. Or, even though his veracity may be above suspicion, he may have had some interest, unknown to us, in deceiving us just this once, and may have succumbed to the temptation. "The unexpected happened"; and in our surprise we begin to see that our "practical" certitude was not certitude at all. Similarly, a jury reaches "practical certitude," on strong circumstantial evidence, that the prisoner in the dock committed the murder of which he stands accused. On this practical certitude the prisoner is condemned to death: and all who have carefully followed the evidence agree

[1] *Cf.* CLARKE, *Logic*, pp. 426-428.

with the verdict. Yet cases have occasionally occurred in which it was afterwards proved beyond all shadow of doubt that the condemned man was innocent of the crime. What do such facts prove? That the human mind is *incapable* of attaining to genuine certitude? No; but only that it is not *infallible* in the weighing and testing of evidence; that even when men have exercised what is commonly considered to be sufficient care, caution, and prudence, they *may* embrace what is erroneous. And such occasional failures will teach prudent men to be still more cautious in deciding grave issues on "practical" certitude.

263. PROBABLE ARGUMENTS: THE ARISTOTELEAN ENTHYMEME.—Among the many kinds of judgments which call for a probable assent, the following are the more important classes: (1) some of our interpretations of the immediate data of our sense experience; (2) some judgments—whether universal, singular, or particular—based on the motive of human testimony; (3) inductive generalizations from sense-experience, by way of enumerative induction, analogy, or unverified hypothesis; (4) conclusions derived by deductive inference from probable premisses; (5) to which may be added judgments reached by the application of the *calculus of probability* (266-8).

These various sources of probability do not reveal any *new forms of inference*. When probability is reached by inference, the obstacles to a *certain* assent lie not in the form but in the matter of the inferential process. With many of those sources of probability, as, for instance, those of the *third* class just mentioned, we are already familiar. Indeed, the main sphere of probability is to be found in induction. Before reaching a certain, scientific knowledge of the inductively established general law, we have to pass through many stages where our knowledge of it amounts to a greater or smaller probability. All those various steps we have already examined: the rough empirical generalizations from enumerative induction, the arguments from example and analogy, suggestions of more or less probable hypotheses, and, finally, the processes of analysis, and the methods of experiment, by which we seek to *verify* those *probabilities* and transfer them into the sphere of *certainties*.

Judgments derived from the *first* source mentioned above do not seem at first sight to involve any inference at all. But sense experience is, as a matter of fact, invariably accompanied not only by judgment or interpretation, but also by inference (238). This

inference is always practically unconscious, but it may be made explicit in a syllogism of the second figure. Thus, if in the shadows of the evening I see an object indistinctly looming into the field of vision, I may think or judge immediately: "That is a horse". But the mental process is really this:—

"A horse presents such and such appearances;
"This object presents those same appearances;
"Therefore this object is a horse."

And it is manifest that this process will produce certitude only where I can be sure that "Whatever presents those appearances is a horse"; in other words, where I can convert the major premiss simply, and so replace the formally invalid syllogism of the second figure by a valid "proof of fact" in the first figure. If I cannot do this, the "inference of perception," as it is sometimes called, will only produce probability. It is clearly of the same form as the argument from analogy, and falls under the class of argument known as the Aristotelean enthymeme, with which we shall deal presently.

The *second* source of probability mentioned above is human testimony. When we cannot secure sufficient evidence, as to the knowledge and trustworthiness of our authority, to produce moral certitude, we must rest content with a probable assent to whatever we accept on such authority: even though the natural yearning of the mind is for a *certain* knowledge of the truth. Certitude is not always attainable; people often have to be content with that high degree of probability which amounts to practical certitude. Hence it has been said that "*Probability is the guide of life*": The farmer sows his fields, the manufacturer erects his machinery, the merchant opens his business, the soldier goes to battle, young people marry, and statesmen legislate. Under the influence of what determining motives? What has the future to offer them? Hopes of success; *Probabilities.*[1]

As a *fourth* source of probability we have set down deductive inference from premisses which are not all certain. If any one premiss of an inference be only probable, the conclusion cannot be more than probable: it must follow the weaker premiss. A *probable argument* may be defined as an argument *whose premisses*

[1] "Tota praesens vita per probabilitatem maxime ducitur. Relationes omnes hominum in familia et in republica viventium, probabilitate fundantur. Qui scribit, qui navigat, qui militat, qui uxorem ducit, et qui leges condit, nonnisi intuitu probabilis eventus operatur."—LEPIDI, *Elementa Philosophiae Christianae*, i., p. 318.

do not warrant a certain, but only a probable, conclusion. This may happen in two ways: (*a*) either because the premisses contain a judgment which is only probably true; or (*b*) because, though the premisses are all true, they do not necessarily involve, but only suggest, or point to, the truth of the conclusion we may seek to derive from them.

(*a*) The former class of argument was described by Aristotle as a *syllogism from probabilities:* *συλλογισμὸς ἐξ εἰκότων*—the *εἰκός* being one of those rough generalizations or "moral universals" commonly accepted as "practical" certainties. Where we have a number of such probable syllogisms depending on one another, in the form of a *sorites* (188), and leading up to a conclusion, we have what is called "*chain* evidence". And as a chain cannot be any stronger than its weakest link, the probability of the conclusion in such a case cannot be greater than that of the *weakest* premiss. It is usually considerably less; for, since each probable premiss inherits, besides its own innate weakness, that of all its antecedents, the conclusion must inherit the combined weakness of them all.[1]

Care should be taken to distinguish *chain* evidence from *circumstantial* evidence; in the latter the various probable signs are independent of one another, and their combination, therefore, forms a cumulation or addition of probabilities which may possibly be so strong as to issue in moral certitude.

(*b*) The second kind of probable argument is that in which the premisses, though true, do not necessarily involve the conclusion sought to be derived from them. This is what Aristotle described as the *syllogism from signs*, or *symptoms*, or *indications:* *συλλογισμὸς ἐξ σημείων*. And to both arguments—the syllogism from probabilities, and the syllogism from signs—he gave the title of *Enthymeme.* The *Enthymeme*, therefore, in Aristotle's meaning of the latter term—the "Aristotelean" enthymeme, as it is now

[1] "For instance, a man is accused of murder. There is very strong evidence that a man just like the prisoner was in the company of the murdered man on the night when the murder was committed. It is *almost* certain that the man who was known to be in his company did the deed. There is, moreover, a strong presumption against the theory urged by the counsel for the defence, that the deceased made an unprovoked attack on his companion on the night in question and met his death from him in self-defence. But it does not follow that the accused should be convicted of murder. For if the probability of the three circumstances pointing to guilt is three to one, the balance of probability is nevertheless rather against than in favour of their being all of them true, and this means that it is more likely that the accused was innocent than that he was guilty."—CLARKE, *Logic*, p. 430.

called—is *an argument from likelihoods or signs* : *συλλογισμὸς ἐξ εἰκότων ἢ σημείων*. He defines it in the *Prior Analytics*, and often recurs to it in the *Rhetoric*, calling it here the "*Rhetorical syllogism*,"[1] for the obvious reason that it does not *convince* by producing scientific certitude, but only *persuades*, and is therefore extensively used by the public speaker whose object is to win the adherence of his audience.

The *σημεῖον* is some *particular fact* which is regarded as probable evidence either of some other particular fact, or of the truth of some general assertion.

For example :—

> "Ambitious men are liberal ;
> "Pittacus is ambitious ;
> "Therefore Pittacus is liberal."

Here the ambition of Pittacus is regarded as a sign of his liberality —one particular fact as the sign of another. If the general principle "Ambitious men are liberal" were certainly true, ambition would be no longer merely a *probable sign*—*σημεῖον*—of liberality ; it would be a *certain evidence*—*τεκμήριον*—[2] of the latter ; and the syllogism would become a valid *proof of fact*, producing certitude ; though it would not be a *demonstration*, or produce *scientific* certitude. An example of a conclusive argument from a *τεκμήριον* would be :—

> "All such combinations of symptoms mean consumption ;
> "Here we have such a combination ;
> "Therefore this is a case of consumption."[3]

Where, as in the former syllogism, the general principle invoked in the premisses is an *εἰκός*, the syllogism may also be rightly regarded as an *argument from a general likelihood or probability*.

The enthymemes just given are in the *first* figure. Here are some examples of the enthymeme in the *second* figure, which also argues from one particular fact to another as signified by the former :—

> "Ambitious men are liberal ;
> "Pittacus is liberal ;
> "Therefore Pittacus is ambitious."

[1] *Anal. Prior.*, II., xxvii. ; *Rhet.*, I., i. and ii. *Cf.* JOSEPH, *op. cit.*, p. 323 n. ; KEYNES, *Formal Logic*, p. 322 ; MELLONE, *op. cit.*, pp. 253-8 ; JOYCE, *op. cit.*, p. 253 ; CLARKE, *Logic*, pp. 356, 429.

[2] *Rhet.*, I., ii.

[3] MELLONE *op. cit.*, p. 258.

Or, again :—

> "Murderers flee from the scene of the crime ;
> "A. B. flees from the scene of the crime ;
> "Therefore A. B. may be the murderer ;" [1]

—where flight is regarded as a sign of guilt. This by itself is a very weak argument; but it may help to form a "coil" of *circumstantial evidence.* For example, suppose that A. B.'s house is searched, and that his clothes are found to be bloodstained, we may add this item to the evidence :—

> "The murderer's clothes must have been bloodstained in the struggle of which there are unmistakable evidences ;
> "A. B.'s clothes are bloodstained ;
> "Therefore A. B. is probably the murderer."

Suppose, further, that we can argue thus :—

> "The murderer's boots made these fresh foot-marks ;
> "A. B.'s boots fit exactly into these foot-marks ;
> "Therefore A. B. is probably the murderer."

Those three "circumstances" together have considerable weight. Now suppose there is observed something very singular about the foot-marks—some altogether peculiar arrangement of the nail-marks, for example,—and that the nails of A. B.'s boots are found to reproduce exactly this arrangement : we at once feel the enormous force of such a circumstance. We conclude that A. B.'s boots were worn by the murderer, for no other boots could, in the circumstances, have produced the foot-marks in question :—

> "The boots worn by the murderer produced these foot-marks ;
> "The boots that produced these foot-marks are A. B.'s boots ;
> "Therefore the boots worn by the murderer were A. B.'s boots."

And A.B. alone was seen to flee from the scene of the crime; and his clothes are blood-stained. Such is a simple example of circumstantial evidence.

It is clear that all arguments from *analogy* and *example* fall into the form of enthymemes in the *second* figure (234).

Where a particular fact is alleged as a sign or indication of the truth of some general principle the enthymeme falls naturally into the *third* figure. For example :—

[1] MELLONE, *op. cit.*, p. 258.

"Socrates was just;
"Socrates was wise;
"Therefore the wise are just."

Here, Socrates may be regarded as an indication or illustration—a σημεῖον—of the connexion between wisdom and justice. The "inductive syllogism," which suggests a universal law from an enumeration of instances (207), is an enthymeme of this kind.

The "probable syllogism" was called an "enthymeme" by Aristotle because it proceeds from what is found by reflection (ἐνθυμήσις; ἐνθυμεῖσθαι) to be a general likelihood, or a symptom of the inferred conclusion. And he called the same kind of argument a "rhetorical syllogism" because orators, whose function it is to persuade rather than to convince, have frequent recourse to it. The enthymeme in the modern sense of this term, might also be called a "rhetorical syllogism," inasmuch as it is not customary in ordinary discourse, and outside formal disputation, to give expression to all the constituent judgments in a process of reasoning. It is not clear why the term *enthymeme* came to be applied to a syllogism with a suppressed premiss or conclusion. Possibly owing to a misinterpretation of Aristotle, *Anal.*, *Prior.*, II. xxvii.—where he says: "If one premiss is stated we have only an [argument from an] indication; if the other is also stated, we have a syllogism".[1] He does not mean here to suggest that the syllogism so stated is not an enthymeme; for it is an enthymeme in Aristotle's sense whether one or both premisses are expressed. In another passage—*Rhet.*, I., ii., § 13—he observes that if one premiss of an enthymeme be such that it is easily supplied by the hearer, there is no need to state it expressly.[2] Those passages may account for the modern use of the term.

264. Estimation of Probability: The Concept of "Chance".—We have already remarked that it is quite impossible to measure the degree of our subjective, mental assent to a probable judgment, by any mathematical calculus. It is, however, sometimes possible to apply such a calculus to the objective factors themselves which point to the truth, or to the falsity, of a judgment. This method of dealing with the data for probable judgments may be considered here as a distinct source of these latter (263).

There are innumerable phenomena around us which happen we know not how or why. We are unable, by induction or otherwise, to discover their *causes*. We do not know the *laws* according to which they happen. Hence we have no *scientific knowledge*,

[1] Ἐὰν μὲν οὖν ἡ μία λεχθῇ πρότασις, σημεῖον γίνεται μόνον, ἐὰν δὲ καὶ ἡ ἑτέρα προσληφθῇ, συλλογισμός.

[2] Τὸ δὲ ἐνθύμημα συλλογισμὸς, καὶ ἐξ ὀλίγων τε καὶ πολλάκις ἐλαττόνων ἢ ἐξ ὧν ὁ πρῶτος συλλογισμός· ἐὰν γὰρ ᾖ τι τούτων γνώριμον, οὐδὲ δεῖ λέγειν· αὐτὸς γὰρ τοῦτο προστίθησιν ὁ ἀκροατής.

and no *certitude*, as to how, when, where, or in what circumstances, they will recur in the future, or have occurred in the past. So far as we know, they occur "irregularly": they just "happen" or "chance" to occur. Hence we speak of the "chances" for or against the occurrence of some such event at a given time and place, or in a given set of circumstances; and we endeavour to calculate those "chances," for and against its happening, and so to "estimate its probability". Special treatises have been written on the nature and rules of this logical calculus of probability or chance[1]; but in a general treatise on logic a brief statement of the leading principles and their applications will be sufficient. The first important concept that calls for examination in this connexion is the concept of *chance*.

Chance is not the negation of *causality*: nothing is casual in the sense of being *causeless*. Every event, everything that happens or begins to be, has a cause. There is a causal connexion between any two events either of which has any positive influence on the production or happening of the other. Induction aims at discovering causal connexions. But there are phenomena, connected *in time and space*, which have no such obvious causal influence on one another. Their concurrence in time or space is a "coincidence," a "chance" concurrence. In seeking for the causes of a given phenomenon we meet several of its concomitant circumstances, and recognize these to be "irrelevant". Their presence or absence has no influence on the phenomenon; the connexion between them and it is not causal but casual—a "chance" connexion.[2]

For example, a person speaks of having "chanced" or "happened" to meet a particular old friend from the country, at a certain street corner in town, at a particular moment when he was just in the act of posting a letter to that friend. It was a "remarkable coincidence," and "unexpected occurrence". Each part of the total phenomenon had its causes, but neither had any influence in bringing about the other, nor can we see in the causes of either part anything that would of its nature have any influence in determining the *coincidence*, in time or space, of the other part. Thus, the coincidence of phenomena not connected as cause and effect, or of various "indifferent" or "irrelevant"

[1] *Cf.* VENN, *Logic of Chance*; BOUDON, *Le calcul des probabilités*; BERTRAND, *Calcul des probabilités*; MANSION, *Sur la portée objective du calcul des probabilités*.

[2] *Cf.* VENN, *Logic of Chance*, p. 245.

elements of the same complex phenomenon, we regard as "due to chance," because they are unexplainable, unaccountable, irreducible to law, by any effort of ours.

Similarly, there are phenomena whose *recurrence* is so irregular, arbitrary, uncertain, devoid of any element of uniformity, that we can entertain no hope of tracing the laws according to which their determining causes combine to bring them about. Thus, a homogeneous, cubic die has each of its faces marked with a different number of spots, up to six. In a dozen throws, the three and the five have turned up three times each, the two and the four twice each, the one and the six only once each : no correspondence of the order in which the figures appear, with the arithmetical series, one to six, or with any other known principle or law : no correspondence with the order of results in any other dozen of throws. So, too, the trump in a deal at cards "chances" or "happens" to be the ace of hearts, or the king of diamonds, etc. Such phenomena we describe as matters of "chance".

Of course, when we say that a definite occurrence of that sort is the result of "chance," we do not mean to deny that the actual result was fully and perfectly determined by the whole combination of causes that brought it about ; we merely mean that *we are unable to see* how those causes necessarily determined or brought about the actual occurrence. We fail to see anything either in the die or in our casting of it, in the cards or in our shuffling of them, in the coin or in our tossing of it, to necessitate the turning of the six rather than of the one, of the ace of hearts rather than of any other card of the pack, of the "head" rather than of the "tail" of the coin. So far as we know the nature of all the antecedents in such cases, we see absolutely nothing in the dice or the cards or the coins, or in our manipulation of them, to bring about any one result out of a large number of equally possible alternatives, rather than any other. And yet the fact that *this* result *happened*, and not any other, is an all-sufficient proof that there were at work influences which determined the occurrence of *it* rather than of any other. But all we know about the antecedents is that they must cause *some* alternative ; and because we do not know *which*, we say that *it* "chanced" to occur. Thus, "chance" is a name for our ignorance.

So, also, the fact that some one particular phenomenon, or part of a phenomenon, accompanies or coincides with some other in time and space throughout our experience—this fact too must

have its sufficient reason, even though we may fail to see it. We can see no connexion between the causes of each. But we are forced to think that if our minds were able to analyse fully, and to trace back clearly to their sources, each set of causes, we should necessarily find between them some connexion capable of accounting for the coincidence. In other words, a mind which understood all things, all reality, all causes, fully and adequately, would see not only the reasons and causes of the isolated elements themselves, but also of the interrelations and connexions of concomitance or sequence between all those elements; and would see how all those causes conspired, each according to its nature, and to the law of its own activity, to produce each of the individual events that appear to our limited intelligences so irregular, so devoid of order, so incapable of being known *scientifically*, or reduced to *law*.[1]

We are forced to those conclusions on the ground that every event has a cause; that the real world is knowable, intelligible, rational, or, in other words, that it offers an ultimate sufficient reason, in explanation of every element and of every coincidence of elements, to a mind capable of comprehending it. It is easy to understand, therefore, that for an Omniscient Intelligence there can be no such thing as chance: the sufficient reason of every event is fully discerned by such an intelligence in the totality of its causes. But such perfect knowledge is for us an unattainable ideal, on account of the limitations of our minds confronted with the complexity of the real world. Not to speak of the self-determining activity of the human will, or of the conscious causality of the animal kingdom, or of vital activities in the organic world, even physical causes in the inorganic kingdom are so complex and hidden in their combinations that we can never hope to attain to such a knowledge of them as will enable us to dispense with the concept of "chance" (267).

But even though for an Omniscient Intelligence there can be no such thing as chance, and even though the concept has its origin in the finiteness of our minds, yet it cannot be regarded as a purely subjective concept; for the ignorance whence it springs has a very real and objective foundation in the complexity and vastness of that world which the finite human mind is ever trying to understand.

Mr. Welton, understanding by "cause" the "sum-total of the determining conditions," gives the following account of the concept of chance.[2] "When in

[1] VENN, *op. cit.*, *ibid.* [2] *Logic*, vol. ii., p. 165.

any particular case we do not know what conditions are operative, we cannot tell, on the one hand, what result will appear, nor, on the other, can we say positively what conditions have produced a certain given event. In such cases we are accustomed to speak of the occurrence as due to chance. But our whole conception of the unity of nature forbids the idea that any element of reality can be really casual. [The turning of any particular face, for example, is casual relatively to the die itself, in the sense that the die itself contributes nothing towards this particular result; and casual relatively to the whole concrete cast of the die, in the sense that *we do not know what combination* of antecedents brings about the result in the concrete case; but not, of course, in the sense that the result has no determining antecedents; it must have them, by the principle of causality. Hence the author continues :] Every detail is, in the strictest sense, necessary, and determined absolutely by conditions—all is causal, nothing casual [for 'casual' here means, equivalently, 'causeless']. Were our knowledge complete, then, the idea of chance would disappear; it is due solely to the imperfection of that knowledge. This imperfection is, of course, greater in some cases than in others; it may affect the event as a whole, or it may only affect some particular aspect of it. But, even with imperfect knowledge, we are often called upon to come to a decision or to act. The question then arises as to what we ought rationally to expect."

265. CONDITIONS FOR THE MATHEMATICAL ESTIMATION OF PROBABILITY.—We have now seen that "chance" or "coincidence" connotes simply the possession of *imperfect* or *incomplete* knowledge about the antecedents or causes of a phenomenon. This is the *first* condition for the application of a calculus of probability to the data of our experience: a knowledge sufficient to assure us, for example, that a phenomenon must happen in some one or other of a definite number of alternative ways, but insufficient to assure us in which of these ways it will happen. We know, for instance, that in any single throw of a die the laws of gravity, inertia, motion, elasticity, action and reaction, the antecedent position of the die and of the box, the intensity, direction, and duration of each rattle, and innumerable other unknown and inappreciable influences—all conspire together for the production of *the actual result* of this particular throw. But we are conscious, on the other hand, that the combination of factors determining this particular result, rather than any other, surpasses our comprehension.[1] We know the *general set of antecedents* which gives rise to some particular result out of a number of possible alternative results. We recognize that while many of them are

[1] *Cf.* WELTON, *op. cit.*, p. 167: "For example, if a penny is tossed it will fall with either head or tail uppermost. Now, which will be uppermost in any particular throw will be exactly determined by such conditions as the position of the coin at starting, how it is grasped in the fingers, the force and direction of the twist, etc. But what special form these conditions will take we are totally ignorant. . . ."

an inseparable portion of the experience, or even exert a positive causal influence on the general result—the die itself and its natural properties, for example,—yet they contribute precisely in the same way to every possible alternative result, and are therefore rightly called "indifferent" causes of any one result in particular. We know, too, that in each particular experience there are some antecedents whose combination determines this particular result rather than any other. But we do *not* know what these are, or, at least, how exactly they will combine. And it is our ignorance on this point that allows and obliges us to estimate the "chances" or "probabilities" which the indifferent antecedents have of being united in a given case with a given concrete result out of many possible alternative ones. In throwing dice, for example, we know that the die itself—so long as it is not "loaded" for the purpose of giving it a "natural inclination" to fall on some particular face—is equally indifferent to each of the six faces. Furthermore, we are unable either to control, or estimate, or reduce to any law, the actual combination of the numerous influences that determine a particular result—forces which emanate with varying intensity from the thrower at each successive throw, and forces which, like gravity, operate independently of him.[1]

There is a *second* condition to which our data must conform, before we can apply to them any definite rules for the estimation of probability. The same general set of conditions and antecedents must be present and operative throughout the whole region of time and space from which these data are drawn. An

[1] "Take such a simple example as that of dropping a stone to the ground. We say, in accordance with the common expression of the causal relation, that if the stone be dropped again just as it was before, it will fall on the same spot. True; and for most practical purposes the thing can be done readily enough; but if perfect quantitative accuracy were required we should soon find that we had undertaken a troublesome task. The stone must be held in exactly the same position as before, for the friction of the air influences its fall; it must be dropped from exactly the same height and over the same spot on the floor; the atmospheric currents, nay the very temperature of the air, must remain unchanged; and so on indefinitely with further demands, as quickly as those already formulated were assumed to be satisfied."

"If it be urged that all this is merely useless subtlety the retort is simple, and, I think, conclusive, *viz.* that many millions of pounds have changed hands in accordance with these conditions of things. It is simply because we cannot do the same thing over again, or calculate how far we shall fall short of doing so, even when our instrument in hand is purposely made of as accurate a shape as possible, that the roulette and the die can be employed for gambling purposes. So impossible is it found to be to spin a top twice with the same velocity, or to discharge a cube twice from the same position, that the fanatics of the gaming table never dream of predicting results from this side, but put their trust in appeals to statistics and other such considerations."—VENN, *Empirical Logic*, pp. 65, 66; *cf. ibid.*, pp. 100, 105.

unknown alteration in those influences would upset our calculations. For example, were the uniform shape of a die, or the uniform density of its texture, to alter imperceptibly, the equality of alternatives which is essential to the calculation of probability would be destroyed. It is, in fact, by determining experimentally whether some one alternative occurs more frequently than it should occur if the chances were equal, that we detect the existence of a "bias" which may lead to the discovery of some causal connexion or law (267).[1] In throwing dice, for instance, the five may be found to turn up 97 times out of 600 trials, and 1003 times out of 6000; from which it is concluded *a posteriori* that the probability in favour of the turning of five lies between $\frac{97}{600}$ and $\frac{1003}{6000}$—or is about $\frac{1}{6}$. The experiment—so far as it goes, for it cannot be carried on indefinitely—verifies the supposition that the sum-total of the influences at work was *indifferent*, and hence made as often for any one as for any other alternative. It is only, therefore, when we have the agencies under our own control, and can experiment with them, that we can secure with absolute certainty the constant prevalence of the same set of conditions. This is secured in all ordinary "games of pure chance". Before inquiring how far it can be assumed to prevail in natural phenomena (268), we will examine the application of the calculus of probability to data that are assumed to fulfil the two conditions just set forth. We may observe here, however, that when the requisite conditions are present, the estimate can be made for past events as well as for future ones. The theory applies independently of time. No doubt, its usual application is to games of chance, dealing with a future event. But when some one, we do not know which, out of a certain definite number of equally possible alternatives *has* happened, we can apply the theory to determine what degree of probability there is that such a *definite* alternative has happened.

"Thus, for example, it is extremely improbable that a hand at whist should consist entirely of trumps. Yet the probability of this is no less than that of any other one definite hand. It is its interesting character which draws special attention to it, and causes us to recognize how enormous are the odds against it. This we do not recognize in the case of other hands. If, then, a person told us he had held a hand of thirteen trumps the previous evening we should probably feel more hesitation in believing him than if he told us he had held a hand consisting of certain definitely named cards. Yet the antecedent improbability would be no greater in the one case than in the other. Were

[1] *Cf.* JOYCE, *op. cit.*, p. 372; VENN, *Logic of Chance*, pp. 78-82.

the person, however, to claim that he had, previously to playing, written down the contents of a certain hand, and that he had been actually dealt that hand, we should probably hesitate to receive his statement just as much as if he told us he had been dealt thirteen trumps; for the previous defining of the hand would have made the odds against it as apparent as in the other case. When we hesitate to believe the statement of such coincidences it is because we feel that the odds against the occurrence were antecedently very great, and we balance that with the odds in favour of the credibility of the witness. If we do not doubt his credibility we receive his statement in spite of its antecedent improbability, for to assume that the extremely improbable is impossible is to fall into a dangerous fallacy."[1]

The data for probable knowledge furnish us with some *certain* knowledge; without some certitude there could be no probability. What we are certain of in regard to such data may be expressed in the form of a disjunctive proposition: *e.g.* "The result of throwing a die will be to turn up either the one, or the two, or the three, or the four, or the five, or the six"; or symbolically, "A is either X_1 or X_2 or X_3 . . . or X_6"—where A represents a particular throw, and X_1, X_2, etc., the possible alternative results. We do not know whether the result of the throw A will be X_4 or not; whether the definite judgment "A is X_4" will prove true. But it *may* be true; we are entitled to entertain some degree of rational expectation that it will be true; and the question now is whether we have any means of measuring the likelihood, the rational expectation we may entertain, of its truth. The mathematical calculus of probability places such a means at our disposal. It aims at expressing probability by means of fractions.

When the chances for and against an event are equal, we are left in *absolute doubt* about its happening; the probability is then expressed by the fraction $\frac{1}{2}$. When the chances are all for and none against, the fraction has grown to unity, and the probability becomes a *certainty* that the event will happen, or has happened. When the chances are none for and all against, the fraction has decreased to zero, and represents certainty against the event. But those are limits towards which the chances may tend indefinitely without ever absolutely reaching them. The event is said to be "probable" or "improbable" according as the fraction is greater or less than $\frac{1}{2}$. Close approximation to the positive and negative limits—to unity or to zero—are spoken of as "moral certainties" and "bare possibilities" respectively.

We have now to examine briefly, in their logical aspect, the

[1] WELTON, *op. cit.*, pp. 169-70, 178-80. *Cf.* BORDEN P. BOWNE, *Theory of Thought and Knowledge*, pp. 187-8.

mathematical principles laid down for the estimation of probability, considering those processes of estimation as modes of reasoning from various combinations of disjunctive judgments.

266. RULES FOR ESTIMATING PROBABILITY.—(1) *Simple events.* The probability of a *simple event* is expressed by a fraction whose numerator is the number of favourable alternatives, and denominator the total number of alternatives.

Thus, if A can occur, or has occurred, in any of n different ways, their sum, A, will represent certainty (for we know A will occur, or has occurred), and is best expressed by unity. The probability of any particular case out of the n is, therefore, $1/n$. Out of n, the total number of alternatives, $(n-1)$ are against, and 1 for, the occurrence of any given alternative. As there are six faces to a die, one only of which is marked 4, and the others differently, the chance of turning some of the others, besides the 4, is $(n-1)$ or five times greater than the chance of turning the 4; *i.e.* the probability for the 4 is $1/n$, or $\frac{1}{6}$, the probability against it $(n-1)/n$, or $\frac{5}{6}$. Again, as there are fifty-two cards in a pack, but each particular value or number is present in four different ways, the probability in favour of drawing some particular value will be $\frac{4}{52}$, and the probability against it $\frac{48}{52}$, as there are four chances in favour of any particular value and forty-eight against. If there be a question of drawing some particular *card*, *e.g.* the ace of hearts, the probability is only $\frac{1}{52}$.

(2) *Compound events.* If an event is *compound*, *i.e.* made up of a number of simple events connected together, these latter will be (*a*) either independent, or (*b*) dependent on one another; hence, two cases arise.

(*a*) *Compound of independent events.* The probability of an event *composed* of a number of *independent* simple events, whether simultaneous or successive, is the product of the probabilities of the latter taken separately.

Suppose, for example, A and B to be two urns, A containing two white balls and one black, say w_1, w_2 and b_1, and B likewise containing two white balls and one black, say w_3, w_4 and b_2: what is the probability of drawing (simultaneously or successively, it matters not) a black ball from each? The draw from A may be represented by the proposition, "If S is A it is either w_1 or w_2 or b_1"; that from B by the proposition, "If S is B it is either w_3 or w_4 or b_2". Now, if we combine both experiments, we find that the total number of possible ways of drawing two balls is 3×3, as any one of the three possible draws from A may combine with any one of the three possible draws from B. The double draw will be expressed by the proposition: "If S is AB it is either w_1 w_3, or w_1 w_4, or w_1 b_2; or w_1 w_3, or w_2 w_4, or w_2 b_2; or b_1 w_3, or b_1 w_4, or b_1 b_2": from which we see that the probability of a double black is $\frac{1}{9}$, *i.e.* the product of the two simple probabilities $\frac{1}{3} \times \frac{1}{3}$. We see, likewise, that the probability of drawing two whites is $\frac{4}{9}$, or $\frac{2}{3} \times \frac{2}{3}$, a further verification of the principle. "Lastly, as the probability of white is in each case $\frac{2}{3}$, and that of black is $\frac{1}{3}$, so [$\frac{2}{3} \times \frac{1}{3} =$] $\frac{2}{9}$ is the probability both that the

drawings will give first white and then black, and that they will yield first black and then white; this is shown to be true by the fact that the final proposition yields two cases in which white is followed by black, and two cases in which black is followed by white."[1]

Similarly, if the first urn contains 3 white and 4 black, and the second 4 white and 5 black balls, the number of possible ways of drawing two balls is $7 \times 9 = 63$, the number of ways of drawing two whites $3 \times 4 = 12$, and of drawing two blacks $4 \times 5 = 20$; consequently the probability of drawing two whites is $\frac{12}{63}$ and of drawing two blacks $\frac{20}{63}$.

It will be seen that there is no difference in principle or theory between this sort of compound, and the simple event itself, the disjunctive proposition expressing the former being a combination of the alternatives of each independent simple event.

(*b*) *Compound of dependent events.* If two simple events are so connected that the occurrence of the first *affects* the probability of the occurrence of the second, the probability of the compound event will be the product of the probability of the first with the probability *of the second as affected by the first.*

For example, what is the probability of drawing two white balls in succession, without replacing the first drawn, from an urn containing two white balls and one black one? If a black is drawn first the estimate is rendered impossible. The probability of drawing a white first is $\frac{2}{3}$. If it is drawn, the constitution of the urn for the second draw is modified by the first draw. There are now only two balls, one white and one black, in the urn; and the probability of drawing the white is $\frac{1}{2}$. The two draws may be represented by the two propositions:—

A is either w_1 or w_2 or b_1
B is either w_3 or b_2.

Where b_2 is the same ball as b_1, and w_3 the white ball that remains after w_1 or w_2 has been extracted. Those two propositions combined will give six alternatives, of which two are *pairs* of whites: "AB is either $w_1 w_3$, or $w_1 b_2$, or $w_2 w_3$, or $w_2 b_2$, or $b_1 w_3$, or $b_1 b_2$"; from which we see the probability of drawing two whites is $\frac{2}{6}$, or the first simple probability multiplied by *the second as affected by the occurrence of the first*, *i.e.*, $\frac{2}{3} \times \frac{1}{2}$.

The same result may be stated in this way: The probability of drawing a white first, and consequently of the second draw taking place at all, is $\frac{2}{3}$. In the hypothesis that it does take place, the probability of drawing the white again is $\frac{1}{2}$. Therefore the probability of drawing two whites is $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$.

(3) *Total probability of events which may happen in a plurality of ways.* When the same simple event may occur in many independent groups of conditions, its total probability is *the sum* of the probabilities that the various conditions will be verified.

For example, what is the probability of turning a "head" in one or other of two consecutive throws of a penny, or (which is the same) in one throw of two

[1] WELTON, *op. cit.*, p. 173, whose treatment is largely followed in the present section.

pennies? The total number of possible combinations of head and tail in either hypothesis is 4 : h_1h_2, h_1t_2, t_1h_2, t_1t_2 ; and out of these four alternatives, three yield a head. The probability is therefore $\frac{3}{4}$. Now this is the sum of the two separate simple probabilities. For, if we toss one coin twice the chance of getting a head the first time is $\frac{1}{2}$; the chance of the second toss taking place at all is contingent on the failure of the first, *i.e.* is $\frac{1}{2}$; and the chance of its securing a head if it do take place is $\frac{1}{2}$. Therefore the total chance from the second toss is $\frac{1}{4}$. Consequently the total chance by both tosses is $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$. If we toss two coins, *A* and *B*, simultaneously, the chance of *A* turning a head is $\frac{1}{2}$, and the event of its failure being combined with the heading of *B* (t_1h_2) is $\frac{1}{4}$; or that of heading *B* $\frac{1}{2}$, and of combining its failure with the success of *A* $\frac{1}{4}$; in either case the total probability of securing a head is $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$. Or again : What is the probability that a throw of two dice, *A* and *B*, will yield any given number between 2 and 12, say 7 ? There are 36 alternatives in a throw. The desired number, 7, can be obtained in six different ways, *viz.* when *A* and *B* are respectively 1 and 6, 2 and 5, 3 and 4, 4 and 3, 5 and 2, 6 and 1. Now the probability of the occurrence of any individual alternative of these six is $\frac{1}{36}$; and as any one of them will yield the desired result, 7, the probability of the occurrence of the latter is the sum of these six probabilities, or $\frac{6}{36} = \frac{1}{6}$.

267. Inverse Probability : Bernouilli's Theorem : Elimination of Chance.—So far, the problems examined have been all of the same general character, namely : Given the conditions capable of realizing indifferently any one of a known number of possible alternative events, what is the probability that a particular alternative, *a*, will occur ? This is called *direct* or *a priori* probability. The inverse problem is the following : Given that a certain event has happened, to which of a number of possible alternative causes is it most probably due ? The event here in question always yields some data for an indirect or *a posteriori* probability, and is, of course, always an actual application or realization of some *a priori* probability. The determination of this *a priori* or antecedent probability will give a probable knowledge of the actual causes from which the phenomenon or event has followed. Thus, for example, an urn is known to contain three balls, but their colour is unknown. We are asked to determine their colour by repeatedly drawing one and returning it. We draw a white : it gives us no reason to conclude anything about the colour of the others. But if we next draw a black we thereby know that the contents of the box may be either *w w b* or *w b b ;* but that these two do not exhaust all the alternatives, another possible one being *w b x*, where *x* may be another colour. If, however, we continue to draw only blacks and whites, the probability against *x* being some third colour (say *red*) rapidly decreases. For if a red ball were there, the probability *against*

its being drawn the first time would be $\frac{2}{3}$, the second time $\frac{2}{3} \times \frac{2}{3}$, . . . the eighth time $(\frac{2}{3})^8$, or less than $\frac{1}{24}$. If, therefore, it has not appeared in eight draws, the probability that there is no such ball in the urn is over 24 to 1,—which would generally be regarded as practical certitude.

Again, suppose it known that the three balls are either black or white, or a combination of black and white in an unknown ratio. This gives four possible alternative contents before the drawings commence, *w w w*, *w w b*, *w b b*, *b b b*; from which we gather that the probability in favour of a particular content of the urn is $\frac{1}{4}$. Now if a white ball be drawn first, it excludes the possibility of *b b b*. As the remaining three alternatives show six possible ways of drawing a white ball, the probability in favour of drawing any individual white ball is $\frac{1}{6}$. Therefore the probability that the white actually drawn was from *w w w*, is $\frac{3}{6}$ or $\frac{1}{2}$, that it was from *w w b*, $\frac{2}{6}$ or $\frac{1}{3}$, that it was from *w b b*, $\frac{1}{6}$. That is how the probabilities stand at the end of the first drawing. If, however, a second drawing brings forth a black ball, *w w w* is excluded, and the alternatives *w w b* and *w b b* are regarded as equally probable. But if a third drawing gives a white, it makes the probability of *w w b* twice as great as that of *w b b*, and its absolute probability therefore $\frac{2}{3}$. For, assuming *w w b* to be real, the probability of securing the result actually attained, *i.e.* of drawing a white, a black, and a white, is $\frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{27}$; whereas the total probability of such a combination from *w b b* is $\frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{2}{27}$, or half as great as the former.

The problem of estimating inverse probability—of determining to which of a number of possible alternative antecedents, or groups of antecedents, a given event or series of events is due—is identical with the problem of determining the true magnitude of a phenomenon by a series of approximate measurements (246, 268); for that problem might be stated thus: "Given a series of registered approximate measurements of a magnitude, what is most probably the true magnitude that has yielded those measurements"?

When it is said that the probability of heading a coin is $\frac{1}{2}$, or of turning up a definite face in casting a die, $\frac{1}{6}$, does this imply any further *certitude* in regard to our data, beyond that already referred to (265)—the certitude that *some* face of the coin, or of the die, will turn up at each throw? It does not, *about any individual throw;* but it does imply further certitude about *the nature of the average* of an indefinite number of throws: granted that all the alternatives are and remain equally probable, we are certain that in an *indefinite* number of throws the coin would be headed *on an average* every second throw, or a definite face of the die turned *on an average* every sixth throw. In other words, we are certain that the *a priori* probability is $\frac{1}{2}$ in one case, and $\frac{1}{6}$ in the other.

Although we cannot be sure that the result of any particular throw will be a head, or a six, we can be sure that if we kept on throwing indefinitely, the heads obtained would be $\frac{1}{2}$, or the sixes $\frac{1}{6}$, of the whole series of trials. The reason for such certitude about the results of a hypothetical indefinite series of experiences is a negative one: because, namely, any other result would be unaccountable, would be an effect without a cause. But in any *definite* series of experiences we are prepared to find that the actual results may *deviate* somewhat from what the *a priori* probability points to—from $\frac{1}{2}$ or $\frac{1}{6}$ in the cases just mentioned. We are justified, however, in expecting further, that (1) the greater the total number of experiences the *more closely* will the actual results *approximate* to the *a priori* probability; though (2) in a larger number of experiences we are prepared to find that the deviation from the *a priori* probability is in itself greater than in a smaller number of experiences. In other words, as the series of trials extends, the *absolute* magnitude of the possible deviations will also *increase;* but its *relative* magnitude, its proportion to the total number of trials, will *decrease:* so that the actual results tend progressively towards the realization of the antecedent probability.

Within what limits, in any given series of experiences, *should the actual results fluctuate around the antecedent probability?* What is the largest deviation that we should expect, in a given set of trials, from a known or assumed antecedent probability? If we could solve this problem, we could in some degree *eliminate chance*, and conclude that *the excessive deviation must be due to the operation of some cause* (264). "If we are able to say," writes Fr. Joyce,[1] "that in a definite number of trials, certain limits will not be overstepped, and if it is discovered that the result is totally at variance with our mathematical estimate, then it becomes clear that we were mistaken in our view as to the nature of the case. . . . A loaded die gives, it is true, very irregular results. Were it not so, it would be detected at once, and so defeat its own purpose. But there comes a point at which, after a certain number of trials, the mathematician is able to say that the appearances of the six exceed the limits of mere chance, and that the very act of throwing must tend to turn that face uppermost."

A noted mathematician, James Bernouilli (1650-1705), devoted many years to the study of this whole question of deviations from

[1] *op. cit.* p. 378.

a priori probability.[1] According to his researches, if, for instance, the antecedent probability of an event is $\frac{3}{5}$ "the odds are 1000 to 1 that in 25,500 trials the event shall occur not more than 15,841 times, and not less than 14,819,—that is, that the deviation from 15,300, the ideally probable number, shall not exceed $\frac{1}{50}$ of the number of trials".[2] Or, again, if from an urn containing white and black balls in the proportion of two white to one black, we were to draw 90,000 times, returning each ball after drawing it, the number of blacks drawn should be between 29,400 and 30,600, showing a deviation of not more than 600, that is $\frac{600}{90000}$, or $\frac{1}{150}$, from $\frac{1}{3}$, which is the antecedent probability. Were we, however, to increase the number of draws 100 times (to 9,000,000), the deviation should increase only ten times, *i.e.* to 6000, the number of blacks drawn lying between 2,994,000 and 3,006,000 —which would be a deviation of only $\frac{1}{1500}$ from the antecedent probability.

We see, then, that as the number of experiences is multiplied, the absolute magnitude of the fluctuations also increases, but the amount of their difference from the antecedent probability diminishes.[3] The first of those two facts enables us to foretell the certain ruin, in the long run, of any one who continues to play at a fair or equal game of chance. Suppose, for example, that two persons, *A* and *B*, play at tossing pennies one at a time, each player putting a penny stake on each toss, *A* betting for head, and *B* for tail, throughout. While *A* is very wealthy, however, *B* has only a shilling. The latter will be financially "ruined" when the number of heads exceeds the number of tails by twelve. Now the probability that this will take place within a certain number of throws can be calculated, and will be found to increase steadily and to tend ever towards certainty according as the total number of throws increases. From the second part of the theorem—that the actual results tend progressively to approximate towards the antecedent probability—we infer that the greater the number of experiences the greater will be the value of the results in point-

[1] *Cf.* CROFTON, art. on "Probability" in the *Encyclopædia Britannica*; BAUDOT, art. on "Probabilité" in the *Nouveau Dictionnaire des Sciences*.

[2] JOYCE, *ibid.*, p. 377.

[3] "Le theorème de Bernouilli revient à ceci : 'Si on fait un nombre indéfiniment croissant d'épreuves, l'écart est infiniment petit par rapport au nombre des épreuves'. Il faut bien remarquer que c'est l'écart relatif qui tend vers zéro, l'écart absolu devient au contraire de plus en plus grand."—BAUDOT, *Probabilité*; *apud* JOYCE, *ibid.*, p. 377, n. 2.

ing with ever increasing probability to the actual alternative antecedent, or combination of antecedents, from which those results actually followed; the greater, too, will be their value in revealing to us the real nature of the supposed alternatives, and in thus "eliminating chance".

268. APPLICATION OF THE CALCULUS OF PROBABILITY TO NATURAL AND SOCIAL PHENOMENA.—The mathematical estimation of probability is best illustrated in games of chance, where it is applied to specially prepared materials. Can we utilize it to estimate probability in regard to the occurrence, or recurrence, of complex natural and social phenomena, the causes of which we know either not at all or only imperfectly? Attempts have been made in various directions to apply it to such data—with, however, no remarkable degree of success. As a matter of fact, the degree of rational expectation we may entertain about the future recurrence of any natural phenomena which we have repeatedly observed occurring in the past, but are unable as yet to refer to its causes,—such probability we never think of basing on the considerations that apply to an artificial game of chance, or of measuring by means of the calculus outlined above. Rather, in such cases, our first endeavour is to detect uniformities of coexistence or sequence even among phenomena which appear at first sight irregular and unconnected; and we do so by compiling statistics, and seeking for hitherto unobserved concomitant variations (243, 249). Research in this direction often leads to the discovery of causes.

Suppose, however, that we are in the presence of a phenomenon which sometimes happens and sometimes does not happen in circumstances of the same general character: we may set ourselves the task of discovering whether or not this set of circumstances as a whole, so far as we know it, is "indifferent" to the occurrence or non-occurrence of the event in question. We are face to face with a result recurring irregularly in certain conditions. We want to find out, if possible, whether it is causally, or only casually, connected with those conditions. For this purpose we see whether or how far we can "eliminate chance" by applying the rule for calculating inverse probability (267). But there are two considerations which reveal the difficulty of such a procedure. One is that we may possibly know very little about even the general set of antecedents to whose combination the event, when it does occur, is really due. The other is that this

general set of antecedents, whatever it is, may be gradually changing in character, without our knowledge (265). Especially in regard to social or partly social phenomena—the frequency of various crimes; the rate of births, marriages, and deaths; political or economic crises, etc.—our uncertainty as to the permanence of their causes and conditions must always render our probable estimates about the future very precarious. Yet this seems to be the only direction in which we can reasonably utilize the calculus: that is, for the purpose of discovering whether or not certain given conditions have only an "indifferent" or "casual" connexion with an event, or whether, on the contrary, we can eliminate chance by detecting a "bias," or, in other words, a causal connexion, between the event and some of those conditions.

Curious attempts have been made, however, by various logicians, to determine mathematically our probability as to the future recurrence of an event, by an appeal to the *number* of times we have observed the event to have occurred in the past. Thus, according to Professor Welton, following Lotze,[1] the occurrence of an event once may be taken as one reason for expecting its recurrence. Apart from that, the chances for and against its recurrence would be equal. There are, therefore, two reasons altogether for expecting its recurrence and one against expecting it; so that the probability of its recurring is $\frac{2}{3}$. Generalizing from this, we see that if an event has occurred m times, the two possibilities of its recurring or not, once again, make the total number of alternatives $m + 2$, of which $m + 1$ are favourable. The probability of recurrence is therefore $\frac{m+1}{m+2}$. Evidently, then, uncontradicted experience should give rise rapidly to a very high probability, which would continue to grow with continued experience. That the sun has risen daily for 5000 years would make the probability $\frac{1826215}{1826216}$ that it will rise to-morrow. "It will be seen," he continues, "that this calculation of probability is the true basis of induction by simple enumeration. The formula shows that with wide and uncontradicted experience the probability that an empirical law which summarizes that experience will hold good in one more case is very high. But it also shows that extension of it beyond the realm of actual experience becomes increasingly uncertain with increase in the width of that extension. For, if the formula is written to show the probability that an event which has occurred m times will happen n times more, it becomes $\frac{m+1}{(m+n+1)}$ —for $m + 2$ in the original formula $= m + 1 + 1$, where 1 is the number of new cases, *i.e.* n—which obviously decreases in value as n is increased. Again, another modification of it shows how the actual experience of the failure of the event, weakens the probability of its recurrence. For if an event has occurred m times and failed to occur n times under circumstances where it might have been expected to happen, then there are already $m + n$ cases; the possibilities that it may or

[1] LOTZE, *Logic*, § 282 (5)—*apud* WELTON, *op. cit.*, ii., pp. 180-82. *Cf.* VENN, *Logic of Chance*, pp. 196 *sqq.*; 358 *sqq.*

may not occur again add 2 more, and thus, the probability for its recurrence is $\frac{(m+1)}{(m+n+2)}$, which decreases as n increases. In this case, the extension to p more cases becomes still more hazardous, as its formula is $\frac{(m+1)}{(m+n+p+1)}$," —decreasing as p increases.

Now, plausible as all this may appear, it can be shown to lead to very questionable conclusions. The mathematical formula $\frac{(m+1)}{(m+2)}$ rightly expresses the probability of drawing a white ball next time from a bag containing "any number, we know not what, of balls each of which is white or black,"[1] after we have drawn a white ball m times successively. But in applying the formula to determine the probability of the recurrence of any natural phenomenon we are assuming "that the universe may be likened to such a bag";[2] an assumption which is at the very least so groundless that we need not be surprised if it leads to some fantastic conclusions.

Again, it is only by experience we can discover whether or how far the degree of our rational expectation of the recurrence of natural events is in accordance with such a formula;[3] and even in so far as we do detect some such accordance, we feel that our probability is not really based on, or measured by, the formula: in applying the formula to our data "we only give the appearance of logic to a conclusion we have otherwise gained. Without consulting experience as to the application of the law, and thus making it superfluous, we should be met by Venn's objection. It has rained for three days; I have given three false alarms of fire; I have fed my chickens three times with strychnine. What is the probability for the fourth case? By the rule it is four-fifths that it will rain the fourth day, that the neighbours will respond to the next alarm, and that my chickens will die the next time."[4]

These conclusions are a sufficient *reductio ad absurdum* of all attempts to employ any such "rule of succession" as that contained in the formula $\frac{(m+1)}{(m+2)}$ for the purpose of determining the probability of the recurrence of a natural event. As a matter of fact, whereas in some cases the repeated occurrence of an event in the past does make its future recurrence more probable, in other cases it has the opposite effect (as in giving false alarms), and in others, again, it has simply no effect: the past facts tell us nothing about the probability of the next occurrence (as in fair games of chance).[5] Hence the original assumption, that repeated occurrences always increase the probability of recurrence, is illegitimate.

We pass next to another doubtful application of the calculus, namely, to the domain of human testimony. Professor Welton says[6] that "the theory of probability is applicable to the credibility of testimony as well as to the prediction of a future occurrence". But, just as in the latter case, so in the former, we have to make arbitrary or hazardous assumptions which render such applications of the theory practically worthless. "Suppose," he writes, in introducing an example,[7] "that two witnesses, the probability of whose accuracy

[1] VENN, *op. cit.*, p. 197. [2] *ibid.* [3] *ibid.*
[4] BOWNE, *Theory of Thought and Knowledge*, p. 190.
[5] VENN, *op. cit.*, pp. 358, 363. [6] *op. cit.*, ii., p. 169. [7] *ibid*, p. 180.

is $\frac{3}{4}$ and $\frac{2}{3}$ respectively, agree in affirming the occurrence of an event . . . whose antecedent probability is $\frac{1}{2}$ . . ." But the whole practical difficulty—the practical impossibility, it may be called—lies precisely in fixing, with any pretension to mathematical accuracy, the degree of probability as to their accuracy in such cases. Of course, if that could be done, the value of their combined testimony would be $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$ that the event occurred, and $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$ that it did not occur: the odds are 6 to 1 in its favour, and its probability is $\frac{6}{7}$. But how can the accuracy of their testimony be so estimated? Is it because they have been found to tell the truth three out of four times, and two out of three times, respectively? But then their carefulness and competence (*scientia*), no less than their truthfulness (*veracitas*), must be taken into account. And, furthermore, the general conditions are so variable, the fluctuations to which human character is subject under the influence of strong motives and temptations are so great and so uncertain, as to render the whole calculation practically worthless (*cf.* 260, 261).

To take another instance from the social sciences: A judge delivers a wrong sentence once in ten times on an average; can he be compared, as Condorcet and Poisson have compared him, to an urn containing nine white balls and one black one? The comparison is scarcely less inaccurate than uncomplimentary. Bertrand's criticism is unanswerable: "The urn is as passive and indifferent to the influences playing upon it as the balls; and the general set of conditions remains ever and always the same throughout the repeated drawings. But the judges listen to the evidence, and consult with one another about it; they hear the same facts, and each bases his sentence, whether it be wrong or right, upon those same facts. And just as one has his reasons for judging rightly, so has the other his reasons for judging in the opposite sense. It is not that he has put his hand, as it were, into the urn of his own mind, and drawn forth by chance an erroneous sentence. No; the influences that lead up to that sentence are of quite a different order from those that determine the drawing of a ball from an urn. He has believed a false witness, or distrusted an honest one, owing to some unfortunate combination of circumstances; or he has been unduly influenced by the pleading of a clever advocate; or he has perhaps been prejudiced by self-interest or some other consideration. The very same objective evidence has elicited just the opposite sentence from his two colleagues; so that the probability of their pronouncing all three the same sentence is not at all to be compared with the probability of drawing three balls of the same colour by three independent draws."[1]

269. Functions of Statistics and Averages: Their Right and Wrong Interpretations.[2]—In the presence of such complex social phenomena as we have referred to already, or of complex natural phenomena like those relating to climate, for example,—where unknown causes are interfering, through undiscovered laws, with those already known,—the compilation of *statistics* and *averages* will enable us to lay down highly probable, or morally certain, judgments, not about the happening of an

[1] J. Bertrand, *Calcul des probabilités*, p. 236.
[2] *Cf.* Mercier, *Logique*, pp. 349-51, 361-70.

individual event at a *definite* point of space and time, or about the happening of a whole class of such events, but about the happening of a *more or less definite proportion* of events of a certain class within *more or less definite* limits of space and time. Suppose, for instance, it has been observed to have rained *on an average* three days in the week during the past twenty years in a certain district. We may be morally certain that if the general climatic conditions of the district continue unchanged we shall have the same average in the future.

But, furthermore, knowing as we do that these results were the outcome of numerous natural causes interacting according to undiscovered laws, we have no reason to doubt that closer and more prolonged and detailed scrutiny of all the climatic conditions of that district, and of other districts, may gradually enable us to bring to light these laws, or some of them. Of course, no induction has yet brought to light the laws that have determined the given condensation of vapour and consequent rainfall; nor has any probable hypothesis as yet suggested what these laws may be. The phenomenon itself is so complex, the antecedent and concomitant conditions of such condensations of vapour in the atmosphere as lead to rain, are so manifold, their action and interaction so intimate and elusive, that it has been impossible, so far, to determine the exact influence of each elementary factor, and to formulate the law of their combined activities.[1] Provisionally, therefore, we have recourse to *statistics*: we catalogue the greatest possible number of instances, note their frequency and their coincidences with other occurrences, in the hope of discovering some clue to a causal connexion (243). We measure the rainfall during the different seasons of the year, per week, per day, per hour, etc., noting the altitude and other geographical conditions, the direction of the winds, etc. We draw up tables of all those various coincidences in the hope that sooner or later we may be able to eliminate the irrelevant circumstances, to trace the recurrence of the phenomenon to its natural causes, and bring to light the laws of their activity.

Or, again, the ratio of male to female births has been investigated, for over 200,000,000 children. For nearly two centuries the numbers of each sex born have been found to be *practically* equal—without any exception whatever of time or

[1] *Cf.* example (of monsoons) quoted above, 222, from Mr. JOSEPH's *Logic*, pp. 444-5.

country. Yet, not *quite* equal: the male births have been found to exceed the female in the ratio of between 104 and 108 males to 100 females. Now those remarkable facts,—that the numbers of births of each sex are practically equal, that there is a slight preponderance in the number of male births, and that this is found to hold good of every race, north and south, east and west, in town and country, among rich and poor alike,—those facts are surely due to the operation of some undiscovered laws of nature. For such a constant persistence of this remarkable ratio, there must be a sufficient reason in the *nature* of the antecedents themselves of those births, even though we have not the slightest suspicion as to what the *natural properties* may be to whose operation, in the antecedents, those uniformities are due.

All we can do in such cases is to note and observe carefully all the circumstances that we may suspect of having any possible influence in determining the nature of the recurring phenomenon. If, going a step further, we try to investigate the concurrence of those circumstances, their variation, the isolated influence of each, we are entering on the employment of the "inductive methods". As soon as some observer detects, amongst all those chaotic surroundings, certain elements which he *supposes* to be the constant, necessary antecedents of the phenomenon in question, he makes a scientific hypothesis. The verification of this hypothesis will be the work of induction proper.

By statistics, therefore, we make a *simple enumeration* of the phenomena to be explained, we reach rough, *empirical generalizations*, which suggest hypotheses that may be later on erected into natural laws for the *explanation* of those phenomena. From statistics we often, therefore, obtain suggestions or indications of the laws underlying complex phenomena, and whose existence had been hitherto unsuspected.

The value of statistics must not, however, be overestimated. When we are investigating the nature and causes of things and events in the natural and social sciences, we are face to face with *facts*. In statistics about those events we are brought face to face with *syntheses*. The statistician must regard his figures as a sort of symbol, whose character and significance are more or less enigmatic; and he must diligently seek out all the probable causes of the facts he has symbolized before him, with a view to their scientific explanation.[1]

[1] *Cf.* Liesse, *La statistique*, p. 49.

Take the case of mortality statistics. A series of deaths is not a phenomenon of precisely the same sort as the drawing of a series of cards from a pack. Each particular death—the death of this particular person at this particular time and place—is the necessary outcome of certain (partially known and partially unknown) natural causes, operating detrimentally in his organism; or it may be the inevitable result of a complex concurrence of natural causes culminating in a fatal accident. The drawing of this particular card from the pack is likewise determined by the concurrence of a number of unknown natural causes. The difference is not in the individual cases: it lies in this, that while the general set of antecedents does not vary in the case of drawing a series of cards, it may vary considerably in a series of deaths. The number of deaths per year in any country depends on many and *variable* factors; and, consequently, the average number of deaths during any period of years furnish a very uncertain basis for conjecturing the average number during another period of the same duration. In Belgium, for example, the average age of the inhabitants in 1829 was 31 years 5 months; in 1856, 38 years 1 month; in 1890, 45 years 1 month.[1]

Tables of mortality form part of the basis on which the business of life insurance companies is conducted. Yet those tables are recognized as only rough approximations to any real constancy: the actuaries take care to *revise* and *readjust* their tables frequently, in order to keep them in agreement with the actual facts.

There is still more reason for caution in making any attempt to explain phenomena of the psychological, moral, or social orders by statistics and averages, or to erect such statistical uniformities into *laws*.[2] During the last century great hopes were aroused of reading the secrets of moral and social phenomena by the application of statistics to their domain. In his *Essai de physique sociale*, published in 1839, Quételet proposed to himself to study, by their consequences, "the natural causes, whether favourable or unfavourable, that influence the development of man". The initiative of this able Belgian scholar opened up the way to a number of highly interesting but extremely delicate researches. He himself brought his observations to bear on the age at which

[1] MANSION, *op. cit.*, p. 32. [2] *Cf.* WELTON, *op. cit.*, ii., p. 198.

people marry. Those observations extended over the years 1841-1865. For example, during the years 1841 to 1846, the number of men who got married in the towns of Belgium between the ages of twenty-five and thirty, were, respectively, 2681, 2658, 2698, 2698.[1] Again, the statistics of crimes showed the same marked constancy. "There is an annual tax [of human beings] which we pay with frightful regularity to the prisons, convict settlements, and scaffolds."[2] "It is well known," writes Professor Welton,[3] "that the number of persons who commit certain crimes, who are born, or who die, in the course of a year bears a remarkably uniform proportion to the total number of the inhabitants of any given country; there is, as we say, a pretty constant *average* preserved in many of the phenomena of social life. For example, something over seventy people out of every million commit suicide every year in England and Wales, whilst in Saxony the proportion is about five times as great as this, and in Ireland only about one-third as large. And these numbers are found to remain very uniform from year to year. Moreover, the averages are found to vary with great regularity according to the months of the year, being highest in June, falling regularly to December, and then gradually rising again; and this occurs year after year. Further, the proportion who commit suicide at different ages remains fairly constant."

Now what does this constancy of averages point to? To a "'law' which must of necessity be fulfilled"?[4] No, but simply "to the fact that social and material conditions remain comparatively unchanged for mankind in general. But it is an error to assume that such a statistical uniformity proves anything beyond its own existence. . . . With regard to the future we can only judge that the same numbers will be found to hold, if the same general conditions remain. . . . But this is mere tautology, for it simply says, If the past be exactly repeated it will be exactly repeated. The very question is whether all the conditions—known and unknown—will remain unchanged. . . . But uniformity in averages . . . involves no necessity for its own continuance. No doubt every element of reality is strictly determined in all its details by its conditions—given exactly those conditions, that result is necessary. But this necessity is concealed by the average, which neglects all the particular characteristics of the individual

[1] QUÉTELET, *Système sociale*, p. 68.
[2] *ibid.*
[3] WELTON, *op. cit.*, p. 198.
[4] *ibid.*, 198-9.

instances. Necessity is only found when conditions are exactly determined, and *ex hypothesi* that is just what is not done in the case of an average. As Sigwart says: 'Such uniformities of numbers and averages are primarily mere descriptions of facts, which need explanation as much as the uniformity of the alternation between day and night; and the explanation can be found only where the actual conditions . . . are forthcoming. But these are the concrete conditions of the particular instances counted, they are not directly causes of the numbers; it is only the nature of the concrete causes which can show it to be necessary for the effects to appear in certain numbers and numerical relations' (*Logic*, Eng. trans., vol. ii., p. 490)."[1]

Inferences from statistics about natural and social phenomena can never, therefore, be more than probable approximations—not laws. Those phenomena result from the combination of numerous unknown or partly unknown causes; and we have no guarantee that these will persist unchanged. No doubt, if the future faithfully resemble the past, the tabulated averages will recur. But who knows accurately *what* has been the past? Or what guarantee have we for thinking that the future will be a repetition of it? Furthermore, we will have observed fluctuations of more or less importance in the number and order of the past events, and we must take warning from them not to expect any greater regularity in the future.

Careful and conscientious scientists, like Quételet, recognize in what a transferred sense the word "law" is applicable to the moral world. He protested against the accusation that according to his theory every year should necessarily produce its crop of crimes in the same number and order, and with the same invariable distribution of each class of crime over the same regions. He objected to the word "invariable," and never used it himself in his writings. On the contrary, he had written expressly: "The laws relating to the condition of the social body are not necessarily invariable: they can change with the nature of the causes that give rise to them".[2]

But hasty and unreflecting people have pushed to absurd extremes the idea of seeking in statistics an *explanation* of the whole vast group of phenomena which constitute the subject-matter of the social, political, and economic sciences, to erect statistical averages and uniformities into laws, and to establish a new science which was to be a sort of *social mathematics*. It was obviously under the influence of such preoccupations as these that Buckle wrote paragraphs like the following: "In a given state of society a certain number of persons must put an end to their own life. This is the general law, and the special question as to who shall commit the crime depends of course

[1] WELTON, *op. cit.*, p. 199. [2] *op. cit.*, p. 15.

upon special laws; which, however, in their total action, must obey the large social law to which they are all subordinate. And the power of the larger law is so irresistible, that neither love of life nor fear of another world can avail anything towards even checking its operation ".[1]

It is only by a complete misconception of the significance of averages that any such "*must*," or any such rigorously necessary "*law*," can be thus read into them. An average is not to be regarded as a secret something which determines events. This blunder is often made in social statistics. After finding a certain average in human affairs, we conclude that some secret fate is at work. By the aid of a little rhetoric we easily persuade ourselves that an event is fully accounted for when "the law of averages" demands it. "There may be an average in birth and death and crime, but after all, the average is not responsible for any of them. It takes something more potent than an average to produce typhoid fever or to crack a safe." [2]

That there is a certain regularity in those social phenomena which result from man's free activity is undeniable. But the regularity is periodical rather than constant; nor is there any exact law governing the duration of the phases or periods. Tabulated returns have been made out, showing the periodical recurrence of economic crises in France, England, and the United States, from the beginning of the century to 1882. The practical coincidence of the dates in those tables clearly indicates similarity of causes and solidarity of their activity and effects. But if the fact of periodicity is certain, the apparition of such commercial crises, and the intervals between them, are variable elements. This variability is the outcome of causes so complex that it is quite impossible to attempt to formulate any sort of a periodic law.[3]

"The free action of human beings," writes M. Bertrand,[4] "as also the action of animals—in spite of what Descartes has said of them—bring into the domain of causality an element inaccessible to the calculus."

The justice of this remark must be apparent to any impartial student of natural and social phenomena. We have already referred to the futility of any attempt to reduce all reality to a system subject to purely mechanical laws. A cosmic system made up of "indifferent" atoms and local motion, capable of exact mathematical measurement, and invariable in the sum-total of its elements

[1] *Hist. of Civilization*, vol. i., p. 25, *apud* WELTON, *op. cit.*, p. 199.
[2] BOWNE, *op. cit.*, p. 188.
[3] The table of cases from 1800 to 1882 is as follows:—

France.	England.	United States.
1804	1803	1803
1810	1810	1810
1814-1815	1815	1814
1818	1818	1818
1825	1825	1826
1830	1830	—
1836-1839	1836-1839	1837-1839
1847	1847	1848
1857	1857	1857
1864	1864-1866	—
—	1873	1873
1882	1882	1882

[4] *op. cit.*, Preface, p. xlix.

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and their possible combinations, would furnish an excellent theatre for the play of "chance". And any isolated and limited portion of it would illustrate the theory of probability as admirably as a game of chance. The ancient Greeks conceived the world as formed by a chance arrangement of atoms—a *concursus fortuitus atomorum*. Many moderns clothe the same crude conception in a more scientific and pretentious terminology. Such a mechanical conception of the universe is only a prejudice. Even in the inorganic world there is a great deal more than mere mechanics. "As Mach puts it; 'Purely mechanical phenomena do not exist . . . [They] are abstractions made, either intentionally or from necessity, for facilitating our comprehension of things. The same thing is true of the other classes of physical phenomena. . . . The view that makes mechanics the basis of the remaining branches of physics, and explains all physical phenomena by mechanical ideas, is in our judgment a prejudice . . . The mechanical theory of nature . . . is an artificial conception.'"[1] And, just as mechanics is inseparable from the physics of the inorganic world, so is the latter inseparable from, and inevitably influenced by, the activities of the organic world—animal sensation and appetition, human knowledge and volition: "Processes, thus, that in appearance are purely mechanical, are, in addition to their evident mechanical features, always physiological. . . . The science of mechanics does not comprise the foundations, no, nor even a part of the world, but only an *aspect* of it."[2]

Besides the laws, therefore, which govern the processes of the inorganic world, there are likewise laws which govern the conscious activities of the brute creation, and laws that govern the free activities of man. But the mode of causality is not the same in these three orders. It is not the same kind of law that connects cause and effect in each. If we describe as a *mechanical* necessity the connexion of cause and effect in a steam engine, we must find another adjective for the necessity, or the law, by which a dog obeys (or disobeys) his master; for sensation and appetite make the dog something more than a machine, and introduce an "indeterminate" element, an element of "uncertainty," into our calculations. And we must find yet another conception of law for the free, self-determining activity of the human will, and for the general uniformity that prevails in human conduct, notwithstanding the "mechanically" disconcerting, but very real, factor of human freedom.

We have already shown (223) that the constancy actually observed in social phenomena is not incompatible with human freedom. We may add here the following eloquent passage from Quételet, which will, perhaps, be found as suggestive and instructive as it is interesting: "Amongst the facts disclosed in my book, the one which has given rise to most alarm is the constancy of crime from year to year. By a comparison of numbers, I believed I had data for inferring, as a natural consequence, that in a given country, under the same conditions and influences, we might expect a repetition of the same facts, a reproduction of the same crimes and the same condemnations. But how was this received? A crowd of timid people raised the cry of fatalism!" The facts, nevertheless, were undeniable; the whole thing was to interpret, to understand them. "Now what do the facts teach us? This, simply,—that in any given State, subject to the influence of the same causes, the effects will

[1] Mach, *Principles of Mechanics*, pp. 495-6; *apud* Welton, *op. cit.*, ii., p. 209.
[2] Mach, *ibid.*, p. 507, *apud* Welton, *ibid.*

not differ appreciably; they will oscillate more or less about some mean. Now, mark well what I have said: subject to the influence of the same causes; so that, therefore, if those causes change, the effects will be likewise modified. But, since the laws and principles of religion and morality are the source of the influences in question, I cherish not only the hope, but—what you perhaps do not—the deepest conviction even, that society can be reformed and ameliorated."

"But, you ask, what becomes of free will? In presence of the facts I am not concerned to debate this much-discussed question, but yet I cannot pass it over in silence, for the simple reason that it seems to me to involve in itself one of the most admirable laws in all creation, a law of conservation which furnishes a new proof of the wisdom of the Creator, a proof, the existence of which you, with your cramped conceptions on the moral organization of man, have been unable even to suspect. To avoid the reproach of denying free will entirely, must we go to the opposite extreme and allow it an absolutely indefinite scope? Or, in that event, what would have become of the whole race centuries ago,—with all the mad follies that have entered the minds of men, and all those evil inclinations that have—even as things are—desolated society? Scourges have come and gone, but man and his faculties remain unchanged, at least so far as our observation serves us. And why? Because the self-same finger that has traced its confines for the ocean has likewise set their limits to man's turbulent passions, and the self-same voice has commanded both: thus far you shall go but no farther. When we have to make up our minds about the simplest matter, do we not find ourselves under the influence of our habits, and our needs, and our social conditions and relations, and of a whole crowd of conflicting motives which drag us to the one side and to the other? Nay, so strong are those influences that we have no hesitation in predicting, even about people whom we know but slightly, or not at all, what decision they are going to take. Why those innumerable forecasts and guesses you are making every day of your life, if you are not convinced beforehand in each particular case that the influence of inherited character and motives, etc., and not free will itself, will determine the issue? Looking out on the world *a priori* you give this free will the very widest latitude; but, when you pass from theory to practice, and talk of what is going on perpetually around you, you flatly contradict your *a priori* self by making your predictions about individuals! Yes, about individuals, in whom motives, etc., can oscillate to such a degree that it would be against all the principles of the theory of probabilities to take them as data for the calculus, or to base even the slightest inductions upon them."[1]

WELTON, *Logic*, ii., pp. 165-80. CLARKE, *Logic*, pp. 356-424 *sqq*. ZIGLIARA, *Summa Philosophica*, i. (42). MELLONE, *op. cit.*, pp. 251-61. JOSEPH, *Logic*, pp. 323. KEYNES, *Formal Logic*, p. 367. VENN, *Logic of Chance*, *passim*. JOYCE, *Logic*, pt. ii., chap. xxiii. BORDEN P. BOWNE, *Theory of Thought and Knowledge*, pp. 178 *sqq*. MERCIER, *Logique*, pp. 350-70. GREISE, *La Statistique*, QUÉTELET, *Système sociale*. *Cf.* works mentioned, p. 269, n.

[1] QUÉTELET, *Études sur l'homme*, pp. 11, 12. For Quételet's attitude towards free will, *cf.* LOTTIN, *Le libre arbitre et les lois sociologiques*, in the *Revue Néo-scolastique*, November, 1911; QUÉTELET, *Statisticien et sociologue* (Louvain, 1912).

CHAPTER III.

ERROR AND FALLACIES.

270. LOGICAL TREATMENT OF ERROR AND ITS SOURCES.—Certitude, probability or opinion, doubt, and error—such are the various states of mind we experience in our search for truth. It is the function of logic, as a practical science, to guard us against the last of these states, and to enable us, as far as may be, to reach the first. Hence, it analyses our processes of judging, reasoning, generalizing, and demonstrating, with a view to discover, and to familiarize us with, the laws and conditions of correct or accurate thought. And we shall be made all the more capable of conforming our own thinking processes to those laws, if we conclude the treatment of our subject by a special study of the more common sources of error, the ways in which we are most likely to fail in the application of logical principles. We shall have a better grasp of the ways in which we *ought* to think, when we have contrasted these with the ways in which we *ought not* to think. Not, indeed, that the knowledge of these latter will be an infallible safeguard against error. Nevertheless, it will certainly be helpful to us,—"Forewarned, forearmed". If, for instance, the conclusion of an invalid argument happens to be true, and to be known by us as true, this very knowledge might throw us off our guard and lead us to accepting the argument as valid: it would be less likely to do so were we familiar with the various ways in which deception may creep into an argument. Or, again, if we know the conclusion to be false we know that there must be something wrong with the argument; but, without a knowledge of the rules of reasoning, and their possible violations, we may be unable to discover what it is precisely that is wrong, especially if the premisses seem true: we may know there is a flaw somewhere, without being able to see *through* it, to explain it, to lay bare the source of it. No doubt, in the course of our work, we have already pointed out, by way of contrast, in illustration

of the various canons and conditions of correct thinking, the more common violations of the latter. But that was only in passing; and the whole subject of *Fallacies* demands for itself a separate and more explicit treatment.

A *systematic classification*, however, of the ways in which the human mind can fall into error, seems to be, from the nature of the case, practically if not absolutely impossible; for error conforms to no laws. Of course it *breaks* laws, the laws that would, if observed, conduct to truth; and hence we may attempt to classify the heads or sources of error according to the laws or canons violated by the mind in reaching the error. This is the plan adopted by Professor Welton.[1] It suffers, in common with other classifications, from this drawback, that the *same* individual example of an error, arrived at through a certain process of thought, may be referred to *different* heads or types of fallacy, may be traced to the violation of *different* logical principles. This difficulty, of referring concrete cases to their proper headings, is present in every scheme—whether of attempted *classification*, or of mere *enumeration* of the various types of fallacy. Apart from this difficulty, however, of referring individual instances to a type, it would seem practically impossible to give an entirely exhaustive catalogue of the types themselves, the kinds of fallacious thought-processes in which the mind may become involved; because there are possible sources of error peculiar to every new department into which we may carry our search after truth. We must, then, only endeavour to notice at least all the more *important* and *common* forms of deception incident to such investigation.

With the *special* sources of error involved in the subject-matter of this or that particular science it is not, of course, the function of logic to deal, but only with *common* sources. Still, this must not be pressed too far. We have already followed the workings of reason a certain distance into various kinds of subject-matter, into the matter of induction, into that of deduction, into the reasoning process known as the Aristotelean syllogism, and into certain analogous forms of mediate reasoning (192); and we have seen that everywhere, even in the mental act of judgment, the form assumed by our thought-process depends to a certain extent upon the matter thought about (10, 81). Hence there will be certain sources of error—certain misconceptions, mistakes, and ambiguities—peculiar to the investigation of this or that particular subject-matter. The reason of this is—partly, at least—because the different kinds of subject-matter call forth different types or forms of reasoning, in accordance with certain axioms that involve conceptions or intuitions of certain systems of relations—such as those of space, time, or quantitative proportion—on which

[1] *op. cit.*, ii., bk. vii.

this or that special form of reasoning is based. In so far as such initial conceptions and the reasoning processes dependent on them are, of themselves and apart from their particular subject-matter, liable to be misapprehended or misapplied, they call for notice in a general treatment of logic. If, for instance, I argue that because "*a* is half of *b*, and *c* is half of *b*, therefore *c* is half of *a*," it is, no doubt, "only a perception of the nature of quantity that reveals . . . the invalidity of the . . . argument"; [1] but this does not remove the fallacy from the jurisdiction of logic, for it is a function of logic to determine what is, and what is not, an axiom or self-evident truth, whatever be the subject-matter in question. The assumption that "things which bear the same quantitative relation to the same thing bear *this same* relation to one another," is a misconception of the truth that such things "bear *a relation of equality* to one another". It is as much the duty of logic to expose this "undue assumption of axioms" as, for instance, to expose the fallacy of arguing that "because all the angles of a triangle are equal to two right angles, and *A B C* is an angle of a triangle, therefore it is equal to two right angles"—a fallacy which may similarly be resolved into the misconception of an axiom, *viz*. the *Dictum de omni*.

271.—ERROR AND FALLACY.—The forms or types of misleading thought-processes examined in logic are usually comprehended under the general title of *Fallacies*. But this term has considerable elasticity of meaning: it has been used in all shades of meaning, from the widest sense of "any erroneous judgment or belief," to the narrowest sense of "the violation of some formal rule of inference".

The former usage is *too wide*. It is not the false judgment or belief itself we should call a fallacy, but rather the causes or sources to which the presence of that error in the mind is due. That something or other, in the origin or progress of the thought-process, which deceived the mind into assenting to a false judgment—that is the fallacy proper. Where the error is reached through a process of inference, it may be due either to the assumption, at some point in the process, of a false premiss, or to the acceptance of a formally inconclusive inference as valid. To restrict the meaning of the term *fallacy* to the latter class of cases, is as inconvenient as to restrict the term *logic* to an exposition of the formal laws of inference, thus making it a science of mere consistency. This usage is *too narrow*. Besides, the distinction between accepting a false premiss and an inconclusive argument is not fundamental. The mind, which, for one reason or another, accepts a formally invalid inference as valid, is by that very fact assenting to a false proposition as true: for the formal force of an

[1] JOSEPH, *op. cit.*, p. 530.

inference can always be expressed in a hypothetical judgment showing forth the necessary dependence of consequent on antecedent: and that judgment, stated in general terms, is the axiom or canon of the form of inference in question. Hence, to accept a formally invalid inference as valid, is to mistake for an evident axiom or canon some judgment that is neither evident nor true. In fact, every logical fallacy may be analysed into the acceptance by the mind of some false judgment as true.[1]

When tacit assent to an implied false judgment, whether this be a canon or a premiss,[2] *leads to further error*, we shall call such false assumption a fallacy; as also the assumption of a false premiss at any stage in a reasoning process: *provided always there is a semblance of validity or truth about the judgment so accepted.* This latter condition is essential to a fallacy. Only by being deceived can the mind be led into error; and it can be deceived only because error can assume for it the semblance of truth. When, therefore, a judgment is *manifestly* false, an argument *plainly* inconclusive, some rule or canon of correct thinking *openly* violated, there is no fallacy, because there is no deception. An argument which openly violates some formal rule of inference is sometimes called a *paralogism*, or, also, a case of "*non sequitur*".

We have said that a false judgment, simply as such, cannot properly be called a fallacy, that the latter is to be sought rather in the grounds and motives which induced us to assent to the false judgment as true. We can, however, distinguish between the *logical* grounds or reasons, and the *psychological* motives, for our assent to a judgment (198, 225). The logical grounds are designated by the general title of *evidence* (248): they appeal directly to the *intellect*, and are its proper object. The evidence for any proposed judgment is either *mediate* or *immediate*. If we assent to a judg-

[1] This is illustrated by Mr. JOSEPH in connexion with the fallacy "*post hoc, propter hoc*": "Nor is it peculiar to this fallacy," he writes (*op. cit.*, pp. 554-5), "that it can be expressed as a false principle. *Equivocation* proceeds on the false principle that a word is always used with the same meaning: *Accident* on the principle that whatever is predicated of a thing may be predicated of its attribute, and *vice versa*: *Secundum Quid* on the principle that what is true with certain qualifications is also true without them. And the fact that these different types of fallacious inference severally depend on a false, or misleading, principle, is what was meant by calling them *loci* of fallacy."

[2] When "the falsity of the premiss can only be ascertained empirically," Mr. JOSEPH (*ibid.*, 532) will not call its assumption a fallacy. This usage would exclude such inductive fallacies as non-observation or mal-observation. We prefer to extend the term to "any false assumption used as a premiss" (*ibid.*, 535).

ment on account of *its known connexion* with some other already known judgment or judgments, it is called a *conclusion*, and its evidence is *mediate*. If, on the contrary, we assent to it on its own account, because of the direct appeal it makes to our intellect as true, it is called a principle, or a *self*-evident, or *immediately* evident, truth.

It needs but little experience of life, however, to convince us that men differ widely in the judgments they accept as self-evident; and this is, in part at least, because men's assents are influenced not by intellectual evidence alone, but also by non-intellectual motives. Their estimate of the evidence is partly dependent on their character and dispositions, their mental habits and training, their likes and dislikes, their will, passions, emotions, etc. (231). These are *psychological* influences, of which logic can take no direct notice. But when there is question of assent to judgments on *immediate* evidence, of estimating this latter, and discriminating between *real* and *apparent* evidence, it is not easy to divide the misleading influences at work into psychological and logical. Against influences supposed to belong to the former class—passion, prejudice, party-spirit, precipitancy, etc., logic can merely give a general warning (203). Only when the source or cause of the deception is *a violation of some logical principle*, can logic deal more effectually with it. Of the source of the deception, the victim of a fallacy is, of course, always and necessarily unconscious: the mind aware of its deception would be no longer deceived. But when deliberate reflection on our thought-processes brings into consciousness the causes which have misled us, these will always appear to have been either inadvertent and indeliberate violations of some of the laws and principles of correct thought, propounded in logic, or else to have been misleading influences of a more subtle and subjective kind, springing from character, prejudice, mental training, etc. The former alone we shall call *Logical Fallacies*.

We may, therefore, define a *Logical Fallacy* as ***a violation of some logical principle, calculated to lead to error by reason of its apparent validity.***

272. SOME ATTEMPTED "CLASSIFICATIONS" OF FALLACIES. "FORMAL" AND "MATERIAL" FALLACIES.—(*a*) The line of thought we have just followed suggests a classification of fallacies made by John Stuart Mill: a "catalogue of the varieties of apparent evidence which are not real evidence".[1]

[1] *Logic*, V., i., § 1.

Assents to false judgments on *immediate* "apparent" evidence, he calls *fallacies of simple inspection*. If the evidence is mediate, we may have *fallacies of inference*. These he subdivides into *fallacies of confusion*, in which there is "an indistinct, indefinite, and fluctuating conception of what the evidence is";[1] and fallacies due to "a false estimate of the probative force of known evidence".[2] The latter may be incident either to deduction or to induction; and in each case it may be due either to the assumption of false premisses, or to a miscalculation of the probative force of true ones. The assumption of false premisses in deduction he identifies with fallacies of simple inspection; wrong estimates of the probative force of deductive arguments he calls *fallacies of ratiocination*. The two corresponding types of fallacy in induction he calls respectively *fallacies of observation* and *fallacies of generalization*. Thus we reach five main classes, some of which are made to yield a number of sub-classes. The following scheme will show forth the various members of the division :—

Fallacies.

(1) *Of simple inspection;* | Of inference;

(2) *Of confusion;* | Of miscalculation of force of evidence.

Of deduction. | Of induction.

(1) *Assumption of false premisses;* | (4) *Fallacies of observation;*

(3) *Fallacies of ratiocination;* | (5) *Fallacies of generalization.*

This classification is based upon an intelligible principle, but it is not exhaustive as it stands; its headings are very general, and we shall find a more convenient plan of treatment than to try to work these out in detail.

(*b*) Bacon's classification of the sources of error, or obstacles to the right "interpretation of nature" is not without interest. He calls them "*Idola*"—*phantoms* which mislead the mind in the processes which subserve *induction*, just as "sophistical reasonings" mislead in ordinary *argumentation*.[3] He reduces them to four great classes: (1) "*Idola Tribus*,"

[1] *Logic*, V., vii., § 1. [2] *ibid.* [3] *Nov. Org.*, i., Aph. xl.-lxviii.

or *Phantoms of the Tribe:* tendencies inherent in human nature itself, exposing men to the danger of interpreting falsely the impressions made upon their senses; (2) "*Idola Specus*," or *Phantoms of the Cave:* sources of error peculiar to the individual, owing to his personal idiosyncrasies and mental habits; (3) "*Idola Fori*," or *Phantoms of the Market Place:* deceptions due to the limitations of language, which is the medium of intercourse and exchange of ideas among men; (4) "*Idola Theatri*," or *Phantoms of the Theatre:* false theories and teachings that have gained currency and are accepted unquestioningly because they happen to be the fashion of the time.

(*c*) Of far greater importance than the classifications of Mill or Bacon is Aristotle's enumeration: not, perhaps, because of any exceptional intrinsic merit, but because it has furnished logic with a universally recognized *nomenclature* of the more important fallacies, and has never been entirely superseded. Aristotle dealt with fallacies in the last book of his *Organon*, under the title of *Sophistical Refutations, Sophistici Elenchi*, περὶ σοφιστικῶν ἐλέγχων. He wrote at a time when public oral discussions and disputations were of the highest educational, social, and political importance, when great issues were decided by the masses on hearing conflicting views championed by special pleaders, when skill in rhetoric, or the art of persuading by plausible reasons, was eagerly ambitioned by public men, and profitably taught by those influential educators of the Athenian youth, the *sophists*. It was with a view to exposing the various dialectical deceits and devices for misleading, which were currently used in those public debates, that Aristotle wrote the περὶ σοφιστικῶν ἐλέγχων. Those "sophistical arguments," or "sophisms," as he called them, were connected in his mind with the intention to deceive or mislead:[1] it was indeed the sophists' own avowed exclusive concern for dialectical victory, irrespective of truth and right, that eventually brought their name and methods and arguments into disrepute. Aristotle's "sophisms," therefore, all regard *inference*, and inference conducted by way of oral disputation. Hence his list is not exhaustive; and, such as it is, it contains many fallacies, arising from the *use of language*, which are nowadays likely to be regarded as trifling; though they must have given some trouble at a time when so much depended upon the general impression produced by public dialectical encounters, for which we have, per-

[1] Kant applies the term *Sophism* to a fallacy used for the purpose of deceiving another, and *Paralogism* to a fallacy which deceives the person who uses it. This ground of distinction is *psychological*, not *logical*. The *logical* nature of a fallacy is independent of the intention of the party using it.

haps, no parallel in modern times, beyond an occasional brilliant display of cross-examination in our law courts.

Aristotle divides fallacies into two groups: (1) *Σοφίσματα παρὰ τὴν λέξιν, sophismata "in dictione,"* fallacies due to *ambiguity in language;* and (2) *Σοφίσματα ἔξω τῆς λέξεως, sophismata "extra dictionem,"* fallacies from sources *other than ambiguity of language.* This division is, of course, exhaustive because dichotomous; but there is no positive bond between the various members of the second or negative class. In the latter he enumerates seven species, in the former six. They are as follows:—

(*A*) Sophismata *in dictione:—*

(1) *Equivocatio*, equivocation, *παρὰ τὴν ὁμωνυμίαν*—due to ambiguity in a single term (*ὄνομα*).

(2) *Amphibolia*, amphiboly (or amphibology),[1] *παρὰ τὴν ἀμφιβολίαν*—due to ambiguity in the construction of a sentence or phrase (*λόγος*).

(3) *Compositio* (or *sensus compositus*), composition, *παρὰ τὴν σύνθεσιν*—due to taking together what ought to be kept separate.

(4) *Divisio* (or *sensus divisus*), division, *παρὰ τὴν διαίρεσιν*—due to separating what ought to be kept together.

(5) *Accentus*, accent, *παρὰ τὴν προσῳδίαν*—due to confusion of meanings differentiated by diversity of accent or quantity.

(6) *Figura dictionis*, figure of speech, *παρὰ τὸ σχῆμα τῆς λέξεως*—due to misinterpretation of the force of a verbal inflexion.

(*B*) Sophismata *extra dictionem:—*

(1) *Accidens*, accident, *παρὰ τὸ συμβεβηκός*—equating subject with attribute.

(2) *A dicto simpliciter ad dictum secundum quid*, and its converse, *a dicto secundum quid ad dictum simpliciter* (no English name for this; called briefly *Secundum Quid*), *παρὰ τὸ ἁπλῶς ἢ πῇ λέγεσθαι καὶ μὴ κυρίως*—ignoring the limitations or special conditions under which a statement is true or false.

(3) *Ignoratio elenchi*, arguing beside the point, *παρὰ τὴν του ἐλέγχου ἄγνοιαν*—mistaking the matter in debate.

(4) *Petitio principii*, begging the question, *παρὰ τὸ ἐν ἀρχῇ λαμβάνειν*—assuming in some form or other the proposition to be proved.

[1] "The Greek word is a *ἀμφιβολία*, which is said to be an *ἀπάτη παρὰ τὸν λόγον*, as distinct from *ὁμωνυμία*, when the ambiguity is in an *ὄνομα* (*Soph., El.* vii. 169a, 22). Hence arose the compound *ἀμφιβολολογία*, which became corrupted into amphibology, as *εἰδωλολατρεία* became corrupted into idolatry."—JOSEPH, *op. cit.*, p. 539, n. 2.

(5) *Non causa pro causa*, false cause, παρὰ τὸ μὴ αἴτιον ὡς αἴτιον —supposing a conclusion to follow from a premiss which is really irrelevant.

(6) *Consequens*, consequent, παρὰ τὸ ἑπόμενον—assuming that a hypothetical proposition is always and necessarily reciprocal.

(7) *Plures interrogationes*, many questions, παρὰ τὸ τὰ δύο ἐρωτήματα ἓν ποιεῖν—asking a question in such a form that a single answer involves more than one admission.

(*d*) Aristotle's distinction between fallacies *in dictione* and fallacies *extra dictionem* is not the same as Whately's division into *logical*, and *non-logical* or *material*. By "logical" fallacies Whately meant those in which "the conclusion does not follow" from the premisses; by "material," those in which the "conclusion does follow" from the premisses. In the former class, the defect of proof lies either in a *manifest* violation of some of the formal laws of the syllogism—*quaternio terminorum*, *undistributed middle*, *illicit major*, *illicit minor*, *negative premisses*, etc., defects which remain even when symbols are substituted for the terms and concepts, and which Aristotle would not regard as sophisms owing to the transparency of the mistake;—or the defect lies in a similar violation *masked in ambiguous language*. The transparent defects Whately called *purely logical*, the cloaked defects *semi-logical* fallacies. The latter he regarded as all alike reducible to *ambiguous middle term*, including in this class all Aristotle's sophisms except the *ignoratio elenchi*, the *petitio principii*, and the *non causa pro causa*. These three he included in his "material" fallacies, by which he understood mistakes due to assuming false or unproven premisses, or premisses which prove the wrong conclusion. Whately's main distinction—between formally inconclusive arguments, and other sources of error—is sound and intelligible. But his nomenclature is objectionable. It is due to his narrow, nominalistic view of the scope of logic. *All* fallacies are *logical*, inasmuch as they are violations of logical principles or canons. Then, although most of Aristotle's *sophismata*, included in Whately's class of "semi-logical" fallacies, do in fact usually lead to formally invalid syllogisms through ambiguous middle terms, yet this is not clear in regard to some; and they certainly may lead to error otherwise as well. Hence the attempt to group them under such a head is unsatisfactory. Finally, on his own view of the scope of logic, Whately should not have dealt at all with what he called "non-logical" or "material" fallacies.

The distinction between a "formal" fallacy and a "material" fallacy is not fixed or clear—any more than that between "formal" and "material" logic. But at all events *in a reasoning process*, we can distinguish between the narrower "formal" or "consistency" aspect, which is independent of the truth of the *premisses* and the *meaning* of the terms used, and the "material" or "truth" aspect. Now, the formal validity of an inference, in this narrow sense, being independent of the subject-matter, *i.e.* of the meaning of the concepts and terms employed, it is only when the invalidity persists with the symbols, *i.e.* when some of the formal laws of reasoning are violated, that the fallacy is a *formal* one. If the fallacy lies in the language, *i.e.* in the *meaning* of the terms employed, in ambiguities of meaning, then its source is in the subject-matter, in the things for which the terms stand, and the fallacy is a *material* fallacy. An ambiguous middle term in a syllogism is, therefore, in this sense a *material* fallacy: when its two distinct meanings are explicitly substituted for it by two distinct terms, we have immediately the *formal* fallacy of *quaternio terminorum*. In this meaning of the expression "material fallacy," all Aristotle's *sophismata in dictione* are, when they enter into an inference, material fallacies; while some of his fallacies *extra dictionem* are formal in the sense that they can be represented in symbols; so that it is a mistake to confound Aristotle's two lists with Whately's semi-logical and material fallacies, respectively: a mistake into which Jevons seems to have fallen.[1]

(*e*) There is, finally, Professor Welton's classification, which we purpose to follow: the classification according to the logical principles violated.

273. FALLACIES INCIDENT TO CONCEPTION.—We shall first consider the fallacies incident to conception (Part I.)—to the processes of forming concepts, of expressing these in terms, of securing clearness and distinctness in thought and language by definition and division, by analysis and comparison of the various logical characteristics of concept and term. Violations of the rules of logical definition and division lead to faulty conceptions, and thence to erroneous judgments and reasonings. Indeed, most of the fallacies incident to conception may be regarded as due to faulty definition. They include many of Aristotle's *sophismata in dictione*.

(*a*) EQUIVOCATION.—The use of the same word in different

[1] *Elementary Lessons in Logic*, xx. and xxi.

senses is the simplest and commonest form of ambiguity in language. It is due to vagueness of conception, resulting from want of accurate definition. Any of three terms of a syllogism may be ambiguous : most usually it is "ambiguous middle" we meet: in which case the syllogism really contains four terms. Many of the stock examples of this fallacy, found in handbooks of logic, are trifling; its most debased and trivial form being the *pun*. But equivocation is really a fertile source of very serious and elusive errors. In the course of any sustained argument, after the manner of the sorites, it may easily lurk unsuspected, owing to the almost imperceptible differences in the varying shades of meaning which may attach to the same term in different contexts. Language is not a perfect instrument of thought. It needs but little reflection on the elasticity of meaning discernible in such terms as "idea," "cause," "law," "nature," "government," "liberty," "money," "free-trade," "socialism," "home rule," etc.—to enable one to realize what an amount of error may be traced to this fallacy as its source. The following few examples will be sufficient for illustration: "Finis rei est illius perfectio; mors est finis vitae; ergo mors est perfectio vitae". "What is rare is dear; a horse for a penny is rare; therefore a horse for a penny is dear." "Criminal actions are punishable by law; prosecutions for theft are criminal actions; therefore prosecutions for theft are punishable by law."

(*b*) The fallacy of the CONCEPT WITH INCOMPATIBLE ATTRIBUTES arises from a vague, unreflecting, careless way of thinking, which can be remedied only by the deliberate analysis demanded by the process of logical definition. To speak, for instance, of an "indivisible portion of matter" is just as self-contradictory as to speak of a "square circle"; while an ultimate analysis of some of our concepts is so difficult that philosophers have at all times disagreed as to whether or not certain combinations of them —such, for instance, as "infinite quantity"—are self-contradictory.

(*c*) COMPOSITIO and DIVISIO, or the fallacies of confounding the *sensus compositus* with the *sensus divisus*. *Composition* is the fallacy of combining what ought to be kept separate; *division* is the converse fallacy, of separating what ought to be kept combined. Aristotle seems to have had in view here the unlawful combination or separation merely of words or phrases. "Three and four are odd and even; three and four are seven; therefore

seven is odd and even" (composition). "Seven and five are equal to eight and four; therefore seven is equal to five, and eight to four" (division). "'Is it possible for a man who is walking not to walk?' 'Yes.' 'Then it is possible for a man to walk without walking'" (composition). "'Though you are not now walking, you can walk?' 'Yes.' 'But if you can thus do a thing when you are not doing it, you can desire a thing when you are not desiring it'" (composition: it is only the *capacity to desire*, not the actual desire, that can coexist with the state of *actually not desiring*).

Of course, the illicit combination or separation of words, as in the cases just given, involves an illicit combination or separation, in our *thought*, of the *objects* denoted. But sometimes the fallacious mental process is not reproduced in the language. *Confounding the distributive with the collective use of terms* is, perhaps, the way in which the fallacy is most usually committed. The showman who announced that children of both sexes would be admitted free, and then proceeded to charge both for boys and girls,—on the plea that none were "children of both sexes,"—combined in *thought* the sexes which the language he used did not necessarily separate, but which were naturally assumed by all to be taken as separate. "All the angles of a triangle are equal to two right angles; *A B C* is an angle of a triangle; therefore it is equal to two right angles" (division). Such instances might also be classified as special cases of *equivocation*, seeing that they turn on the employment of an ambiguous term in different senses.

The spendthrift who thinks that, because he can prudently spend the portion *A*, or the portion *B*, or the portion *C* . . . of his revenue, he can therefore prudently spend *A*, and *B*, and *C* . . ., is guilty of the fallacy of composition; the miser who argues that, because he cannot prudently subscribe to charities *A*, and *B*, and *C*, and *D* . . ., he cannot prudently subscribe to any of them, is guilty of the converse fallacy, of division. The person who argues that protective tariffs would benefit all the industries of a country because it can be shown that such tariffs benefit this or that special industry, is guilty of the fallacy of composition. The person who argues that because a knowledge of this, that, and the other science, benefits the community, therefore every citizen should be taught all the sciences in the schools, is guilty of the same fallacy. "'Does one grain of corn make a heap?'

'No.' 'Do two?' 'No.' . . . 'Do two million?'" This example gave the name *sorites* to this class of fallacy among the Greek sophists. Exactly similar is the example called the *calvus*: "'Does pulling one hair from a man's head make him bald?' 'No.' 'Does pulling two?'" etc. They may be regarded as examples of the fallacy of composition. Here is an instructive example of the fallacy of composition, from John Stuart Mill's work on *Utilitarianism*:[1] "No reason can be given why the general happiness is desirable, except that each person, as far as he believes it to be attainable, desires his own happiness. This, however, being a fact, we have not only all the proof the case admits of, but all which it is possible to require, that happiness is a good: that each person's happiness is a good to that person, and the general happiness, therefore, a good to the aggregate of all persons". Apart from all the ambiguities (in the terms "good," "desirable," "happiness") which make the reasoning in this passage worthless, it is also vitiated by the fallacy of arguing that because *A* desires *his own* happiness (or what is "good" to *himself*), and *B* his own, and *C* his own . . . therefore *A* desires the happiness of (or what is "good" to) *B*, *C*, *D* . . . (as well as his own), and similarly *B*, and similarly *C* . . .; and that therefore the general happiness is desired by all, and is therefore a "good" to all.

(*d*) ACCENTUS.—By the fallacy of *accent* (or *prosody*) Aristotle meant the mistake of using a wrong tonic accent, or stress of the voice, in pronouncing the written Greek word: the written language had, in his time, no signs to mark the differences of stress and breathing in speech. The same mistake can arise in Latin from giving a wrong quantity in pronouncing the written word: "Omne malum est fugiendum; pomum est malum; ergo fugiendum". It was distinguished from equivocation, perhaps because words differently pronounced, though spelled the same way, are scarcely the same word; and the ambiguity was regarded as confined to written language. It is nowadays generally understood to embrace all ambiguities of meaning which turn on change of *emphasis* in speech: the commandment "Thou shalt not bear false witness against thy neighbour" may be made to bear different meanings according to the word emphasized. De Morgan would regard as an instance of this fallacy the introduction (without notice), into a quotation, of italics not in the original, or the

[1] p. 53.

omission of italics that were in the original, or any similar attempt to convey more or less than the context guaranteed in the original.

(*e*) By the fallacy of FIGURA DICTIONIS, *Figure of Speech*, Aristotle designated the erroneous supposition that the same inflexions, or roots, or other modifications of grammatical form in words, always imply similar kinds or modifications of meaning: that "*poeta* is feminine because Latin words of that form are usually feminine"; that "a man who walks on the whole day tramples on the whole day"; that "*important* is a negative notion because *impotent* is negative"; that when a person "is resolved" to do a certain thing he does not act freely but is passive, because when he "is beaten" or when he "is flattered" he is passive; that a wooden "image" is unreal because what is "imaginary" is unreal; that paronyms like "artist," "artisan," and "artful" must have similar meanings (confounding etymology with connotation). J. S. Mill gives as an instance "the popular error that *strong* drink must be a cause of *strength*"—which, if it were true, would be equally, if not *eo ipso*, true of strong poison! But Mill has himself fallen a victim to the fallacy in a passage in his *Utilitarianism*,[1] which has since become classical in this connexion: "The only proof capable of being given that an object is visible is that people actually see it. The only proof that a sound is audible is because people hear it: and so of the other sources of our experience. In like manner, I apprehend, the sole evidence it is possible to produce that anything is desirable, is that people do actually desire it." The force of this argument rests upon the assumption that the termination *-able* in "desirable" has the same sort of meaning as the termination *-ible* in "visible" and "audible"; but this assumption is false, for though "visible" and "audible" mean "what *can be*" seen and heard, "desirable" in the context means what *ought to be* desired, not merely "what *can be* desired"; and, therefore, since Mill's argument only proves that what people actually desire *can be* desired, while purporting to prove that what they do desire is desirable in the real sense of the word, *i.e. ought to be* desired, the argument is also an excellent example of the fallacy of *ignoratio elenchi* (275, *A*, *a*). This example, therefore, also illustrates the fact that the same individual argument may be referred to different types of fallacy. The *figura dictionis* itself may be regarded as a special sort of *false analogy* (275, *B*, *b*).

[1] p. 53.

274. FALLACIES INCIDENT TO JUDGMENT AND IMMEDIATE INFERENCE (Part II.).[1]

(*a*) THE SELF-CONTRADICTORY JUDGMENT.—Many of the fallacies incident to judgment have their source in vagueness of conception, so that the difference between them and those already enumerated is not fundamental. We may commence with the self-contradictory judgment. The mind cannot, of course, consciously accept what it sees to be a contradiction ; but here, as in the case of the self-contradictory concept, it accepts what is really contradictory just because it does not think clearly. Following the tendency to make general assertions where only particular assertions are justified, we may be betrayed into the self-contradictory statement that "every rule has exceptions". By laying down this as universally true we thereby claim that it has no exceptions: therefore some rules have no exceptions: hence our statement contradicts itself. Much ingenuity has been expended in showing that the old sophism of the *Liar* (ψευδόμενος) does not really disprove the universal applicability of the law of contradiction.[2] "Epimenides, the Cretan, says that all Cretans are always liars." On the hypothesis that this assertion of his is true, he too must be lying when he makes the assertion, *i.e.* the assertion itself must be false on the very hypothesis on which it is true. The statement from his mouth contradicts itself. The essential characteristic of an assertion or proposition is its claim to be true. If we assume that the proposition "All Cretans always lie" is objectively true, it is a contradiction to suppose at the same time that Epimenides can assert this proposition.

(*b*) AMPHIBOLY is the fallacy arising from ambiguity in the structure of a sentence. As equivocation is ambiguity in terms, so amphiboly is ambiguity in propositions. Latin is much exposed to this fallacy in the construction of the accusative with the infinitive: "Aio te Eacida, Romanos vincere posse" is the well-known reply of the oracle of Apollo to Pyrrhus; "τό βούλεσθαι λαβεῖν με τοὺς πολεμίους" is one of Aristotle's examples; the witch's prophecy in Shakespeare's *Henry VI.*, "The duke yet lives that Henry shall depose," is yet another example.

This form of ambiguity may easily arise in English from care-

[1] The fallacies incident to *mediate* inference (Part III.) have been sufficiently considered in connexion with the rules of the syllogism and the various laws and canons of hypothetical and disjunctive reasoning.

[2] *Cf.* KEYNES, *Formal Logic*, fourth edit., pp. 457 *sqq.*

lessness about the proper sequence of words in constructing the sentence. Here are a few instances: "How much are twice four and seven?" (It may be fifteen, or twenty-two.) "I accomplished my business and returned the day after." "Lost a valuable umbrella belonging to a gentleman with a curiously carved head." "Lord Salisbury will reply to Mr. Gladstone's recent speech at the Guildhall." "Wolsey left at his death many buildings which he had commenced in an unfinished state."

(*c*) The fallacies A DICTO SECUNDUM QUID AD DICTUM SIMPLICITER, and A DICTO SIMPLICITER AD DICTUM SECUNDUM QUID, have this in common, that they confound what is true absolutely with what is true only under certain restrictions and limitations. The former consists essentially in arguing from a statement which is true with certain limitations or qualifications, as if it were true absolutely, always, and apart from those qualifications. Aristotle illustrates it by examples which are apparent violations of the principle of contradiction, *e.g.* "arguing that an object which is partly white and partly black is both white and not white". This is confounding "white in a certain respect" (*secundum quid, πῇ*) with "white absolutely" (*simpliciter, ἁπλῶς*). Similarly, to argue that "we should never give alms, because giving alms to professional tramps promotes idleness," is to commit this fallacy. So, also, the argument that "because alcoholic drinks are pernicious they should be forbidden," would be regarded as an instance of the fallacy by those who hold the alleged premiss to be true only *secundum quid, i.e.* of immoderate quantities. Some instances of this fallacy might be classified under the head of *illicit generalization* (275, *B*, *c*): they are attempts to extend a statement beyond the special circumstances in which it is true.

A similar fallacy is committed by arguing from one special case to another special case, regardless of circumstances which invalidate the inference. It might be described as the fallacy *a dicto secundum unum aliquid ad dictum secundum aliquid aliud;* it is really a sort of *false analogy* (275, *B*, *b*). "He who takes life in sport is cruel; therefore he who eats flesh encourages cruelty." The story is told in the *Decameron* of the servant who brought to table a stork *minus* one of the legs. To his master's inquiry about the other leg he replied that storks have only one leg each. Master and servant settled the dispute by adjourning after dinner to a field where a number of storks were standing each on one leg. When the master shouted they put down each its other leg

and flew away. "But," replied the servant, "if you had shouted to the stork at dinner he would have shown his other leg too."

The fallacy *a dicto simpliciter ad dictum secundum quid* is generally regarded as the converse of the one just explained. It consists essentially in the assumption that what holds true *normally, as a general rule*, will be true *always*, and *without qualification*, of *any individual case or cases*, irrespective of special circumstances that may alter these cases. Here "simpliciter" means "generally speaking". The "moral" universal, admitting of exceptions, is misinterpreted as a strict, *necessary* universal, and applied to the exceptions. It is forgotten that "circumstances alter cases". Some special case is wrongly regarded as an instance of a principle, when the case in question is not really an instance, owing to the presence in it of special conditions not contemplated by the principle. It is the illicit application of a general rule to a special case which does not fall under the rule. The fallacy, therefore, occurs in the process which is the special function of the syllogism: the application of general principles to particular cases. And it can be committed by misinterpreting either the scope of the principle or the nature of the case. "The employment of labour is beneficial to the community, therefore it will benefit the community to find some occupation or other, no matter how useless, for unemployed workmen." "Water boils at 100° centigrade; therefore it will cook an egg in a few minutes at the top of Mont Blanc." "It is unjust to interfere with a man's private property; therefore State interference with land tenure, by compulsory sale or otherwise, is unjust." "Thou shalt not kill; therefore war is never lawful; or the killing of animals for human food." "What you bought yesterday you ate to-day; you bought raw meat yesterday; therefore you ate raw meat to-day." "This piece of raw meat," remarks De Morgan in his *Logic*,[1] "has remained uncooked, as fresh as ever, a prodigious time. It was raw when Reisch mentioned it in the *Margarita Philosophica* in 1496; and Dr. Whately found it in just the same state in 1826." What is predicated about the subject in the major premiss is true of that subject "*simpliciter*," but not of that subject "*secundum quid*," *i.e.* in its state of rawness.

The fallacy *a dicto simpliciter ad dictum secundum quid* is commonly identified nowadays with the fallacy of the *accident*, and the fallacy *a dicto secundum quid ad dictum simpliciter* is

[1] p. 251.

described as the "*converse fallacy of the accident*". The examples, too, by which the fallacy of the *accident* is usually illustrated, fail to distinguish the latter from the fallacy we have just been considering.[1]

(*d*) ACCIDENS. The precise nature of this fallacy—"παρὰ τὸ συνβεβηκός," "*per accidens*"—has not been clearly determined. It appears to be the mistake of assuming that whatever is predicable of a subject is also predicable of its "accidents," *i.e.* of attributes that are not commensurate with that subject; or, conversely, that the "accidents" of a given predicate are also, and equally with the latter, predicable of its subject. Here are some of the examples and solutions offered by Aristotle. "'Do you know Corsicus?' 'Yes.' 'Do you know the man approaching you with his face muffled?' 'No.' 'But he is Corsicus, and you said you knew him.'" To be "a man approaching with his face muffled," is an accident of Corsicus; and it does not follow that because Corsicus is known, this accidental state of him is known. "Six is a few; and thirty-six is six times six; therefore thirty-six is a few." But it is accidental to thirty-six to be regarded as a few *groups*; hence though "fewness" may be predicated about an accidental condition of thirty-six it cannot be predicated of thirty-six itself. "To call you an animal is to speak the truth; to call you an ass is to call you an animal; therefore to call you an ass is to speak the truth." The fallacy here lies in the minor premiss, in the assumption that if "animal" can be predicated about a given subject, "ass," which is an *accident* of this predicate, can likewise be predicated of it. (The *species*-notion is always an *accidens* of the *genus*-notion: the *fundamentum divisionis* must be an *accidens* of the *genus*: *some* animals are asses, but an animal need not necessarily be an ass). Sometimes, of course, a subject (or predicate) and one of its accidents may be *de facto* commensurate. In such cases the fallacy does not occur: we know *from the subject-matter* that the subject (or predicate) may be validly replaced by its commensurate accident. For instance, although the fallacy is committed in arguing that all plane rectilinear figures have the sums of their interior angles equal to two right angles, because this latter is true of all plane triangles; yet the fallacy is avoided in arguing that because all right-angled triangles have the square on the hypotenuse equal

[1] *Cf.* JOSEPH, *op. cit.*, p. 547; WELTON, *op. cit.*, ii., p. 256; *Palaestra Logica*, p. 86.

to the sum of the squares on the remaining sides, the same is true of all triangles inscribable in semi-circles: for in this case we know that the *accidens* "inscribable in a semi-circle" is commensurate with the subject "right-angled triangle"; that what we can predicate of the former we may predicate of the latter; that the proposition, "right-angled triangles are inscribable in semi-circles," being a *reciprocal* proposition, is simply convertible. Wherever the fallacy occurs it will be found to lie in the assumption that some proposition is *simply* convertible, which is really convertible only *per accidens*. Hence the name of the fallacy. Take the inference: "This dog is yours; this dog is a terrier; therefore this terrier is yours";—evidently a valid inference. But is there not illicit process of the minor term? Apparently there is; and apparently the conclusion should be "*a* terrier (or *some* terrier) is yours"; but really the minor premiss contains the information "this dog is *this* terrier"—which justifies the definite conclusion.

We have seen that the fallacy *a dicto simpliciter ad dictum secundum quid* is committed when we attach the predicate of a genus to some subject which is not really contained in, or subordinate to, that genus. We now see that the fallacy of the *accident* is committed by unlawfully equating a genus and its subordinate notions (species or individuals), either as subjects of the same predicate ("All men are rational; all angels are rational; therefore all men are angels"); or as predicates of the same subject ("All men are rational; all men are bipeds; therefore, all bipeds are rational").

(*e*) CONSEQUENS. The fallacy of the *consequent* is the mistake of inferring the truth of the antecedent from the truth of the consequent, or the falsity of the consequent from the falsity of the antecedent, of a hypothetical proposition.[1] It is therefore illicit conversion, or contraposition, or inversion, based on the erroneous supposition that the hypothetical judgment is always reciprocal. "If a religion can elevate the soul it will survive persecution; therefore if a religion survives persecution it must elevate the soul; and if it does not elevate the soul it will not survive persecution." "This man has no visible means of support; therefore he is a professional thief." There is an ob-

[1] Another mistake is that of interpreting the antecedent of a hypothetical as necessarily giving a *causa essendi* of the consequent: it need only give a *ratio cognoscendi*, a symptom, or effect, of the latter.

vious analogy between the simple conversion of the categorical "*A*" proposition in the fallacy of the *accident*, and the attempt to argue from consequent to antecedent in the hypothetical proposition. The establishment of laws of nature in the form of reciprocal hypotheticals is an ideal at which science aims; and when we know from the subject-matter that a given hypothetical is reciprocal we can derive from it the inferences just mentioned; but, apart from such knowledge, the mere form of the hypothetical does not guarantee them. The fallacy is committed in inductive research when an hypothesis is regarded as proved by the mere fact that it explains the very phenomenon to account for which it was invented: it falls into the form "If *A* then *C*; but *C*; therefore *A*": an inference which is formally invalid, inasmuch as it ignores the possibility of a plurality of antecedents or causes.

Two very common and dangerous forms of the fallacy of the *consequent* are the assumptions (*a*) that we refute or disprove a thesis by showing that the arguments alleged in support of it are unsound (whether by reason of false premisses or of formal invalidity); (*b*) that the arguments in support of a thesis are necessarily sound (both materially and formally) because the thesis itself is true (148). In regard to (*a*), we must remember that the only way to *disprove* or refute a thesis is by *positively proving* its contradictory. Suppose that two premisses, *A* and *B*, are advanced in proof of a conclusion, *C*. He who advances the argument asserts two things (1) "If *A* and *B*, then *C*" (the formal validity of his syllogism), and (2) "*A* and *B* are both true" (its material validity). Now, evidently, if we merely show that his argument is formally invalid ("If *A* and *B*, not necessarily *C*"), or that his premisses are not true ("Not both *A* and *B*"), we do not thereby establish the proposition "Not *C*": except, indeed, the combination of (1) and (2) are known to furnish *the only possible ground* of *C*. Similarly, in regard to (*b*), we have to bear in mind that a true conclusion is often defended by false or inconclusive reasons—a good cause is often supported (and injured) by bad arguments. A true conclusion does not guarantee the formal validity of any argument that may be alleged as proving it; nor, when the argument is *formally* valid, does the true conclusion guarantee the *truth* of the alleged premisses: *unless these be the only ones* from which the conclusion in question can follow; and this latter is not guaranteed in any particular case by the mere formal validity of the argument.

(*f*) Plures Interrogationes. The fallacy of *many questions* consists in so putting a question that a single answer will involve more than one admission. In its simplest form it combines many questions into one, and insists on a categorical "Yes" or "No" for answer: "Is he a socialist and a conspirator? Yes or No?" More frequently, the question is single, but is based upon a certain admission which it wrongly assumes as already made: "Have you given up your intemperate habits?" The traditional example, "Have you cast your horns?" gives the fallacy the name of *cornutus*. Such traps must be met by a distinction between the various parts of the question ("*Distinguo*"), or by a denial of the assumed admission ("*Nego suppositum*"). The fallacy is rather frequently committed by giving reasons and arguments in explanation of some *supposed* fact which is not really a fact at all. It is open to question whether, for example, it is not committed by those who endeavour to show *how* protective tariffs encourage the industries of a country; or *how* communication is effected with the souls of the dead; or *how* dowsers detect subterranean springs.

The policy of "tacking," in the American Legislature, is a sort of practical application of this fallacy. "The President of the United States can veto bills, and does veto them freely; but he can only veto a bill as a whole. It is therefore not uncommon for the Legislature to tack on to a bill which the President feels bound to let pass a clause containing a measure to which it is known that he objects; so that if he assents, he allows what he disapproves of, and if he dissents, he disallows what he approves." [1]

On analysis, this fallacy will be found to involve *misinterpretation of an alternative or disjunctive judgment.* The alternatives enumerated or suggested are tacitly and erroneously assumed to be exhaustive, when they are really not so. The alternatives that a person "either has or has not" given up his intemperate habits, is *apparently* exhaustive, but is not *really* so, for the *desitive* proposition is not simple, but compound (95). "Is he a socialist and a conspirator? Yes or No?" implies that the alternatives "He is both or he is neither" are exhaustive. We have, in other words, a wrong application of the *principle of excluded middle.* The disjunctive premiss of a dilemma should enumerate all the alternatives permitted by the subject-matter; and failure to secure

[1] Joseph, *op. cit.*, p. 557.

this complete enumeration is the most common source of the inconclusive dilemma (185).

A cognate fallacy is that of supposing that the alternatives enumerated in the alternative judgment are always *mutually exclusive* (145).

275. FALLACIES INCIDENT TO METHOD (Part IV.).—We may perhaps conveniently divide these into two main classes: (*A*) *Fallacies incident to deduction or proof;* and (*B*) *Fallacies incident to induction or discovery.*

(*A*) Fallacies incident to deduction or proof:—

(*a*) IGNORATIO ELENCHI is the fallacy of proving the wrong conclusion. By an ἐλέγχος Aristotle meant an argument by which a disputant established the contradictory of his opponent's conclusion, thereby refuting the latter; and the disputant committed this fallacy if he proved anything other than the exact contradictory of his opponent's thesis—if, in other words, he mistook the proposition he had to establish. The fallacy is nowadays understood to include all cases of "arguing beside the point," "proving the wrong conclusion," "missing the point at issue"—whether deliberately and with intent to deceive, or not. The argument used in such cases may be perfectly valid; but it is not to the point, and herein lies the fallacy. To argue that a particular branch of study—for example, the study of the Irish language—should not be included in the curriculum of our schools, on the plea that it will never earn "bread and butter" for nine-tenths of those who study it, would be a typical instance of the fallacy. Even though the study of Irish might be useless as a means of earning a livelihood, it might be highly desirable on other grounds. Again, to point out a disadvantage or difficulty against some practical proposal—some social reform, for instance—is not to prove it impracticable or undesirable: for this it would be necessary to prove that the difficulties or disadvantages against it *outweigh* the advantages of carrying the proposal into effect. Or, again, to show up the weakness of an adversary's arguments is not to prove his contention unsound: such an assumption, involving the fallacy of the *consequent*, would be also *ignoratio elenchi.*

This fallacy, of confusing the point at issue in some way or other, is of most frequent occurrence in every domain of argumentation. Pleadings at law, political debates in parliament or on the platform, newspaper controversies on questions of public

interest, discussions in books and periodicals, whether on theology, philosophy, science, art, literature, etc.—furnish an unfailing supply of examples. It is a favourite device with those who have to support a weak cause. The attorney for a defendant is said to have handed the barrister his brief marked "No case; abuse the plaintiff's attorney". Discussions on topics of great and urgent practical importance—religious, ethical, social, political, educational, administrative, etc.—naturally stir up deep and strong personal feeling; and hence they tend to stray from a calm, impartial consideration of the merits of the question, and to confuse the issues by irrelevant personalities and recriminations. Such displays offend not merely against the requirements of courtesy and good taste, but also against the canons of logic, as being instances of fallacious reasoning—of *ignoratio elenchi*. They will be duly discounted by those who can recognize them for what they really are: substitutes for real argument, betrayals of weakness or defeat. The writer or speaker who is clearly conscious of holding a well-reasoned position can afford to be calm, courteous, patient, provided he is addressing intelligent people; but if he wants to carry the crowd against the demagogue, he cannot afford to despise the power of rhetoric, or to dispense with the art of oratory.

There are many minor forms of the fallacy. The *argumentum ad baculum* is an appeal to physical force. The *argumentum ad populum*, or "appeal to the gallery," for the purpose of exciting the feelings, or arousing the passions, of the crowd, is the favourite device of the mob-orator. The *argumentum ad ignorantiam* is the fallacious reasoning that is made to pass muster owing to the ignorance of those to whom it is addressed. The *argumentum ad verecundiam* is an appeal to the people's veneration for authority, in matters that should be decided by reason, and on their own merits. The appeal *ad misericordiam* is any argument to show that a person deserves *pity*, when proof of his *innocence* is demanded. Socrates refused to have recourse to it, though urged by his friends to do so, when condemned to death by his judges.

The *argumentum ad hominem*, or "*tu quoque*" style of argument, is a fairly common form of the *ignoratio elenchi*; it includes such practices as personal abuse, recrimination, charges of inconsistency, etc. If the personal character of one party is relevant to the trustworthiness of his allegations, it will not, of course, be *ignoratio elenchi* on the part of the other party to impeach the former's veracity. In cross-examination, a counsel can lawfully shake the

credibility of a hostile witness by showing that the latter has a criminal record, and that his evidence is unreliable. Sometimes, too, we may be quite satisfied to show that, whatever about our own position, that of our adversary at all events is unsustainable, being *inconsistent with the latter's own principles or admissions.* This is refuting him "out of his own mouth," from his own admissions in theory or in practice, and without committing *ourselves* to the truth of such admissions (254, *b*). Thus, Christ silenced those who blamed Him for curing on the sabbath by asking them which of them, if his ox or his ass had fallen into a ditch, would not pull it out on the sabbath.

(*b*) PETITIO PRINCIPII, or "*begging the question,*" is the fallacy of assuming as a premiss, in some form or other, either the very proposition to be proved, or a proposition which can be proved only by means of the latter. It is, therefore, a fallacy incident to *demonstration, proof, scientific explanation.* The student will be familiar with it already, from what has been said on the *Nature of Inference* (195-6) and on the *Uniformity of Nature* (224); but it can be so hidden and harmful that it calls for special analysis. The title of the fallacy recalls the language of the dialectical disputation (205), in which the disputant *sought* his premisses, for the refutation of his adversary, among the latter's admissions. If he endeavoured to get his adversary to admit the very point in dispute, or used this as a premiss against the latter, or used some other proposition which he could establish only by means of such an admission, his refutation would be sophistical: he would have "begged the question"; he would have assumed as a premiss [1] a proposition which he could not legitimately assume for his purpose.

Aristotle distinguishes five ways in which the fallacy can occur: (1) by assuming the very proposition itself to be proved, usually under cover of synonyms; (2) by assuming, for the proof of a particular proposition, a universal principle *which cannot be itself established except through a knowledge of that particular*; (3)

[1] The universal propositions accepted as starting-points of disputation were called by the Scholastics *principia* (*cf.* WELTON, *op. cit.*, ii., p. 283). These were either self-evident axioms, or demonstrable and universally admitted truths. The fallacy of assuming as a *principle* something which is not such, is dealt with below as "*Undue Assumption of Axioms*". It is not quite the same as "begging the question," *i.e.* assuming either the conclusion to be proved, or some proposition which can be proved only by means of this conclusion: this would be better called *Petitio Quaesiti*, or *Petitio Quæstionis*; but the traditional name, *Petitio Principii*, is too well established to be disturbed by any alteration of usage.

by assuming a particular to prove the universal which involves it; (4) by assuming successively, in parts, the proposition to be proved; (5) by assuming, *without independent proof*, a proposition which is the reciprocal of the proposition to be proved. This *last* is too simple to need notice. It is arguing, for instance, that Cork is south of Dublin because Dublin is north of Cork. The *fourth* mode is merely a variety of the first. Aristotle instances the attempt to prove "that the art of healing is knowledge of what is wholesome and unwholesome" by assuming it successively to be a knowledge of each. The *third* mode is really the inductive fallacy of supposing that *enumerative induction* establishes a universal truth (207). We have, therefore, to consider the first two modes, which are the really important ones.

(1) The very proposition to be proved is rarely assumed as a premiss, except under cover of some circumlocution. As simpler examples we may take the following: "The House of Lords is out of date because an upper chamber in England is an anachronism". "The bill before the house is well calculated to elevate the character of education in the country, for the general standard of instruction in all the schools will be raised by it."

De Morgan, in his *Budget of Paradoxes* (p. 327), gives an interesting example, from an attempt at squaring the circle, by a Mr. James Smith, in a work entitled *Nut to Crack*. Smith attempted to prove that the ratio of circumference to diameter is $3\frac{1}{8}$, by *assuming* that it is so, and showing that every other ratio is, *on this hypothesis*, absurd: "I think you will not dare to dispute my right to this hypothesis, when I can prove by means of it that every other value of π will lead to the grossest absurdities; unless indeed you are disposed to dispute the right of Euclid to adopt a false line hypothetically, for the purpose of a *reductio ad absurdum* demonstration in pure geometry". He thus confounds his own fallacious procedure with Euclid's process of indirect proof. He argues that "if $3\frac{1}{8}$ be the right ratio, then all other ratios will be wrong; but they will be wrong, *on the hypothesis;* therefore the hypothesis is right"; whereas he should have shown, *independently of his hypothesis*, that they will be all wrong. He argues that "If A is true, B will be false; but B will be false *if A is true;* therefore A is true"—instead of arguing "If A is true, B will be false, but B is false, therefore (since B includes all suppositions other than A) A is true." In the *reductio ad absurdum* Euclid argues that "If A is false B will be true; but B is false; therefore A is true." "Euclid assumes what he wants to *disprove*, and shows that his *assumption* leads to absurdity, and so *upsets itself*. Mr. Smith assumes what he wants to prove, and shows that *his* assumption makes *other propositions* lead to absurdity. This is enough for all who can reason." (De Morgan, *ibid.*).

The example just given suggests a method of procedure which is an exceedingly seductive form of the fallacy, a method which apparently entraps honest reasoners themselves just as frequently as it is knowingly used by dishonest

reasoners with intent to deceive. Some theory is put forward as a thesis to be established—the extreme evolution hypothesis, for instance, including abiogenesis; or the theory of transformation of species by "natural selection"; or the view that all religious belief originated in a primitive feeling of fear which personified the forces of nature; or the doctrine that the known universe is purely mechanical, belief in free-will and in purpose or design in nature being an illusion of the mind; or the agnostic attitude that miracles, revelation, the supernatural, are all alike impossible. . . . The advocate describes and expounds his theory; interprets relevant facts in the light of it; gets his reader around gradually to look at these provisionally from his own point of view; shows as plausibly as possible how the facts may be seen to fit in with his theory, or to corroborate it; insists on reading the facts *through* the theory, on describing—and, so, colouring—these in terms of the theory; substitutes, as far as may be, for the facts themselves, the interpretations he has given them from the theory; then ventures to show how what he calls the facts (thus *coloured*, *interpreted*, *manipulated*, as they have been) are inconsistent with any other view, and will admit of no other explanation than his own; and so insinuates the conclusion that the theory advocated is the true one. The serious student of religious, philosophical, and scientific literature, who happens to have cultivated the faculty of thinking logically for himself, and of testing what he is asked to accept, will be amazed at the facility and frequency with which writers deceive themselves, or their readers, or both, with pages, or chapters, or even volumes, of such solemn question-begging argumentation. It is not that such writers naïvely believe, or make believe, that an hypothesis must be true merely because it can give a plausible explanation of the facts. Rather they try to persuade themselves and their readers to look at the facts only through the coloured glass of the hypothesis, and, by such means, to believe that there is no other way in which the facts can be intelligibly apprehended or explained. The *petitio principii* is committed by gratuitously interpreting all the facts only in the light of the preconceived theory. The whole process may also be regarded as an illustration of the fallacy of the *consequent*: arguing from the truth of the consequent to the truth of the antecedent.

When the fallacy of *petitio principii* is committed in a single step of inference, it is called the *hysteron proteron* (ὕστερον πρότερον). It is usually concealed by the use of synonyms. Sometimes the judgment, expressed in abstract terms, is given as a reason for itself in concrete terms: "Opium induces sleep because it has a soporific quality". Sometimes the fallacy is committed by a single "question-begging epithet"—generally either laudatory or condemnatory: as when, to prove that some new measure ought to be resisted, we call it an *innovation*, or contend that it would call into question an *irrevocable* law. The *hysteron proteron* may be expressed symbolically by the syllogism, "M is P; S is M; $\therefore S$ is P," where M is a synonym for S, or for P: the syllogism would then read "S is P; S is S; $\therefore$ S is P," or "P is P; S is P; $\therefore S$ is P"; thus showing one premiss as a tautology and the other as

identical with the conclusion. "Whatever has a soporific quality induces sleep" is a tautology; "opium has a soporific quality" is the same as the conclusion to be proved: "Opium produces sleep".

When the conclusion is separated by more than a single step of inference from the assumption, the fallacy is called the *circulus in demonstrando*, *circulus vitiosus*, or *arguing in a circle*. Thus, if we were to prove the immortality of the soul from its simplicity (as Plato does in the *Phaedo*), and then prove its simplicity from its immortality (as he does in the *Republic*), we should be arguing in a circle. It may be expressed thus: "M is P, S is M, $\therefore$ S is P; and M is P because S is P and M is S."

(2) The second, and perhaps more common, form of *petitio principii*, consists not in assuming the very conclusion itself to be proved, but in assuming, *without independent proof*, some wider principle which involves the latter, and which *could not have been proved or established otherwise than through a prior knowledge of the latter.* To this head we may refer all cases wherein the assumed proposition, whether wider than the conclusion or not, is such that it could not have been known or established otherwise than through a prior assumption of the truth of the conclusion.

We have already dealt with Mill's contention that *all* syllogism commits this form of *petitio principii* (195-6). We saw there, however, that the fallacy is really committed only when a premiss is assumed which could not be established otherwise than through a knowledge of the conclusion; that this is usually the case when the premiss in question is a mere *collective* proposition; but that *genuine universal* premisses can be known with certitude independently of any knowledge of instances to which they are applicable, and otherwise than by enumeration of these instances.

A few examples will suffice here to illustrate this form of the fallacy. Galileo accuses Aristotle of having committed it in the following argument: "The nature of heavy things is to tend towards the centre *of the universe*, and of light things to fly from it; experience proves that heavy things tend towards the centre *of the earth*, and that light things fly from it; therefore the centre of the earth is the centre of the universe". Spencer, in his work on *Education*, after distinguishing two values in any branch of study—value for knowledge imparted, and value as a

mental training,—proceeded to argue that the branches he had shown to have the first value must have the second value likewise: on the ground that it "would be utterly contrary to the beautiful economy of nature, if one kind of culture were needed for the gaining of information and another kind were needed as a mental gymnastic".[1] This is simply assuming as universally true what he wanted to prove of the cases he examined. Here are a few briefer instances: "The imposition of legacy duties is justifiable, because all property passing by will ought to be taxed"; "His cruelty may be inferred from his cowardice, for all cowards are cruel"; "A table of logarithms must be entertaining, for all books are so".

If our assent to the principle of the *Uniformity of Nature* were based upon simple enumerative induction, and if at the same time *all* induction depended for its validity on a prior belief in the *Uniformity of Nature*, we should never be able rationally to justify this belief, or, consequently, to put our trust in any single inductive generalization (apart altogether from this other damaging fact, that *enumerative* induction can never beget *scientific* certitude). Yet Mill contends that the *petitio principii*, apparently involved in this attitude, is only apparent, not real. Unfortunately, it is real. And when Mill attempts to show that the position is free from the fallacy, so far from succeeding, the attempt only involves him in the fallacy afresh. His line of argument has been examined already (224), but the matter is of such importance that a reference to it in the present context will not be superfluous.

According to Mill, the principle of the *Uniformity of Nature* is the "ultimate major premise of all induction"; communicating its reliability to all inductions; without which none would be valid; which, therefore, we must hold for certain, antecedently to all inductions that are scientific, that give certitude. How, then, do we come to give a certain assent to this principle? on what grounds? by what process? We reach it, Mill answers, through a vast induction *per enumerationem simplicem*, by which we accumulate and generalize "many laws of inferior generality," each of which was reached by a like process of generalizing from enumerated instances [2]. But how can we make an induction *per enumerationem simplicem*, *before* we are sure of the principle, if *all* induction *presupposes* certitude about the principle? And, anyhow, is not *enumerative* induction so admittedly weak that of itself it can never carry us beyond an *empirical generalization*, to the certitude we need for the principle of uniformity as a basis for *scientifically certain* physical laws? Mill replies that most *enumerative inductions* do presuppose the principle established; that, therefore, this process does not and cannot *in ordinary cases* establish a law; but he attempts to show that the enumerative induction by which we reach the principle in question is different from all other lesser enumerative inductions, and *does* give us certitude about the principle. And here is how he proceeds: The *wider* the field of experience over which we generalize by enumerative induction, the *safer* this process becomes (a state-

[1] *apud* WELTON, *op. cit.*, ii., p. 284.

[2] *Cf.* WELTON, *op. cit.*, ii., pp. 42, 43.

ment which is true *only on the assumption that natural causes act uniformly*): so that when the domain of generalization embraces *all* experience, as it does in the case of this principle, enumerative induction can beget certitude, and "the distinction between empirical laws and laws of nature vanishes" (*Logic*, III., xxi., § 3). But how does he prove, without assuming uniformity in nature, the assertion that *the safeness of the generalization grows with the extent of its domain?* His only attempt at proof is the observation that the wider the field of experience on which a generalization is based the more likely we are to meet with adverse instances, if any such occurred. *But this already supposes belief in uniformity*, for why should we attempt at all to generalize beyond experience did we not believe in uniformity? Granted this belief, an adverse instance would convince us that our supposed causal connexion *was not really causal*, because not uniform; but without such belief an adverse instance would be really devoid of all significance for us. *Granted* the uniformity of nature, the wider our *actual* uncontradicted experience of a given sequence, the more likely it is to be a *causal* sequence; but if we have yet to prove that natural causes must act regularly, not capriciously and chaotically, how can any such experience of itself guarantee any generalization, even a single step beyond itself? Or what can be the use of comparing all *actual* experience in time and space, no matter how extensive, with *all possible* experience, in the hope of concluding directly from the former to the latter? Yet this is what Mill does: "If we suppose, then, the subject-matter of any generalisation to be so widely diffused that there is no time, no place, and no combination of circumstances [within or beyond actual experience] but must afford an example either of its truth or of its falsity, and if it be never found [within actual experience] otherwise than true, its truth cannot be contingent on any collocations, unless such as exist at all times and places [even outside actual experience]" (*ibid.*) In other words, the domain of experience to which the principle refers is so wide, being all *possible* experience, that *every* time and place must afford an instance *either of its truth or of its falsity;* but it *has been found to be true* at all times and in all places *within actual* experience; therefore it is true of all times and places *beyond* our experience! The premisses give no right to any such conclusion.

(*c*) UNDUE ASSUMPTION OF AXIOMS. All *proof* presupposes, and proceeds ultimately from, self-evident truths, called *principles* or *axioms:* some of which are "common" principles, of universal application, while others are "proper" to the subject-matter of this or that special science (252). Now axioms are indemonstrable; their evidence is immediate, *i.e.* embodied in themselves; so that their truth must be apprehended by an act of intellectual intuition or vision. The theory of all this is easy enough. Nothing could be more reasonable, or more necessary, than the demand of logic, that truths which are really axioms ought to be admitted by all to be such, and that, conversely, no judgment which is not really an axiom should be accepted or allowed to pass current as such. The violation of this latter demand involves

the fallacy under consideration; but the violation of the former demand involves a corresponding fallacy, which might be called *Undue Rejection of Axioms.* The same considerations apply to both mistakes; and when we set them down as logical fallacies we assume, of course, that the intellect, the faculty which apprehends truth and estimates evidence, is essentially similar in all normal human beings; that it is similarly affected in all by the same kind and degree of evidence; that it is, therefore, similarly impelled in all to assent to really self-evident truths. In a word, we assume that a knowable reality forms the object of human science, and that all normally constituted minds behave in the same way towards this reality. This assumption, itself, is not, perhaps, an axiomatic truth, but is rather one of those postulates or assumptions which are indispensable to all research, and which are justified only by actual human experience. The only "proof" of the contention that man *can* discover some truths with certitude, lies in the fact that he *has* discovered some; and to doubt man's capacity in this regard would be to paralyse the mind, and so destroy completely the path to any truth whatsoever.

But, granting that man can discover truth, and, consequently, that some truths are really self-evident to all normal minds, the question as to which truths are really axioms, and which are not, is a grave and momentous question. While no one has ever seriously doubted the self-evident character of the fundamental laws of thought, and of certain abstract principles of mathematics and metaphysics, there has always existed an abundance of controversy as to the *ultimate significance* of such *abstract truths of the ideal order*, when applied to the *concrete data of sense experience* for the purpose of obtaining a rational interpretation of the universe in which we live (224, 229-32).

The settlement of these controversies, the elimination of errors as to which judgments are self-evident axioms, which are only postulates justifiable by experience, which are even unjustifiable, erroneous, or misleading, assumptions —all this belongs to epistemology, and not to logic; though, in our treatment of *Induction*, *Demonstration*, and *Scientific Explanation*, we have been afforded opportunities of glancing at some of the conflicting philosophies in which the discussion of these problems has issued. That view of the universe, which is embodied in materialism, materialistic monism, phenomenism, sensism, positivism, agnosticism, apparently accepts, as an indisputable truth or axiom, the false and unjustifiable assumption that whatever transcends the scope and range of our *sense faculties* is unreal, worthless, and inadmissible.

In denying the existence, or at least the cognoscibility, of suprasensible reality in any mode or form, it is making an unjustifiable application of the maxim: *Entia non sunt multiplicanda praeter necessitatem* (231). At the opposite extreme lies the philosophy of idealist or spiritualist monism, which regards even the material universe as a mere manifestation or expression of the thought-activity of One Immanent Spirit or Mind: a view which appears to accept as axiomatic this other false and misleading assumption, that whatever is real is intelligible in terms of *abstract* thought, and that whatever cannot be totally included in this ideal domain is illusory and unreal.

Prepossessions in favour of certain broad, general views or theories about things, dispose us to exaggerate the evidence for these views, or to set down as evidence what is really not evidence at all. And, by dint of refusing to see things otherwise than in the light of these theories, we may gradually persuade ourselves that the latter are self-evident, axiomatic. Thus it is that questionable postulates are wrongly allowed to assume the rank of axioms in the minds of those who entertain them. It is not so much because of formal fallacies in our conscious reasoning processes that profound philosophical errors are so prevalent. These are due rather to an unquestioning and uncritical acceptance of doctrines, beliefs, and opinions, which happen to appear plausible to us on account of our own individual mental development, and of the special intellectual atmosphere in which we have been trained from our earliest days: *phantoms of the cave*, *of the theatre*, and *of the market place*.

For the human mind, the domain of really self-evident axioms is very limited; and they are all abstract. But there is a larger domain of what may be called concrete truths of "common sense," in reference to which all individual minds are not equally receptive. Education; intellectual, moral, and religious training; mental companionship with books and teachers; character, habit, and disposition; likes and dislikes; passions and prejudices;—all these are agencies of enormous influence in moulding the individual mind for the right or wrong discernment of evidence: for the reception or rejection of important truths of the *concrete* order, truths that have a direct bearing on life and conduct, and which, though not strictly axiomatic in the sense of mathematical principles, nor on the other hand capable of strict and cogent demonstration, are nevertheless such that the normal mind, unimpeded by any one-sided bias, will unhesitatingly assent to them, and will act with entire reasonableness in doing so. Against these sources of deviation from the healthy, normal state of mind, logic has no

infallible safeguard to offer. It can merely emphasize the importance of these influences, point them out to us for our consideration, and call our attention to the undeniable fact that though the faculty for discerning truth is of the same nature in all men, nevertheless different men are so differently disposed in mental habits and equipment that what appears self-evident to one may well appear not only inevident, but utterly untrue, to another.

While it would be wrong to say that the "normal mind" is itself an abstraction, never to be met in real life, it must be admitted that there is great variety in the attitudes of different minds, face to face with the same real universe, and that in our actual experience of life we meet many strange mixtures of scepticism and credulity. But from the historical fact that philosophers in every age have propounded conflicting and irreconcilable solutions of the most momentous problems concerning man and the universe, it would be a mistake to conclude—as some have concluded—that truth on these grave matters is unattainable, and that therefore the inquiring human mind is fated to move, however reluctantly, towards the dark, final bourne of doubt and agnosticism. For we have a right to conclude merely that the human mind is both finite and fallible, that there is no royal road to knowledge, that truth must be *sought after*, that the more precious the truth, the more diligent must be the search, that it is only by a cautious, painstaking, persevering application of the mind, deception and error can be avoided. The historical study of the workings of human thought upon philosophical problems, its gropings and findings, its gains and losses, its advances and aberrations, cannot fail to convince the unprejudiced student that man *is* capable of attaining to sufficient truth about his own origin, nature, and destiny, and his proper place in the universe, to guide his life aright, if he only has the will to do so. It will, no doubt, convince him at the same time that the work of discovering truth, and *living up to it*, is noble and elevating, if difficult and sometimes even arduous. He will be surprised at first to discover that so many great minds have greatly erred in regard to the fundamental truths of human life. But, according as he realizes the complexity of the problems they had to face, the conflicting evidences they had to weigh, the traditional beliefs or disbeliefs in which they were trained, and all the *objective* sources of error that surrounded them, his surprise will gradually diminish. And it will be likely to disappear altogether if he fixes his attention on the *subjective* sources of error by which even great minds may be disturbed and led astray. These *phantoms of the tribe* need only to be mentioned, to make us realize something of their dangerous influence.

On the part of the intellect, *sloth* is a source of much error: it is the cause of undue *haste* in the search for truth. Doubt is an irksome state of mind; suspense is unpleasant; while assent brings rest. Assent is the goal to which inquiry is the path. But the path is through a land of pitfalls and will-o'-the-wisps; and the passage to a *right* assent is often laborious. Hence the temptation to move along hastily and without due care, to stifle misgivings, and to cut short the search by resting in assent to some position the truth of which is not really guaranteed by the evidences our inquiry may have brought to light.

This is a violation of that canon of method which counsels us to proceed *step by step* in the discovery of truth (203).[1]

Another obstacle to the attainment of truth, one that is rooted in the will rather than the intellect, is the prejudice created by the habit of a long-standing and cherished belief. "Prout unusquisque affectus est ita judicat," says the author of the *Imitation of Christ:* the wish is father to the thought. There lurks no small danger to the cause of truth in the fear of having our habitual mental attitude in any way disturbed, of being unceremoniously robbed of what we have always complacently accepted for the truth. It is decidedly unpleasant to have cherished beliefs rudely exploded. Habit is tyrannical, as St. Thomas well observes;[2] and requires not a little courage to break with it. Hence our eagerness to accept whatever falls in with the habits we have formed, and to give it an unhesitating welcome. "We like to hear people talk of all things in the way we have been accustomed to think of them and to hear them talked about."[3]

Yet another source of failure is the *absence of a disinterested love for the truth.* A well-known French psychologist, M. Henri Joly, has some suggestive remarks on this subject: "Very often," he writes, "*we do not find the truth because we do not seek it.* . . . For we do not seek the truth when we give to the investigation of facts and questions a mere superficial, half-hearted attention; when pride prompts us to imagine that by a simple glance we can see well and see all; when we are too impatient for the gratification of an idle curiosity; when a hasty half-truth pleases us better than a full truth brought to light laboriously; when we stubbornly cling to an hypothesis for the sole reason that we have invented it; when we obstinately adhere to an opinion simply because we committed ourselves to it in the beginning and are unwilling now to acknowledge our error; when, finally, our estimate of things is influenced less by what they are in themselves than by the way they affect our interests, our passions, our sympathies, our prejudices, our likes and dislikes".

"But *why do we not seek the truth? Because we do not love it sufficiently.* I do not mean that we positively love its opposite, which is error; but that we are not ready to dare all, and to sacrifice all, for the sake of truth. In the field of science we pitch our *camps* and form our *parties;* we bring to all our discussions a *party spirit* if we are disciples, the spirit of *personal vanity* if we speak for ourselves. We prefer new and striking hypotheses to truths

[1] *Cf.* St. Thomas, *Summa Theol.*, ii. ii., Q. 53, *a.* 3.

[2] "Ea quae sunt consueta, libentius audiuntur et facilius recipiuntur. Dignum enim videtur nobis, ut ita dicatur de quocunque, sicut consuevimus audire. Et si quae dicantur nobis praeter ea quae consuevimus audire, non videntur similia in veritate his quae consuevimus audire. Sed videntur nobis minus nota et magis extranea a ratione propter hoc quod sunt inconsueta. Illud enim quod est consuetum, est nobis magis notum. Cujus ratio est, quia consuetudo vertitur in naturam; unde et habitus ex consuetudine generatur, qui inclinat per modum naturae. Ex hoc autem quod aliquis habet talem naturam vel talem habitum, habet proportionem determinatam ad hoc vel illud. Requiritur autem ad quamlibet cognitionem determinata proportio cognoscentis ad cognoscibile. Et ideo sec undum diversitatem naturarum et habituum accidet diversitas circa cognitionem. . . . Sic igitur quia consuetudo causat habitum consimilem naturae, contingit quod ea quae sunt consueta sint notiora."—St. Thomas, *In II. Met.*, Lect. 5.

[3] *ibid.*

already old. Above all, we want to make a name ; and so our zeal for the truth is gradually replaced by anxiety to embrace up-to-date opinions, or to attract attention by the fearlessness of our views and the brilliancy of our writings. We criticize our adversaries and are glad if we can make them contradict themselves ; we controvert their arguments and build up elaborate demonstrations of our own : and we take greater delight in all this than in the discovery and possession of the truth. But in all this we are showing a greater love for our own views, for our own selves and interests, than for truth : and, as St. Augustine well says, he that does not love the truth will not find it : '*Sapientia et veritas nisi totis animi viribus concupiscatur, nullo modo inveniri poterit*'."[1]

He who has a disinterested love for truth will prize it more highly than originality. "A great man," says M. Emile Boutroux,[2] "will not aim at being novel or original, but at finding out the truth." He will discuss, but in order to prepare the way for the truth, not to assert his own superiority : according to the admirable sentence of St. Ignatius, *Rationes modeste afferantur eo animo ut suus veritati sit locus, non ut in ea re superiores inveniantur.*

When such a one sets himself to examine a system of philosophy he will try to get at the author's point of view, and to enter into the latter's thoughts ; he will not engage in a mere search for weak points, with the unworthy idea that the more he disparages others the more renown he will win for himself. Instead of revelling in an author's apparent inconsistencies, he will examine the latter's views with a ready impartiality, and try to reconcile them, remembering that every error contains a soul of truth of which it is a perversion or exaggeration. Such criticism will be invaluable to himself, as a test whereby to control his own personal convictions.

At the same time, there is the opposite extreme to be avoided : *impartiality is not indifference.* He is no lover of truth who admits indiscriminately all sorts of opinions, who affects to regard them from a superior height with a condescending sort of sceptical curiosity, as if they had all the same value for the individual and the same significance for society. Whether the indifference of the *dilettante* springs from pride or from sloth, it is a betrayal of truth and a crime against reason. Love of truth is hatred of error. We cannot embrace the former without condemning the latter. An easy toleration of all sorts of conflicting opinions in regard to religion and philosophy, is not unfrequently regarded in our own day as the hall-mark of enlightenment and broadmindedness, whereas it is in truth a mark of mental imbecility, or intellectual indolence.

(*d*) NON CAUSA PRO CAUSA, OR "FALSE CAUSE". By *causa* (*αἴτιον*) Aristotle here meant not a cause in the ordinary sense of *causa essendi*, but *a reason*, a *causa cognoscendi.* The fallacy consists in assigning as a reason for some conclusion a proposition which is really irrelevant to that conclusion. Aristotle contemplated especially cases in which this occurred in the

[1] H. JOLY, *Nouveau cours de Philosophie. Logique*, pp. 312-13. For instructive views on the same subject, *cf.* BALMES, *Art d'arriver au vrai*, chap. xxii ; OLLÉ LAPRUNE, *De la certitude morale ; La philosophie et le temps present ; Les sources de la paix intellectuelle.*

[2] *Études d'histoire et de la philosophie*, p. 8.

reductio ad impossibile, or *indirect proof* (169, 254, *b*). In this process we disprove a thesis by showing that the assumption of its truth would lead to absurdities; or we prove a thesis by showing that the assumption of its falsity would lead to absurdities. Now, the fallacy under consideration is committed if the absurdities do in reality follow not from the assumption made, but from some extraneous and irrelevant proposition which we have foisted into our argument. The absurd conclusion is wrongly sought to be fathered upon the initial assumption. Hence Aristotle calls the fallacy also "*Non per hoc*," "*Non propter hoc*," which is the answer by which the fallacy ought to be met: "Your conclusion is indeed absurd (or impossible), not, however, because of your assumption about my thesis, but quite independently of it". The following instance is from Father Joyce's *Logic* (p. 281): "Thus, if we suppose the sophist's opponent to have affirmed that the death penalty for murder is just, the sophist might argue as follows: 'The position leads to an absurdity: for granting that the death penalty for murder is just, and that punishment is to be held just in so far as it is efficacious as a deterrent, then it follows that it would be equally just to inflict the death-penalty for pocket-picking'. Here the original statement has nothing to do with the conclusion obtained. This follows from the principle that the justice of a punishment is measured by its efficacy as a deterrent,—a principle which is in no way connected with the statement that the death penalty for murder is just."

Aristotle includes under the head of this fallacy all cases in which a conclusion is drawn from premisses which are quite irrelevant to it, and which, for want of any better classification we describe as cases of "*Non Sequitur*": "arguments so foolish and inconsequent, that they cannot even be said to simulate cogency; these cannot be positively characterized, but must be lumped together by the mere negative mark of inconclusiveness".[1]

As the fallacy of *non causa pro causa*, in Aristotle's sense, was peculiar to dialectical disputations, and is of comparatively rare occurrence nowadays, its name has been transferred pretty commonly to the *inductive* fallacy of mistaking for the cause of an event something that is not really the cause, the fallacy of *Post* (or *cum*) *hoc, ergo propter hoc* (*infra, B. c.*). This mistake, of confounding temporal sequence or coexistence of facts with causality,

[1] JOSEPH, *op. cit.*, p. 529.

is not quite the same as confounding a temporal sequence or co-existence of judgments in the mind with the logical consequence of conclusion from premisses. But little ambiguity can arise from giving the name of *false cause* to the inductive fallacy in question. And this mistake of confounding mere sequence or coexistence with causality or consequence is one of the many modes of the fallacy of *Illicit generalization* which will be examined presently.

(*B*) Fallacies incident to induction or discovery :[1]—

(*a*) In induction we pass from *observation* of particular facts, through *analogy*, and *hypothesis*, to the discovery and verification of *general laws*. Here, then, the first possible source of error will be IMPERFECT OBSERVATION, and the fallacy may be either negative or positive in character.

(1) NON-OBSERVATION is the fallacy of *overlooking something that ought to have been observed.* The function of observation is to *select* and *isolate* the facts from which we hope to bring to light some causal law (238). Hence we may either fail to notice *instances* pertinent to the kind of fact we are investigating, or fail to notice *influences* that are really operative in the instances actually observed.

Prejudice in favour of some preconceived theory is the most potent cause of neglect to observe pertinent instances. The process and product of our observation are profoundly influenced by the unconscious interference and intermixture of our previous knowledge and beliefs. "The opponents of Copernicus argued that the earth did not move, because if it did, a stone let fall from the top of a high tower would not reach the ground at the foot of the tower, but at a little distance from it, in a contrary direction to the earth's course; in the same manner (they said) as, if a ball is let fall from the mast-head when the ship is in full sail, it does not fall exactly at the foot of the mast, but nearer to the stern of the vessel. The Copernicans would have silenced these objectors at once if they had *tried* dropping a ball from the mast-head, since they would have found that it does fall exactly at the foot, as the theory requires: but no; they admitted the spurious fact and struggled vainly to make out a difference between the two cases".[2] The opponents of the new

[1] *Cf.* WELTON, *op. cit.*, ii., pp. 261-77, whose treatment is here closely followed.

[2] MILL, *Logic*, V., iv. § 3.

theory *neglected* to secure and examine an experimental *instance*, because they assumed that if examined it would corroborate the old one; the Copernicans were not sufficiently scientific to question this; while the inferences of both parties alike reveal to us how comparatively elementary was the scientific knowledge that prevailed in those days about the laws of motion.

It is important in the next place to note, and to guard against, the *natural tendency to neglect* NEGATIVE *instances*. Striking instances of a coexistence or sequence impress us, drawing off our attention from the cases in which the coexistence or sequence has not occurred; and so we are tempted to see a causal connexion in what may be merely casual. Remembering the few instances in which our dreams have been "fulfilled," and ignoring the far more numerous instances in which they have not, we are tempted to think that dreams are really prophetic. It has been already pointed out (238) that non-observation, whether of instances or of operative influences, does not prove their non-existence.

Non-observation of *operative influences* is one of the greatest dangers to induction. The accuracy of our hypotheses and generalizations depends upon the success with which we isolate our phenomena, eliminating *only* what is unessential, and taking into account *all* that is essential, to their occurrence. It was commonly believed in the seventeenth century that a wound could be healed by the application of a certain salve or powder *to the instrument* that caused the wound, the latter being kept clean and cool during the process of healing. The cure was attributed to the "sympathetic" influence of the salve, rather than to the unobserved recuperative forces of the patient's constitution when left to act in favourable conditions. As De Morgan well remarks,[1] "If we remember the dreadful notions upon drugs which prevailed, both as to quantity and quality, we shall readily see that any way of *not* dressing the wound would have been useful". This form of fallacy is particularly prevalent and difficult to avoid in exploring the causes of complex social, economic, political, and religious phenomena.

(2) MAL-OBSERVATION is the wrong interpretation of what falls immediately under sense perception. It is due to the misleading interference of unconscious inference arising from habitual beliefs, prejudices, and mental tendencies. Many people believe

[1] *Budget of Paradoxes*, p. 66; *apud* WELTON, *op. cit.*, ii., p. 264.

they have seen ghosts when they have really seen tombstones or stray donkeys. It requires some reflection to realize what a vast amount of inference is inseparably bound up in sense perception. The success of conjuring, ventriloquism, etc., is based on our partial incapacity to separate, from what we actually see or hear, our own misleading inferences, and our habitual associations of ideas. In all such cases of error, it is in the *interpretation* the mistake is committed, not in the *perception* itself: the sense-impression is always what it ought to be in the circumstances. So it is when, looking from a train which is stationary at another which is passing close by, we imagine that the latter is at rest and our own train in motion. So, too, we imagine that we see the sun moving around the earth, and the stars revolving around the pole, when it is really the diurnal revolution of the earth on its axis that causes those appearances. Our only safeguard against mal-observation is a lively advertence to its ever-present dangers, together with the fullest knowledge we can acquire on the subject-matter under investigation.

(*b*) FALSE ANALOGY.—We have seen that analogy is a fertile source of scientific hypothesis (234). An analogy or resemblance which is only apparent, not real, is called a *false analogy*. The points of similarity are not due to the operation of some common cause. Hence, an hypothesis based on such an analogy will be misleading, and must be abandoned; and this often happens in scientific research. Or, again, the observed analogy may indeed be real, but may have been assumed by the inquirer to be more, or less, extensive than it really is; he may misinterpret it and extend its scope unduly in one direction, or fail to apprehend its real application in another direction. An analogy whose scope and weight are thus wrongly estimated, may be called an *imperfect analogy*; the hypothesis based upon it will need to be remoulded before it can be verified, and so transformed into a law. This, too, frequently occurs in science: indeed it may be regarded as the usual procedure in inductive research. Non-observation of operative influences is the most frequent cause of imperfection in our analogies.

It is a common mistake to miss the point of an analogy, that is, to misinterpret its real significance, to base upon it an inference, conclusion, or hypothesis, other than that to which it really points. If the just man shows skill in regard to the possession of property, it does not follow (as the Platonic Socrates humor-

ously argues in the *Republic*) that because the thief also displays skill in this direction the just man must be a thief. The metaphorical use of language is an unfailing source of such sophisms. The relations of the *mother*-country to its colonies, of the *head*, or *heart*, of the body to the metropolis of a country, of the governor of a state to the pilot of a ship, of the political community or society to the individual *organism*—are all cases in point (234). Within due limits they may yield legitimate inferences; but they become fallacious if the analogy be pressed too far. In all cases, differences must be noted and weighed no less than resemblances. Finally, it must be remembered that mere analogy as such never amounts to *proof*.

(*c*) ILLICIT GENERALIZATION. The tendency to generalize from insufficient data, is perhaps the principal pitfall of the unscientific mind. The "man in the street" is fond of making "sweeping statements". The general assertion is always simple and brief. It brings a feeling of rest and satisfaction; and so, the plain man is inclined to take refuge in it—prematurely. But the failing is not peculiar to him. The many causes which account for undue haste in assenting to conclusions—impatience or indolence in the laborious work of inductive research, the habit of *a priori* reasoning, the influence of prejudice, conscious and unconscious, insufficient knowledge and equipment for accurate investigation, etc.—all these are constantly working mischief in every domain of science.

The great leaders of science are, indeed, seldom betrayed into going beyond their premisses. Hazardous, sensational prophecies in the name of science, have no attraction for their prudent and well-trained minds. But the smaller and narrower type of mind is itself misled, and misleads others. The work of *popularizing* science, *i.e. true* science, of spreading *truth* among the masses, is unquestionably a most praiseworthy work. But there are many half-educated camp-followers of science, who, actuated by less laudable motives, are constantly popularizing *travesties of science*; who misrepresent its conclusions; who palm off false and improbable hypotheses on unsuspecting people as established truths; who are attracted less by what is true than by what is sensational; who are influenced less by the scientific spirit of impartial inquiry than by their own likes and dislikes; to whom the laborious search after truth is a less congenial task than that of attacking whatever is opposed to their own preconceived

notions. In this dissemination of error, the fallacy of *illicit generalization* plays a large part. The wildest guesses, the merest speculations of the scientist, are proclaimed as established truths of science; and verified laws are extended beyond their rightful domain. This fallacious procedure has special reference to the formation and verification of scientific *hypotheses*, and to the establishment of scientific truths or laws. It may assume a variety of forms.

In the process of verifying an hypothesis, and so setting it up, and yielding our assent to it, as a general principle or law, we sometimes succumb to the temptation of ignoring *extreme cases*, which, if taken into account, would necessitate a modification and restatement of our general conclusion. "The application of the *extreme case* is very often the only test by which an ambiguous assumption [or generalization] can be dealt with. . . . Where anything is asserted which is true with exceptions, there is often great difficulty in forcing the assertor to attempt to lay down *a canon by which to distinguish the rule from the exception*." Yet everything depends upon this, "for the question will always be whether the example belongs to the rule or the exception". But "when one case is brought forward which is certainly an exception, the assertor will, in nine cases out of ten, refuse to see why it is brought forward. He will treat it as a fallacious argument against the rule, instead of admitting that *it is a good reason why he should define the method of distinguishing the exceptions*."[1] Such an attitude is, of course, in flagrant antagonism to the method by which hypotheses can be accurately moulded into scientific laws (234, 240). A scientific law should be so stated as to admit of no "exceptions" other than those for which the statement of it makes express provision: "A rule may have exceptions, it is said; but this is hardly a correct statement. A rule with exceptions is no rule, unless the exceptions *be definite and determinable:* in which case the exceptions are exclusions *by another rule*."[2]

The fallacy to which *empirical generalizations* (247) are most exposed, is that of wrongly interpreting their scope and import: of mistaking them for *established universal laws*, and so extending them to cases which they do not, or at least may not,

[1] De Morgan, *Formal Logic*, pp. 270-1; *apud* Welton, *op. cit.*, p. 271 (italics ours).

[2] *ibid.*, p. 272.

really cover. Mere *observation* of an uninterrupted uniformity can never of itself transform the latter into a law; only scientific inductive *analysis* can achieve this result. To generalize from mere observed uniformity of sequence is to confound *sequence* with *consequence*, and so to run the risk of setting down as *causal* a connexion which may be merely *casual:* the fallacy already referred to as *post hoc, ergo propter hoc.*

The conclusion of a *merely enumerative* induction can never be safely extended to instances that differ notably in their circumstances from those actually observed. In the social sciences, politics, and economics, most generalizations are only rough and empirical; it would therefore be a mistake to extend "such a generalization founded upon a survey of the social conditions of any one country at any particular time to other times and other peoples".[1] This might also be brought under the head of *false analogy.* The latter fallacy, and indeed many other of the forms of fallacy already examined, involve *illicit generalization:* which is thus a rather extensive *locus* of fallacies.

It may be also regarded as involved in uncritical and indiscriminating appeals to men of supposed great authority in matters of human science: the fallacy lying in the assumption that because such men have won great fame by their writings or researches in certain departments of investigation they must therefore be authorities in *every* domain. Such procedure is certainly a violation of scientific method; but it might be classified as a sort of *undue assumption of axioms*, or of positions not sufficiently proved, as well as under illicit generalization. "A striking example of this fallacy," writes Professor Welton, "is found in the intellectual idolatry with which the Schoolmen regarded Aristotle." But this is less than historical justice to the *Schoolmen:* it is really true only of the mediaeval *Averroistic commentators* of Aristotle.[2] Nor need we go to the Middle Ages for examples of such idolatry: the cult of Kant, of Hegel, of Darwin, in modern times, would furnish fairly apt illustrations of undue deference to the authority of an individual.

It has been already observed that the whole inductive process is, in the main, a process of generalization: so that all fallacies incident to induction are likely to involve illicit generalization in some shape or form. All the difficulties of the process of in-

[1] WELTON, *op. cit.*, p. 273.
[2] DE WULF, *History of Medieval Philosophy*, pp. 228, *sqq.*, 379, *sqq.*

ductive analysis and generalization have been pointed out already in the course of the various chapters devoted to an examination of the process. It will therefore be sufficient here to recall very briefly a few of the principal ones.

In *establishing* a general causal law there is always the possibility of not having included all the really operative influences, or of having included some that are not really operative; and in *applying* the law there is the danger that we may fail to detect the presence of counteracting conditions. Intermixture of causes in the concrete world of physical and social phenomena makes the application of laws to actual facts a matter of extreme delicacy. In the process of applying laws to new facts, for the purpose of explaining these latter, we are constantly being brought face to face with all sorts of anomalies and exceptions. These always demand further analysis: which will determine whether they are apparent or real exceptions, whether they are due to the interfering influence of other known causes, or prove to us that the statement of our law is too wide, or too narrow, and so needs further adjustment and rectification. Thus, the work of applying laws invariably contributes to a more accurate and definite conception of the latter. "For example, to state that the boiling-point of a liquid depends on the temperature would be to omit the equally essential condition of atmospheric pressure. Thus, to say that water boils at 100° C. is wrong; it boils at that temperature under the pressure of one atmosphere; *i.e.*, the normal atmospheric pressure at the sea-level. Up a mountain the boiling-point is different.[1]

Needless to say, the exact formulation of scientific laws, and of the conditions under which they are applicable in the concrete, is a matter of much greater complexity, as also of much greater moment, in the human sciences, than in the physical sciences. There, the facts are not amenable to quantitative measurement. Operative influences which take the form of human motives of action are more elusive than mechanical, physical, chemical, or physiological energies. Facts of mind and facts of matter do not belong to the same order; nor can the evidence, on which the scientific knowledge of such facts is based, be expected to conform to the same order of critical canons and requirements in both cases. And this obvious consideration is

[1] WELTON, *op. cit.*, p. 275.

sometimes neglected in attempts to set up the same sort of ideal for all departments of science (203).

Finally, it is not always easy to determine, whether in the physical or in the human sciences, what are the determining or causal factors, and what the determined elements or effects, in the complex phenomena actually before us; and the difficulty is increased by the undoubtedly common fact of *interaction*, the fact that the elements are often, by their mutual interaction in the reality, partly causes and partly effects (243). As an illustration of the difficulty of discriminating between cause and effect we are informed "that meteorologists are not agreed whether the copious and sudden downfalls of rain which usually attend thunder-storms are the cause or the effect of the electric discharge. The common opinion is that they are the effect, but Sir John Herschel held that they were the cause."[1] And, as illustrating the interaction between cause and effect, a thoughtful and suggestive passage from Sir G. C. Lewis's *Methods of Observation and Reasoning in Politics*[2] will be amply sufficient: "Thus," he writes, "habits of industry may produce wealth, whilst the acquisition of wealth may promote industry; again, habits of study may sharpen the understanding, and the increased acuteness of the understanding may afterwards increase the appetite for study. . . . The general intelligence and good sense of the people may promote its good government, and the goodness of the government may, in its turn, increase the intelligence of the people, and contribute to the formation of sound opinions among them. Drunkenness is in general the consequence of a low degree of intelligence, as may be observed both among savages and in civilized countries. But, in return, a habit of drunkenness prevents the cultivation of intellect, and strengthens the cause out of which it grows. As Plato remarks, education improves nature, and nature facilitates education. National character, again, is both effect and cause: it reacts on the circumstances from which it arises. The national peculiarities of a people, its race, physical structure, climate, territory, etc., form originally a certain character, which tends to create certain institutions, political and domestic, in harmony with that character. These institutions strengthen, perpetuate, and reproduce the character out of which they grew, and so on in succession, each

[1] WELTON, *op. cit.*, pp. 275-6. [2] vol. i., p. 375; *apud* WELTON, *op. cit.*, p. 276.

new effect becoming in turn a new cause. Thus, a brave, energetic, restless nation, exposed to attack from neighbours, organizes military institutions: these institutions promote and maintain a warlike spirit: this warlike spirit again assists the development of the military organization, and it is further promoted by territorial conquests and success in war, which may be its result—each successive effect thus adding to the cause out of which it sprung."

WELTON, *Logic*, II., bk. vii. JOSEPH, *Logic*, chap. xxvii. JOYCE, *Logic*, pt. i., chap. xvii. MILL, *Logic*, bk. v. MELLONE, *Introd. Text-book of Logic*, chap. x. MERCIER, *Logique*, pp. 245-68. JEVONS, *Elementary Lessons in Logic*, xx., xxi.; *Palaestra Logica*, chap. x. BOWNE, *Theory of Thought and Knowledge*, pt. ii., chap. xi. KEYNES, *Formal Logic*, pp. 457 *sqq*.

QUESTIONS.

PART IV.

CHAP. I.—How do we come to assent to the premisses used in deductive reasoning? Distinguish three classes of general truths in reference to reasoning. How far should logic deal with the methods of investigation to be observed in the special sciences? Illustrate historically the influence of the sciences on logic. Indicate some other departments of human research which claim from logic equal recognition with physics. Define *Logical Method, Analysis, Synthesis.* On what basis are sciences classified as deductive or inductive? "There is one and *only one* scientific method": State the general rules of method. Explain the use of analysis and synthesis in teaching. Describe the "Scholastic Methods of Exposition and Debate". What are the advantages and the defects of a purely Scholastic training?

CHAP. II.—How do we ascend from particular facts to metaphysically necessary principles? Give the widest meaning of the word *Induction*, and its Greek equivalent. What is a *propositio per se nota—in se—quoad nos—quoad aliquos—quoad omnes?* Distinguish between the "induction" of "necessary" truths in mathematics, and "physical" induction. Distinguish between *enumerative* and *scientific* induction; between "complete" and "incomplete," "formal" and "material". Give examples of the so-called "inductive syllogism" which concludes by complete enumeration. State and explain Aristotle's definition of it. What are its drawbacks? When complete, is it scientific? What relation does it bear to the ordinary (deductive) syllogism? Will incomplete enumeration, as such, demonstrate the general law ("*M is P*")? Why? What efficacy did Aristotle attribute to it? What useful purpose does it serve? Show that Aristotle and the Scholastic philosophers of the Middle Ages were acquainted with *scientific* induction. Account for the widespread error on this point. What name did the Scholastics, after Aristotle, give to scientific induction? On what principle did Scotus base generalization from particulars? Why did the Scholastics not make any progress in physical induction? Explain the teachings of (1) Roger Bacon; (2) Francis Bacon; (3) Newton; (4) Whewell; (5) J. S. Mill; (6) Jevons—on induction. Explain, and illustrate by an example, the various steps in the inductive process. Compare deduction with induction (1) as methods; (2) as inferential processes. Would you describe any form or forms of reasoning as specially characteristic of induction? How do you understand the description of induction as an "inverse process"?

CHAP. III.—Should logic *formulate* the rational principles which form the *grounds* of our ascent from facts to laws? Should it *justify* those principles? Mention some of the more important notions involved in physical induction. Formulate the "Principle of Sufficient Reason". Has it a *real*

as well as a *logical* application? Is the ultimate *reason* of logical principles itself *real?* Describe (*a*) the Phenomenist, (*b*) the Hegelian, views of *Reality*. Is the *Causa Essendi* always also the *Causa Cognoscendi?* What is the Scholastic view about Reality? State the "Principle of Causality". Define "Cause". Distinguish it from "Condition". Explain the Aristotelean fourfold division of "Cause". What class of cause is mainly sought in physical induction? Show, by an example, how each of the four causes is sought by induction. Do inanimate causes act "for ends"? Is there evidence of plan, purpose, design, in the action of physical causes? Define "Essence" and "Nature". Explain the use, and various meanings, of the word "Law," in induction. Contrast the "teleological" with the "mechanical" conception of the universe. Can the latter be explained without recourse to "final," "formal," and "material" causes? Where are the two latter properly sought? Is the view that an event is produced by "efficient" causes incompatible with the view that it is due to "final" causes? Give the traditional definition of "Efficient Cause". Distinguish various kinds of efficient cause. Are all efficient causes "necessary" causes? What notion have modern writers substituted for that of "efficiency"? Are all causes, of which we can have knowledge, necessarily perceptible by our senses? Give an account of the empirical view of efficient causality. What did Mill mean by "*invariable* antecedent"? By "unconditional"? By "necessary"? Is "necessity" "invariability"? Is it "unconditionalness"? Can we, according to Mill, really know an antecedent to be "unconditionally invariable"? Give Mill's three statements of the meaning of "Cause". What did he mean by "unconditional" antecedent? *Must* the "cause" have disappeared, or can it have disappeared, before the "effect" appears? Explain "*Cessante causa cessat effectus*". How are cause and effect connected? Explain "*Actio et Passio sunt idem numero Motus*". Is this "*motus*" identical with the *agens*, or the *patiens?* or the two latter with each other? May all physical efficient causality be resolved into local motion, or change of spatial relations? Can one event have many "partial" causes combining to form one "total cause"? Can the same kind of event have different total causes? Why does the popular mind so conceive a "necessary cause" that, although the latter can have only one effect, yet this one effect may have several different "necessary causes"? Which of the two—cause or effect—is conceived by the popular mind in the more abstract state? Does the scientist make the concept of "cause" more abstract, or less abstract, than it is in the popular mind? Is plurality of causes consistent with the scientific concept of total cause? What do you understand by the *immediate* (total) cause? by the *determining* cause? In order to "explain" an effect by its causes, how far back along the converging chains of efficient causality must the scientist go? How near to the effect must he come?

CHAP. IV.—State the "Principle of the Uniformity of Nature". To what class of causes does it mainly refer? Distinguish between two different senses in which it may be interpreted. Understood hypothetically, is it *analytic*, *a priori*, metaphysically necessary, self-evident? Do propositions which express ordinary physical laws imply the existence or occurrence of the facts and phenomena to which they refer? State the principle of uniformity categorically. Is our belief in this uniformity physically, or meta-

physically, certain? Is this belief involved in induction? Is it a synthetic, or an analytic, judgment? What right have we to infer *general* uniformity from *observed partial* uniformity? Does the validity of induction *presuppose* belief in the existence of God? Does it rest ultimately on this belief? Is the (categorial) principle of uniformity reached by induction. Must it be reached *antecedently* to the establishing of any narrower law of nature? How does Mill account for our belief in the uniformity of nature? Are the inductions by which we establish special laws of nature, enumerative? Is "belief in uniformity" a "presupposition" of "rigorous induction"? Account for Mill's failure to derive scientific certitude from induction. What kind of "unity" is discernible in the physical universe? In order to reach certitude by means of induction, must we *postulate* that the universe is intelligible? that *physical* phenomena are connected by *metaphysical* relations? What sort of certitude do we reach by induction, and why? Is the principle of uniformity the "major premiss" of the inductive process? Is the latter an inference at all? Compare the function of the principle of uniformity in induction, with the function of the *Dictum de omni et nullo* in deduction. Is belief in uniformity involved in the use of deduction? Why are the grounds of this belief not discussed in the logic of deduction?

CHAP. V.—What is the function of hypothesis? Is every supposition in science an hypothesis? Must an hypothesis be true in order to be useful? Describe the various kinds of hypothesis distinguished by logicians, and compare them with one another. In an hypothesis of cause, must the supposed cause be itself a phenomenon? Must it be at least picturable by the imagination? How far, or in what sense, must it be capable of detection? What are so-called "occult" causes? Are any hypotheses admissible in philosophy though not admissible in science, or *vice versa?* Explain the rôle of analogy in suggesting and verifying hypotheses. Are all legitimate hypotheses capable of *rigorous* verification? In what does this latter consist? Can hypotheses be verified by cumulative evidence? What do you understand by "Consilience of Inductions," and "Extension of Hypotheses"? Discuss the significance of *simplicity* in an hypothesis. Why do philosophers differ as to what the ultimate systematic conception really is, which would best explain the totality of human experience? Indicate briefly the conditions for a legitimate scientific hypothesis. Besides analogy, indicate some other sources of hypotheses. Explain and illustrate the argument from analogy. How is the force of such an argument to be tested? How are such arguments formally expressed? Expound and illustrate the teaching of Aristotle on Example and Analogy.

CHAP. VI.—Show that all observation involves selection, judgment, and inference. Compare experiment with observation. What is the aim of perceptual analysis in induction? Give an outline of the process of experimental analysis of facts, indicating the causes of its complexity and difficulty. Explain the significance of "exceptions," and the reason for repetition of instances. What is to guide us in marking off the field for observation and experiment? What two principles of elimination underlie all applications of the analytic process? Enumerate the various ways in which the process can be conducted. Does logic enable us to determine which of these "methods" we are to apply in a given inductive inquiry? Formulate and illustrate each

of the methods. Compare "agreement" with "difference". Explain and illustrate two ways in which these may be combined. Is the joint method of difference and agreement the strongest of all the methods? Distinguish between qualitative and quantitative methods. Compare the method of concomitant variations with the methods of agreement and difference. To what classes of phenomena is the former specially applicable? What are its limitations? How is it subserved by statistics? Compare the method of residues with the methods of agreement and difference. What are its characteristics? What are the various ways in which causes or effects may be conjoined? What is the common aim of all the "methods"? Can they be adequately expressed by any system of symbols? Explain the nature of measurement and its influence on the progress of science. "All measurement is relative and only approximate"? What are the sources of inaccuracy? By what means is this latter minimized? Distinguish between verification and explanation, between fact and law, between empirical laws, derivative laws, and laws of nature.

PART V.

CHAP. I.—Enumerate and define the various elementary notions subsidiary to the treatment of *science* and *certitude*. Describe and compare the three kinds of certitude. Compare metaphysical with physical laws; necessary with contingent truths; categorical with hypothetical necessity; knowledge of possible essences with knowledge of actual facts. What is our justification for applying abstract metaphysical principles to the interpretation of the world of concrete fact? From what standpoint did Aristotle obtain his conception of the ideally perfect form of human knowledge? From what standpoint is the modern conception formed? Are these two views in necessary conflict? Explain, according to Aristotle, the nature and requirements of scientific demonstration. Explain "The middle term expresses the cause". Discuss the distinction between what is φύσει or λόγῳ πρότερον, and what is πρὸς ἡμᾶς πρότερον. How, according to Aristotle, do we reach the former? Do deductive applications of inductively established laws yield "science" in the Aristotelean sense of this term? Compare demonstration with scientific explanation. How do we "explain" a fact, a *law* or uniform series of facts? Distinguish between "popular" and "scientific" explanation. Have actual facts or events the same kind of necessary connexion with other facts or events as a conclusion in geometry has with its premisses? What is the difference? Are the latter dependent on the Divine Will, and the former on the Divine Intellect only? What is the only ultimate explanation of all actual facts? Contrast the Hegelian with the Theistic view of explanation. Should logic teach us how to "discover" truths, as well as how to "prove" truths already discovered? Compare discovery and proof by induction, with discovery and proof by deduction. Does the middle term of a demonstrative syllogism give the "only possible" cause of the conclusion? Compare the ultimate principles of deductive demonstration with the ultimate explanatory laws reached by induction. Assign three meanings to the expression, "moral certitude". Can certitude be based on "moral," "human," "rational," propensities? Do any of these underlie belief that is based on human authority? On what conditions does the latter warrant certitude?

Compare "belief" with "science". Can human testimony be an ultimate motive for certain assent? State some rules of historical criticism for ascertaining (*a*) the knowledge, (*b*) the veracity, of human testimony. Classify the chief sources of historical information. How are the authenticity and integrity of documents ascertained? Discuss the value, and the limitations, of oral traditions; of the *argumentum ex silentio;* of the argument from prescription.

CHAP. II.—Define opinion and probability. Are there grades or degrees of truth, or of probability, in our judgments? Are there degrees of firmness in our opinions or assents? Can opinion ever pass into certitude by the aid of cumulative evidence? Distinguish between various kinds of probable judgments. What is a "Probable Syllogism"? What is *chain* evidence? How does it differ from *circumstantial* evidence? Are probable arguments from authority of much practical utility? Define the Aristotelean Enthymeme. How does it differ from the Modern Enthymeme? What did Aristotle understand by σημεῖον, τεκμήριον, and εἰκός, respectively? Give examples of Aristotelean Enthymemes in the first, second, and third figures, respectively. Distinguish between "causal" and "casual". What is a "chance coincidence," or a "chance recurrence"? Are there "coincidences" or "coexistences" that are ultimately unexplainable? When is a phenomenon said to be "due to chance"? Would the possession of perfect knowledge eliminate the concept of chance? What is the object of this concept? Indicate two essential conditions for the application of the theory of probability to any group of phenomena. Explain the formulæ: "If S is A it is X," and "S is either A_1X_1, A_2X_2, A_3X_3, . . ." Does the theory apply equally to past and to future events? State and illustrate the rules for estimating the probability of (*a*) simple events; (*b*) compound independent events; (*c*) compound dependent events; (*d*) the total probability of an event that may happen in various ways. What is "inverse probability"? How is it applied in measuring magnitudes? in eliminating chance? Explain *Bernouilli's Theorem.* How may the calculus be applied to natural and social phenomena? What are the obstacles to its application? Can it measure our probability as to the recurrence of a natural event? or the credibility of human testimony? How do statistics subserve inductive inquiry? Discuss the connexion between "statistical uniformities" and "laws," between free will and the regularity of social phenomena.

CHAP. III.—Discuss the possibility of classifying the modes, or the sources, of human error. Distinguish between *error* and *fallacy.* Define fallacy, sophism, paralogism. Describe the classifications of Mill, Bacon, Aristotle, and Jevons, respectively. Discuss the division of fallacies into formal or logical, semi-logical, and material. Are fallacies *in dictione* formal or material? Explain and exemplify each of the fallacies incident to conception; to judgment. Compare the fallacy of *Secundum Quid* with that of *Accidens.* Illustrate the various forms of (*a*) *Ignoratio Elenchi;* (*b*) *Petitio Principii.* Distinguish between axioms and postulates. Why do men differ so much in their views upon the great questions of religion and philosophy? What are the *subjective,* and what the *objective,* obstacles to the attainment of truth? Explain the fallacy incident to indirect proof. What fallacies are to be avoided in (*a*) observation, (*b*) analogy, (*c*) generalization?

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